TEXT BOOK OF PHYSICS

DUNCAN AND STARLING

HEAT LIGHT AND SOUND

Kenneth
Rossier
Phi Kappa Sigma
Evanston Ill

1 ft lb = 1.35 x 10^7 ergs

1,980,000 ft lbs = 1 HP for 1 hr

4.18 x 10^7 ergs wasted for every calorie wasted

1 HP = 33,000 ft lb per sec

Energy = $h\nu$

h = Planck's constant

ν = frequency of radiated energy

A TEXT BOOK OF PHYSICS

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A TEXT BOOK OF PHYSICS

FOR THE USE OF STUDENTS OF SCIENCE & ENGINEERING

BY

J. DUNCAN, Wh. Ex., M.I.Mech.E.

AND

S. G. STARLING, B.Sc., A.R.C.Sc.

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A TEXT BOOK OF PHYSICS

FOR THE USE OF STUDENTS OF SCIENCE AND ENGINEERING

BY

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PARTS II, III, AND IV
HEAT, LIGHT AND SOUND

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PREFACE

THE preparation of this volume was undertaken to meet a demand that has been growing for some years past for a text-book of Physical Science which should connect more intimately than has hitherto been usual the scientific aspects of Physics with its modern practical applications. The reader must be left to judge how far the authors have succeeded in thus combining the outlooks of the man of science and the engineer.

The contents have been selected to meet the requirements of various classes of students: those preparing for Intermediate and other examinations of London and other Universities; and those entering for appointments in the Army, Navy, and Civil Service, or offering themselves for examination in Electrical Engineering (Grade I.) by the City and Guilds of London Institute.

The book has been arranged in parts, in accordance with the divisions of the subject found convenient in most schools and colleges. Part I., Dynamics, comprises the sections of Mechanics and Applied Mathematics usually studied, and includes sections on motion, statics, and the properties of fluids. Part II., Heat; Part III., Light; Part IV., Sound; and Part V., Magnetism and Electricity; deal respectively with the principles of these subjects and their applications.

Complete courses of laboratory work have been provided in each Part. Many physical laboratories are equipped with apparatus differing in some respects from the instruments here described, nevertheless the guidance given will enable intelligent use to be made of other forms of apparatus designed for the same or similar purposes.

Attention is directed to the experimental treatment of dynamical principles, because its neglect, which is unfortunately common,

makes it difficult for a student to secure a thorough and systematic knowledge of physical science. The complete course of experimental work has been devised to meet both the requirements of the physicist and of the engineer; in cases where the methods of treatment adopted by these differ radically, the teacher or student may choose the experiment which best suits his special needs.

In Part V., the treatment of the Dynamo, Telegraph, and so on, is that which follows naturally and logically from the earlier theoretical principles explained; technical considerations of design and construction have been omitted as unsuitable in a text-book of Physics.

A large number of worked-out examples have been included to assist the student to understand the text and to solve the exercises at the ends of the chapters. Many of these exercises have been taken, with the permission of the authorities to whom grateful acknowledgments are made, from examination papers, the source being given in each case. Questions marked L.U. are from examination papers of the London University and those with C.G. from papers of the City and Guilds of London Institute.

Answers have been supplied in the case of numerical exercises, but it is too much to hope that these are entirely free from errors. The authors will welcome any corrections which readers may send to them.

The authors are glad of this opportunity to express their indebtedness to Prof. Sir Richard Gregory and Mr. A. T. Simmons for constant assistance and invaluable hints while the book was in preparation and passing through the Press.

J. DUNCAN.
S. G. STARLING.

1918.

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PART II

HEAT

CHAPTER XXIII

TEMPERATURE

Temperature.—On touching in succession two pieces of iron, one piece having been exposed for some time in the sun's rays and the other piece shaded from them, it will be noticed that the first piece is hotter than the second. Our sense of hotness is quite different from the sense of touch, which enables us to distinguish roughness and smoothness, hardness, etc. Of two bodies, the hotter is said to be at a higher **temperature.** Take two bodies, A and B, and place them in contact; **if A is at a higher temperature, heat will flow from A to B.**

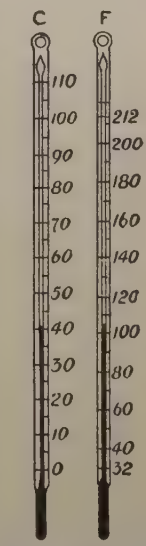

FIG. 333.—Centigrade and Fahrenheit mercury thermometers.

The sense of hotness is not always to be relied upon in determining which of two bodies is the hotter. If pieces of wood and iron, both at the temperature of the room, be touched in succession, the iron appears to be colder than the wood. Hence the necessity for employing an instrument in the determination of temperatures; such instruments are called **thermometers.**

Mercurial thermometers.—In the commonest type of thermometer, reliance is placed upon the expansion in volume which takes place when mercury is raised in temperature. In Fig. 333 two mercurial thermometers are shown. These are made by blowing a bulb at the lower end of a fine-bore glass tube; clean dry mercury is introduced and heated to drive out any air. The top end of the tube is then sealed so that the contents consist of mercury and vapour of mercury. The quantity of mercury is adjusted so that the mercury stands some distance up the tube at ordinary temperatures. If the

bulb, which contains the greater part of the mercury, is brought into contact with a hot body, the mercury is warmed and expands, as is made evident by its level rising in the tube.

The glass walls of the bulb and tube also expand, but to a much smaller extent than the mercury. Elevation of the level of the mercury is due to the difference in expansion of mercury and glass when both are heated through the same range of temperature. Mercury has high expansive properties, and arrives quickly at the temperature of any body with which it is brought into contact; hence it is a very suitable material for the purpose.

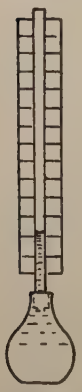

FIG. 334.—Expansion of water.

In manufacturing mercurial thermometers, it is usual to blow a small bulb at the top of the stem; this minimises the danger of the mercury expanding to an extent which would fill the whole tube, when a pressure would be exerted which would probably burst the thin-walled lower bulb.

EXPT. 57.—**Expansion of water when heated.** In Fig. 334 is shown a small glass flask fitted with a rubber stopper and a glass tube; a paper scale is attached to the tube. The flask is filled with water, preferably coloured, and the stopper is inserted and pushed in so that the water level rises a little in the tube. Place the flask in a vessel containing some hot water. Note that the water level falls slightly at first owing to the expansion of the glass (which becomes hotter first), and then rises steadily as the water in the flask rises in temperature and expands. No further rise in level takes place when the temperature of the water in the flask becomes equal to that of the water in the vessel. The action is similar to that in a mercury thermometer, the only difference being in the time taken.

EXPT. 58.—**Unequal expansion of water and alcohol.** Arrange another apparatus similar to that used in Expt. 57. The flasks should be equal in size, and the bores of the tubes should be equal. Charge the first flask as before with water and the other with alcohol, and push the stoppers in until the water and alcohol stand at the same height when at the temperature of the room. Place both flasks in the same vessel of hot water. After some time, the levels of the liquids will cease rising and it will be found that the final levels differ, showing that water and alcohol do not expand to the same extent when heated through the same range of temperature.

Fixed points and graduation of thermometers.—If the bulb and part of the stem containing mercury be immersed in a mixture of

clean ice and water, it will be found that the level of the mercury remains steady while the ice continues to melt. A mark is made on the stem at this level and is called the **freezing point.**

If the bulb and part of the stem containing mercury be immersed in steam coming from water boiling under standard barometric pressure of 760 mm., it will be found that the mercury level again remains steady. Another mark is made on the stem at this level and is called the **boiling point.**

The freezing and boiling points are called the **fixed points** of the thermometer; other temperatures are measured by reference to these points.

In the **Centigrade thermometer** (Fig. 333) the freezing point is marked 0 and the boiling point 100. The Centigrade scale of temperature is constructed by dividing the stem between the fixed points into 100 equal parts or **degrees** (written 100°). In the **Fahrenheit thermometer** the fixed points are marked 32 and 212 respectively, and the stem between these points is divided into 180 degrees. These scales may be extended above the boiling point and below the freezing point. Temperatures lower than zero on either scale are denoted by a negative sign. Thus $-15°$ C. means 15 Centigrade degrees below freezing point, $-15°$ F. means 15 Fahrenheit degrees below 0° F., or $(15+32)=47°$ F. below freezing point.

The **Réaumur scale** of temperature is not much used; in this scale the freezing and boiling points are marked 0° and 80° respectively. Other boiling points now used in thermometry as fixed points are: naphthalene, 218° C.; sulphur, 444·6° C.; zinc, 928° C.

Conversion of temperatures.—To avoid risk of error in converting from one scale of temperature to another, the method used in the following example should be employed:

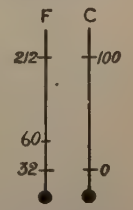

FIG. 335.—Conversion of thermometric scales.

EXAMPLE.—Find the temperature Centigrade corresponding to 60° F.

Sketch two thermometers side by side (Fig. 335); mark these F and C respectively and place the fixed points on each, putting corresponding marks opposite each other. Mark the given temperature of 60° F. on the F thermometer. Inspection shows that this temperature is $(60-32)=28$ Fahrenheit degrees above freezing point. Since 180 Fahrenheit degrees are equivalent to 100 Centigrade degrees, the number of Centigrade degrees equivalent to 28 Fahrenheit degrees is given by

$$\text{Centigrade degrees} = 28 \times \tfrac{100}{180} = 15{\cdot}5.$$

The given temperature of 60° F. therefore corresponds to 15·5° C.

EXPT. 59.—**Freezing point error of a thermometer.** Arrange a funnel and beaker on a retort stand (Fig. 336). Remove some shavings from a block of ice and put them into the funnel. Insert the thermometer to be tested, and pack the ice closely round the bulb and stem up to the level of the freezing point graduation. Bring the eye to the level of the top of the mercury column, and take readings at intervals. Note the final steady reading; this may be taken as the true freezing point. The freezing point error of the thermometer is the difference between the final steady reading and 0° or 32°, according as the instrument is graduated in Centigrade or

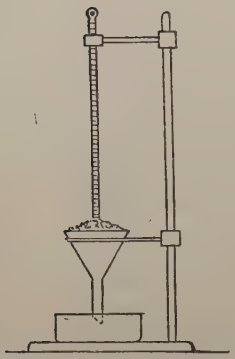

FIG. 336.—Apparatus for determining the freezing point of a thermometer.

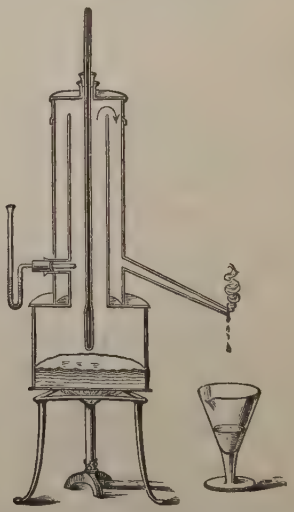

FIG. 337.—Apparatus for determining the boiling point of a thermometer.

Fahrenheit degrees. The correction to be applied is equal to this difference, + or − according as the observed temperature is lower or higher than the graduation mark at the fixed point.

In carrying out this experiment, it should be noted that the temperature, as shown by the thermometer, remains steady during the whole time that the ice is melting.

EXPT. 60.—**Boiling point error of a thermometer.** In Fig. 337 is shown a small copper boiler having a double copper tube attached to the cover. The thermometer used in Expt. 59 is placed in the inner tube after having been pushed through a cork which fits a hole in the top cover. The water is brought to boiling, and the steam passes up the inner tube, thus surrounding the thermometer, then down the outer tube, and is discharged

at the bottom. The object is to steam-jacket the inner tube containing the thermometer, and to ensure that this tube shall be at the same temperature as the steam. A small glass U gauge contains water, and is connected to the outer tube; when the water stands at the same level in both limbs of the gauge, evidence is provided that the pressure of the steam is equal to that of the atmosphere. The apparatus is called a **hypsometer.**

The thermometer should be arranged so that the boiling point graduation is just above the level of the cork. If the thermometer has a long stem, this condition cannot always be complied with, since the bulb of the thermometer must be situated well above the surface of the boiling water in order to prevent drops of water being thrown on to it.

After the water has been giving off steam freely during a few minutes, take readings of the thermometer. Read also the barometer. The temperature of steam coming from boiling water depends on the pressure, which in this case is equal to that of the atmosphere as shown by the barometer. By consulting the table on p. 533, the temperature of steam at this pressure will be found, and the boiling point error of the thermometer is equal to the difference between the observed boiling temperature and that shown in the table. The correction to be applied to the thermometer is equal to this difference, + or − according as the observed temperature is lower or higher than the correct temperature.

If the same thermometer be immediately retested for the freezing point error, it will be found probably that this error has altered somewhat. This is owing to the glass bulb and stem, which have expanded considerably during the boiling-point test, failing to return to the original volume. The initial volume will be recovered if sufficient time be given, some months probably. Hard glass is less liable to this effect than soft glass.

The hypsometer is sometimes used instead of the barometer for ascertaining the pressure of the atmosphere. The apparatus is set up at the required place (e.g. up a mountain), the temperature of the steam is observed, and the table (p. 533) gives the pressure of the atmosphere.

EXPT. 61.—**Graduation errors of a thermometer.** Assuming that the thermometer stem has been graduated by marking the freezing and boiling points, and then dividing the distance between these marks into the required number of equal parts, and also that the expansion of mercury is proportional to the rise in temperature throughout the range of the thermometer, it follows that the effect of any variation in the bore of the stem will cause incorrect indications of temperature to occur at different parts of the scale.

If a standard thermometer is available, for which the graduation errors are known, the thermometer under test may be examined for graduation errors by comparison. Suspend both thermometers with their bulbs immersed in a beaker containing water. Gradually raise the temperature of the water, and take simultaneous readings of the thermometers at

intervals of, say, 10°, being careful to stir the water well before taking the readings. Note these readings thus:

Standard thermometer.		Thermometer under test.	
Observed temp.	True temp.	Observed temp.	Correction.

Columns 1 and 3 are filled in from the observations; column 2 from the known errors of the standard thermometer; column 4, obtained by taking the differences of columns 2 and 3, shows the corrections to be applied to the thermometer under test at various parts of the scale. Draw a correction curve by plotting columns 3 and 4.

EXPT. 62.—**Variations in the bore of a thermometer stem.** Detach a thread of mercury having a length of about 10 scale divisions of the stem. This may be done by directing a small sharp-pointed flame at the place where the break in the mercury thread is required. By inverting the thermometer the detached thread may be brought to any part of the stem. If the bore is uniform, then the detached thread will have equal lengths at all parts of the stem.

Assuming that a Centigrade thermometer is being tested, shake the thermometer until the detached thread occupies approximately the space lying between 0° and 10°; read the length of the thread in stem-scale divisions. Repeat the operation in the spaces lying between 10° and 20°, 20° and 30°, etc., throughout the range of the scale. Let the thread lengths be a_1, a_2, a_3, etc. The mean thread length A is given by

$$\mathrm{A} = \frac{a_1 + a_2 + a_3 + \text{etc.}}{\text{number of readings}}.$$

Assuming meanwhile that both freezing and boiling point corrections are zero, the corrections C at other parts of the scale may be obtained in the manner indicated in the following table:

Range.	Values of a.	Values of C.	
		Temp.	Corrections.
—	—	0°	$C_0 = 0$
0° – 10°	a_1	10°	$C_{10} = A - a_1$
10° – 20°	a_2	20°	$C_{20} = A - a_2 + C_{10}$
20° – 30°	a_3	30°	$C_{30} = A - a_3 + C_{20}$
30° – 40°	a_4	40°	$C_{40} = A - a_4 + C_{30}$
...	...	...	...
90° – 100°	a_{10}	100°	$C_{100} = 0$

Plot temperatures and values of C. To obtain a complete correction curve, plot on the same diagram the known freezing and boiling point corrections, first reversing their signs, and join these points by a straight line. The intercepts between this line and the graph give the true corrections for any part of the scale.

Proportions of a thermometer.—The volume of the bulb required for a thermometer depends upon the length of stem considered to be suitable for the range of temperature intended to be measured, upon the bore of the stem, and upon the kind of fluid to be employed in the thermometer. The volume of fluid which would fill the stem between the marks placed at the extreme temperatures to be measured, represents the increase in volume of the fluid in the bulb when heated through this range in temperature. The change in volume of one cubic centimetre of the fluid when heated through this range being known, a simple calculation gives the volume of bulb necessary to provide the required expansion.

Types of thermometers.—**Alcohol** is sometimes used in thermometers instead of mercury. Mercury solidifies at $-39°$ C. and alcohol at $-130°$ C.; hence an alcohol thermometer may be used for much lower temperatures than is possible with a mercurial thermometer. Alcohol expands more than mercury for a given rise in temperature; hence the alcohol thermometer is more sensitive than the mercurial thermometer. Alcohol wets glass, and the thread of alcohol has therefore no tendency to stick in the stem. Mercury does not wet glass and is apt to move with jerky action. Alcohol boils at 78° C. approximately, and is not suitable for temperatures above 50° or 60° C. For the same reason, the upper fixed point cannot be found in the manner described on p. 315, and the stem of an alcohol thermometer must be graduated by comparison with a standard thermometer.

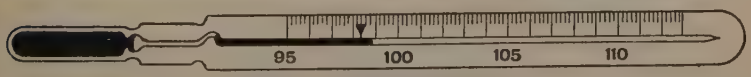

FIG. 338.—A clinical thermometer.

Clinical thermometers are specially adapted for measuring the temperature of the human body. The body of a person in health varies only slightly from 98·4° F., and the stem of a clinical thermometer is graduated from about 95° to 110° F. The bulb of the instrument is placed in the mouth or under the armpit of the patient for a minute or two, and then withdrawn in order that the temperature may be read. There is a constriction in the stem near the bulb (Fig. 338) which prevents the mercury returning to the bulb; hence

the reading may be taken at leisure. The broken thread of mercury is then reunited by jerking the instrument so as to drive the mercury downwards.

Maximum and minimum thermometers register the highest and lowest temperatures occurring during the interval elapsing between successive settings of the instrument. In Six's thermometer (Fig. 339) a long bulb A is filled with alcohol and is connected to another bulb D by a bent tube; the bulb D also contains alcohol, which, however, does not fill the bulb completely, a space being left for expansion. The tube between B and C contains mercury, which separates the alcohol in the tube between A and B from the alcohol in the tube between D and C. The positions of the ends of the mercury thread at B and C are indicated by small steel indexes having light springs to prevent slipping due to their weights, but not strong enough to prevent the indexes being pushed along the tubes by the mercury thread. Change of volume of the alcohol in the bulb A will cause movements of the mercury thread. The minimum temperature occurring will be indicated by the position of the index controlled by B; the maximum temperature is shown by the index at C. A small magnet applied to the outside of the tube enables the indexes to be reset in contact with the mercury.

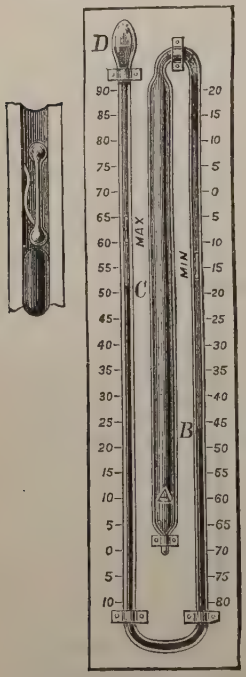

FIG. 339.—Maximum and minimum thermometer.

Sensitive thermometers are used for measuring small differences in temperature in cases where the change in temperature is important, and a knowledge of the actual temperature is not required. An example is shown in Fig. 340. The short stem is graduated to show a few degrees only, and each degree is subdivided into tenths. The top of the tube is bent over so as to form a chamber into which some of the mercury may be shaken; sufficient

FIG. 340.—Sensitive thermometer.

mercury is left in the bulb to enable the lower temperature to be read, the level being then near the foot of the scale. The higher temperature is then read, and the difference in the readings gives the required difference in temperature. The advantage of this plan consists in greater sensitiveness being obtained without the necessity of employing a very long stem, which would be broken easily.

Precautions to be observed in using thermometers.—Do not attempt to force the thin-walled bulbs through corks. A thermometer may be injured if subjected suddenly to great changes in temperature. No thermometer should be employed where there is risk of its being exposed to temperatures higher than that to which it is graduated, otherwise the bulb may be burst by the pressure of the expanding mercury. The bulb should not be subjected to fluid pressure much in excess of that of the atmosphere; there is risk of collapse. Even if collapse does not occur, the diminution of volume of the bulb caused by the external pressure leads to false readings of temperature. In Fig. 341 is shown a means of obtaining the temperature of steam in a pipe or other closed vessel. A metal cup closed at the inner end is screwed into the pipe and is charged with oil or mercury which comes quickly to the temperature of the steam. A thermometer inserted in the cup will indicate the required temperature.

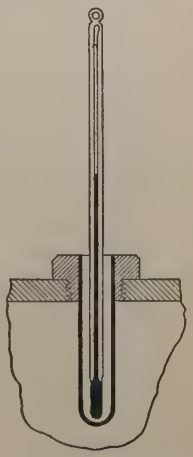

FIG. 341.—Temperature in a steam pipe.

Differences in temperature at two parts of a metal pipe may be obtained by securing two thermometers with their stems lying along the pipe; flannel is then wrapped round the pipe over the bulbs. Both thermometers will then be under like conditions, and the difference in their readings may be taken as the difference in temperature of the contents of the pipe at the two places.

Measurement of high temperatures.—Under ordinary atmospheric pressure, mercury boils at 357° C.; hence ordinary mercurial thermometers can be used only for temperatures somewhat lower than this. High temperatures may be stated sometimes with sufficient accuracy by reference to the known melting temperatures of certain substances. Thus we may say that the temperature of a body is about that of melting lead (320° C.) if the temperature be such that a small piece of lead in contact with the body just melts. Naphthalene, sulphur and tin may be used in this way. The method has

been employed for rough determinations of the temperature of furnaces. Substances used in this way are called **thermoscopes.**

The temperature of a flue or furnace may be estimated by inserting a piece of platinum, copper, or other substance, allowing it to remain for some time so as to come to the temperature of the furnace, then removing and plunging it into water. The calculations involved in this method will be found in Chapter XXVI.

The electrical resistance of platinum wire varies regularly with the temperature of the wire. This fact enables high temperatures to be measured by observation of the electrical resistance of a platinum wire exposed to the hot gas or liquid. Instruments used for measuring high temperatures are called **pyrometers.** In **thermo-couple** pyrometers advantage is taken of the varying strength of electric current set up in a circuit consisting of two dissimilar metals, such as platinum and iridium, in contact with one another, when the junctions are at different temperatures. Optical pyrometers are also used in modern metallurgical operations.

Exercises on Chapter XXIII.

1. Describe, with sketches, the construction of an ordinary mercurial thermometer.

2. A small spherical glass vessel having a fine stem contains cold water which stands about half-way up the stem. If the bulb is plunged into hot water, state what will happen to the water level in the stem; give a full explanation.

3. Define the terms (*a*) freezing point, (*b*) boiling point of a thermometer. What is meant by the scale of temperature of a mercury thermometer?

4. Convert the following temperatures: (*a*) 140° C. to F.; (*b*) −5° C. to F.; (*c*) −273° C. to F.

5. Convert the following temperatures: (*a*) 100° F. to C.; (*b*) 10° F. to C.; (*c*) −60° F. to C.

6. There is a certain temperature which has the same reading on both the Centigrade and Fahrenheit thermometers. Find this temperature.

7. Give sketches of the apparatus required, and explain how to determine the freezing point error of a thermometer.

8. Answer Question 7 for the boiling point error of a thermometer.

9. In testing the boiling point error of a Fahrenheit thermometer, the observed reading was 211·6 degrees. The barometer at the same time reads 76·2 cm. of mercury. Find the boiling point error. (See the table, p. 533, for any quantities required.)

10. What precautions should be observed in using a mercury-in-glass thermometer?

11. What is meant by a thermoscope? Give some examples of substances which may be used as thermoscopes.

12. An accurate Centigrade thermometer registers 15·5°, while a faulty Fahrenheit thermometer, hanging beside it, registers 61·5°; what is the correction to be applied to this latter reading? Sen. Cam. Loc.

13. Explain what is meant by a scale of temperature. What properties would guide you in the selection of a liquid for use in a thermometer? What would determine the dimensions you would give to the parts of the thermometer?

14. Describe and give a sketch of a sensitive mercurial thermometer. Describe how to graduate such a thermometer with a Centigrade scale.

15. Describe, with reference to a sketch, the construction of a maximum and minimum thermometer.

16. You are supplied with a thermometer and its corrections at freezing point and boiling point. Describe how to obtain the corrections at other parts of the scale, and how to plot a complete correction graph.

17. A Fahrenheit and a Centigrade thermometer, hanging side by side, indicate 110° and 45° respectively. Describe how you would find out which thermometer was wrong, and what was wrong with it.

CHAPTER XXIV

EXPANSION OF SOLIDS

Expansion.—Most substances expand when the temperature is raised, and contract when cooled. Numerous practical examples may be cited. Wheel tyres of iron are made a little smaller in diameter than the wooden wheel; expansion occurs when the tyre is heated, and it may then be slipped on to the wheel; the wheel is then submerged in water, and the contraction of the cooling tyre binds the whole firmly together. Steam pipes become longer when steam enters them, as may be demonstrated by the following experiment:

EXPT. 63.—**Expansion of a steam pipe.** A small boiler A (Fig. 342) is connected by rubber tubing B to a copper tube C. This tube is about

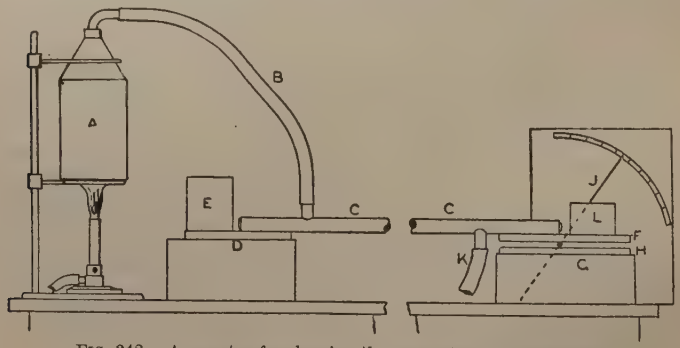

FIG. 342.—Apparatus for showing the expansion of a metal tube.

3 feet long and is plugged at both ends. Branches are soldered near each end on opposite sides of the tube; the steam enters the tube through B and is discharged freely through K. Brass plates D and F are soldered to the ends of the tube. D rests on a block and is held down by a weight E; F rests on a small roller made from a portion of a steel knitting needle.

The roller is supported by a brass plate H, which is fixed to a block G; a light pointer J is attached to the roller and travels over a graduated scale. On permitting steam to flow through the pipe, expansion takes place and will be evidenced by the pointer moving over the scale.

EXPT. 64.—**Unequal expansion of metals.** Take two flat bars of equal size, say 12 inches by 0·75 inch by 0·125 inch, one of iron and the other of copper, and rivet them together, flat to flat. The composite bar so formed should be straightened at the temperature of the room. On heating the bar it will be found to have become bent in the process, the copper being on the convex side, showing that copper expands more than iron when both are heated through the same range of temperature.

Coefficient of linear expansion.—The **coefficient of linear expansion** of a substance may be defined as the increase in length which a bar of unit length undergoes when its temperature is raised through one degree.

Let α = the coefficient of linear expansion.
L = the original length of the bar.
t = the elevation of temperature.

Assuming that the expansion per degree is uniform throughout the range of temperature, we have

Increase in a bar of unit length $= \alpha t$.

Increase in a bar of length $L = L\alpha t$.

Final length of the bar $= L + L\alpha t$

$$= L(1 + \alpha t). \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

COEFFICIENTS OF LINEAR EXPANSION.*

(per degree Cent. at ordinary atmospheric temperatures).

Material.	α.	Material.	α.
Lead	$27{\cdot}6 \times 10^{-6}$	Nickel	$12{\cdot}8 \times 10^{-6}$
Zinc	26·0 ,,	Wrought iron / Mild steel	11·9 ,,
Aluminium	25·5 ,,	Cast iron	10·2 ,,
Tin	21·4 ,,	Platinum	8·9 ,,
Brass	18·9 ,,	Glass	7·8 to 9·7 ,,
Gunmetal	18·1 ,,	Masonry	4 to 7 ,,
Copper	16·7 ,,	Timber	3 to 5 ,,
Nickel steel, 10 per cent. Ni.	13·0 ,,	Nickel steel, (Invar) 36 per cent. Ni.	0·9 ,,

* See *Physical and Chemical Constants*, by Kaye and Laby (Longmans).

Coefficient of superficial expansion.—This coefficient may be defined as the increase in area which a plate of unit area undergoes when its temperature is raised through one degree.

Consider a square plate having edges of unit length. Using the same symbols as before, we have

$$\text{Final length of each edge} = 1 + \alpha t,$$

$$\text{Final area of the plate} = (1 + \alpha t)^2$$

$$= 1 + 2\alpha t + \alpha^2 t^2.$$

Now α is always very small ; hence the term containing the square of α may be neglected, giving

$$\text{Final area of the plate} = 1 + 2\alpha t ;$$

$$\therefore \text{ Change in area of the plate} = (1 + 2\alpha t) - 1$$

$$= 2\alpha t. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence we may infer that the numerical value of the coefficient of superficial expansion for a given substance is double that of the coefficient of linear expansion for the same substance.

Coefficient of cubical expansion.—The **coefficient of cubical expansion** of a substance is the increase in volume which unit volume undergoes when the temperature is raised through one degree.

Consider a cube of unit edge, and use symbols as before.

$$\text{Final length of each edge} = 1 + \alpha t.$$

$$\text{Final volume of the cube} = (1 + \alpha t)^3$$

$$= 1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3.$$

Neglect the terms containing the square and cube of α, giving

$$\text{Final volume of the cube} = 1 + 3\alpha t ;$$

$$\therefore \text{ Change in volume of the cube} = (1 + 3\alpha t) - 1$$

$$= 3\alpha t. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Hence the coefficient of cubical expansion is three times the coefficient of linear expansion of the same substance.

EXPT. 65.—**Coefficient of linear expansion of metal rods.** The modification of an apparatus designed originally by Weedon and illustrated in Fig. 343 has been found to give good results. A is a double copper trough having the space between the outer and inner boxes packed with asbestos. A large hole is cut in each end of the inner box, and circular discs B, B, made of thin copper slightly corrugated, are soldered over the holes. A smaller hole, coaxial with the large one, is cut in each end of the outer box. C is the rod under test, and is made of such a length as to require a little pressure to get it into position, when its ends bear against the centres of the corrugated discs B, B. Two metal stools D, D assist in supporting the rod. Any expansion in the rod will push the discs outwards. The disc at the left-hand end bears against a fixed stop E, made

of glass rod. The expansion of the rod C is measured at the right-hand end by means of a micrometer F, having a glass rod G fixed to its point and bearing against the corrugated disc when the micrometer is advanced.

The trough is supported on two rods, and arrangements are provided which enable it to slide easily. The trough is pushed towards the left by means of a spring, and thus the left-hand corrugated disc is kept bearing firmly against the fixed stop E.

The tank is filled with cold water up to the level of an overflow outlet M. The temperature of the water is taken by means of three thermometers T, T, T; steam for heating the water is supplied through a copper pipe U, which is fitted with a stop valve.

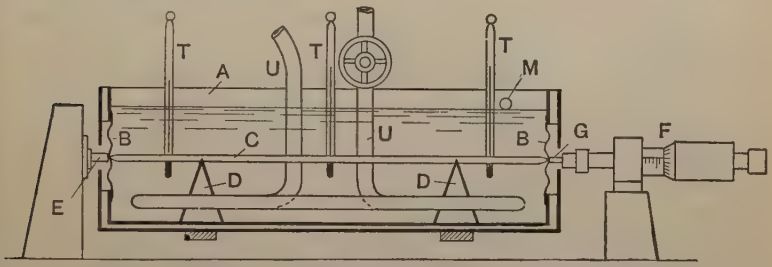

FIG. 343.—Apparatus for determining the coefficient of expansion of rods.

Rods of iron, steel, copper, brass, etc., are supplied. Select one and measure its length as accurately as possible. Place it in position in the trough, having first screwed the micrometer out of contact with the disc. Reduce the temperature of the water to 0° C. by adding ice shavings until some remain unmelted, or melt very slowly. Advance the micrometer until contact with the disc is obtained, and take the reading; read also the three thermometers, and take the mean as the temperature of the water. Screw back the micrometer (care must always be taken to do this before raising the temperature of the bath). Admit steam and raise the temperature to about 10° C.; stir the water, and take the micrometer and thermometer readings as before. Repeat this operation for every 10° up to 100° C. Tabulate the readings as follows:

EXPT. ON THE LINEAR EXPANSION OF A (name of material) ROD.

Length of rod = L mm.

Temp. Cent.	Micrometer reading. mm.	Expansion from 0° C. mm.
0°	a_1	0
t_2	a_2	$a_2 - a_1$
t_3	a_3	$a_3 - a_1$
etc	etc.	etc.

Plot columns 1 and 3. A straight-line graph indicates uniform expansion and constant value of the coefficient of expansion. Select a point on the graph, and read the temperature t and the expansion l for this point. Calculate the coefficient of expansion α from :

$$\text{Expansion} = l = L\alpha t ;$$

$$\therefore \ \alpha = \frac{l}{Lt}.$$

Compensated pendulums.—The time of vibration of a simple pendulum is given by $t = 2\pi\sqrt{l/g}$ (p. 224), and it is essential that l should not alter in any pendulum, otherwise the clock under its control will lose or gain time. If a simple metal rod is used for supporting the pendulum bob, variations in temperature will cause the pendulum to become longer or shorter. For this reason the pendulum rod is sometimes made of wood, which expands but little when the temperature is raised. There are several methods of compensating the pendulum rod for change in temperature, probably the best known being **Harrison's gridiron pendulum**, shown in outline in Fig. 344. The pendulum is suspended at A, and the bob B is supported by five iron rods C_1, C_2, C_3, and four brass rods D_1, D_2. These rods are attached to cross bars in such a way that the expansion of all the iron rods tends to lower the bob, and the expansion of all the brass rods tends to raise the bob.

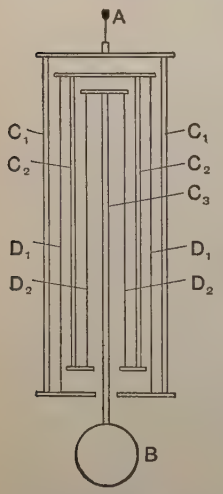

FIG. 344.—Harrison's gridiron pendulum.

The coefficient of linear expansion of brass is about 1·5 times that of iron ; hence two rods, one of brass and one of iron, will expand equal amounts for the same rise in temperature if the brass rod has a length of $\frac{2}{3}$rds that of the iron rod. Referring to Fig. 344 the downward movement of the bob is equal to the expansion of three iron rods, and upward movement is equal to the expansion of two brass rods, and the total lengths of iron and brass bear an approximate ratio of 3 to 2. Hence the length of the pendulum remains practically constant.

In the **Graham compensated pendulum** (Fig. 345) an iron rod is suspended at its upper end, and has a closed cast-iron vessel

attached to its lower end and containing mercury. Expansion of the rod lowers the cup, and hence the centre of gravity of the whole pendulum. Expansion of the mercury takes place upwards in the cup, and hence raises the centre of gravity of the pendulum. By suitably adjusting the quantity of mercury, the centre of gravity of the pendulum will remain at a constant distance from the point of suspension.

Fig. 345.—Graham's mercurial pendulum.

In **chronometer escapements**, the vibrating balance wheel which controls the instrument is compensated for expansion due to changes of temperature. The arrangement is illustrated in Fig. 346. Each spoke supports a separate portion of the rim of the wheel; expansion of these spokes will cause the points A to recede from the centre of the wheel. The segments of the rim are constructed of two strips of different metal, that having the higher coefficient of linear expansion being placed on the outer circumference. Expansion will therefore cause the segments to take a smaller radius of curvature, and will diminish the radius at which the small loads B revolve. When in proper adjustment, the effect of the expansion of the spokes in increasing the radius is nullified by the expansion of the rim segments.

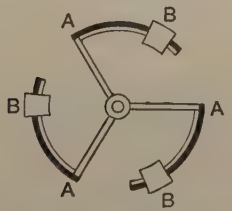

Fig. 346.—Balance wheel of a chronometer.

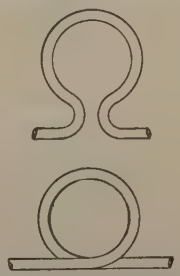

Fig. 347.—Expansion loops for pipes.

Expansion of pipes and rails.—In the case of long metal pipes for conveying gas, increase in length due to alterations in atmospheric temperature may be provided for by making a loop, or circle at intervals in the pipe (Fig. 347). The elasticity of the metal permits the loop to bend easily, and thus to take up the expansion of the straight portions of the pipe.

In long steam pipes which are liable to contain water due to the condensation of some of the steam, a better plan is to cut the pipe

and introduce a stuffing box and gland. In Fig. 348 one portion of the pipe has an enlarged part in which the other portion of the pipe may slide. There is a stuffing box packed with asbestos, or other material which is squeezed in by means of a gland, and thus prevents leakage of steam.

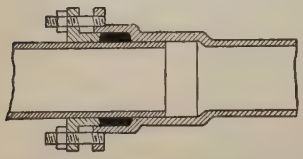

FIG. 348.—Expansion joint for a steam pipe.

On railways the separate portions of rail do not butt closely end to end, but are laid with a small interval between so as to permit of expansion (Fig. 349). The joint is made by means of two fish-plates, A, A, one on each side of the rails, and four bolts. The holes in the rail are slotted as shown on the right-hand rail so that the bolts will not interfere with the rail sliding between the fish-plates. The fish-plates bear on the top and bottom of the rails, as shown in the section, thus preserving level the top surface on which the wheels run.

Lines of rail on which electric tramways run are used for electrical conductors and are generally welded end to end, thus forming a

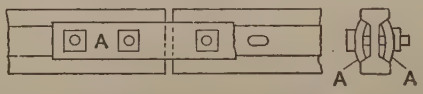

FIG. 349.—Expansion joint for rails.

continuous rail. Such a procedure is rendered possible by the fact that only the top surface of the rail is exposed to atmospheric alterations in temperature. The bulk of the rail is underground, and its temperature varies to a comparatively small extent.

Stresses produced by change in temperature.—Suppose an elastic rod having a length L to be raised in temperature $t°$ C., and that free expansion is permitted. The rod will extend by an amount Lαt, where α is the coefficient of linear expansion. Let the ends of the hot rod be held rigidly, and let the rod be cooled again to the initial temperature. It is evident that the forces required to hold the rod extended will have the same value as those required to produce an extension Lαt at constant temperature.

Let P = the pull required.
A = the cross-sectional area of the rod.
E = Young's modulus.

Then $\text{stress} = \frac{P}{A}$, and $\text{strain} = \frac{L\alpha t}{L} = \alpha t$,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{P}{A\alpha t}, \quad \text{(p. 156)},$$

or $$P = EA\alpha t. \quad \ldots\ldots(1)$$

Supposing the rod to be heated and at the same time held rigidly between abutments which prevent entirely any change in length. These conditions may be imagined to take place as follows: first allow the bar to expand freely on heating; then, maintaining constant the temperature, apply forces to the ends, and let these be sufficient to compress the bar back to its original length.

Length of bar before applying the forces $= L(1 + \alpha t)$.

Change in length produced by $P = L\alpha t$.

$$\text{Strain} = \frac{L\alpha t}{L(1 + \alpha t)} = \frac{\alpha t}{1 + \alpha t}.$$

Now $$E = \frac{\text{stress}}{\text{strain}} = \frac{P}{A}\left(\frac{1 + \alpha t}{\alpha t}\right);$$

$$\therefore P = \frac{EA\alpha t}{1 + \alpha t}. \quad \ldots\ldots(2)$$

Exercises on Chapter XXIV.

(Values of the coefficients of expansion required in the following questions are to be taken from the Table on p. 325.)

1. Give any two examples you may have noticed of the expansion of metals, and explain, with sketches, how the effects of the expansion were eliminated.

2. A bridge constructed of mild steel is 250 feet in length. If the temperature ranges from −10 to 45 deg. Cent., find the alteration in the length of the bridge.

3. The following record relates to an experiment made in the apparatus described in Expt. 65 (p. 326). The rod was of mild steel 20 inches in length.

Temp. C. -	10·2	25·0	35·0	47·0	57·5	65·0	75·5	85·5	100
Micrometer reading, inches	0·1502	0·153	0·1552	0·158	0·160	0·1612	0·1635	0·165	0·1681

Plot a graph showing temperatures and micrometer readings. Choose

two points on the graph, and from the readings at these points deduce the average value of the coefficient of linear expansion of the rod.

4. The pendulum rod of a clock is made of wrought iron and the pendulum swings once per second. If the range in temperature is 30 deg. Cent., find the alteration in length of the pendulum.

5. Calculate the length of a brass rod which will expand in length to the same extent as an iron rod 3 metres in length when both are heated through the same range of temperature.

6. A sheet of lead has an area of 12 square feet at 15° C. Find the area when the temperature is raised to 30° C.

7. A circular flat sheet of thin wrought iron is coated thickly with tin on one side only, and is then heated. Describe and explain any effect which may be observed.

8. A tape used for measuring distances is made of steel and is correct at the temperature of 15° C. If the tape is used for measuring a distance of 2000 feet when the temperature is 10° C., what will be the total error on the measured distance due to the expansion of the tape ?

9. A tube made of thin aluminium has a mean diameter of 40 cm. at 15° C. Find the mean diameter when the temperature is raised to 100° C.

10. Platinum wire can be fused into glass without the glass cracking, or the wire becoming loose during cooling and subsequent changes in temperature. Explain why this is possible.

11. A rod of iron, 12 feet long and 1 inch in diameter, is heated from 15° to 165° C. The rod is held forcibly at its new length and is cooled again to 15° C. Find the pull in the rod. Take $E = 30 \times 10^6$ lb. per square inch.

12. Using the same rod and temperatures as in Question 11, the rod is prevented from expanding in length during heating. Find the push exerted by the rod when the temperature of 165° C. is reached.

13. A ball of cast iron has a volume of 120 cubic inches at 20° C. Find the change in volume when the temperature is raised to 110° C.

14. Describe and give sketches of any apparatus you have used for finding the coefficient of linear expansion of a metal. Show how the results would be calculated.

CHAPTER XXV

EXPANSION OF SOLIDS AND LIQUIDS

Change of density caused by expansion.—During expansion the mass of a given body remains constant and the volume increases. Hence the density, *i.e.* the mass per unit volume, diminishes. The following applies more particularly to solids and liquids ; applications to gases will be found in Chapters XXX. and XXXI.

Let d_1 = the density of a substance at a given temperature, grams per c.c.

d_2 = the density when the temperature is raised through t degrees C.

β = the coefficient of cubical expansion.

Then, taking an initial volume of v_1 cubic centimetres,

Volume occupied at the higher temperature $= v_2 = v_1(1+\beta t)$.

Mass of the body at the higher temperature $= v_2 d_2 = v_1 d_1$;

$$\therefore (1+\beta t) d_2 = d_1,$$

$$d_2 = \frac{d_1}{1+\beta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If d_1 and β are known, the density at the higher temperature may be calculated from this equation. Equation (1) may be written thus :

$$1+\beta t = \frac{d_1}{d_2},$$

$$\beta = \left(\frac{d_1}{d_2} - 1\right)\frac{1}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This result indicates that the value of β may be found by determining the densities of the substance at two different temperatures. The method is specially applicable to liquids.

Expansion of a vessel.—When a vessel containing a liquid is heated, both vessel and liquid expand. The observed or apparent change of volume of the liquid is the difference between the actual change in volume of the liquid and the change in volume of the vessel. If

these happened to be equal, no change in volume of the liquid would be observed.

Suppose that a vessel of thin glass is used, and that the coefficient of cubical expansion of the glass is G. Imagine the vessel to contain a piece of glass which fills it completely, so that the outer shell, which constitutes the vessel, fits closely at all places. If the volume of this piece of glass is v, and if the temperature is raised through t degrees, the volume becomes $v(1+Gt)$.

As the piece of glass and the vessel form practically one piece of glass, it is clear that the shell will still fit closely at the higher temperature, *i.e.* the expansion of the piece of glass contained in the vessel is the same as the expansion of the volume contained by the empty shell. Hence

Change in volume of the vessel $= vGt$. .....(1)

The **coefficient of apparent cubical expansion** of a liquid is the coefficient of the expansion of the liquid relative to the vessel. The **coefficient of absolute expansion** of the liquid refers to the expansion of the liquid which would be observed if the liquid were contained in a vessel incapable of expansion.

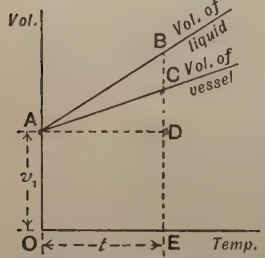

FIG. 350.—Apparent and absolute expansion.

Relation of the apparent and absolute coefficients of expansion.—In Fig. 350 volumes are plotted as ordinates and temperatures as abscissae. Let a glass vessel full of liquid have a volume v_1 at 0° C. The volume of the liquid at any other temperature t is represented by the ordinate EB, and the volume of the vessel at the same temperature by the ordinate EC. The apparent change in volume of the liquid is the difference of these ordinates, viz. BC. Draw the horizontal line AD; then DB is the absolute expansion of the liquid, and DC is the absolute expansion of the vessel.

Let β_a = the coefficient of absolute expansion of the liquid.
β = the coefficient of apparent expansion of the liquid.
G = the coefficient of absolute expansion of the glass.

Then Volume of liquid = BE $= v_1(1+\beta_a t)$.
Volume of vessel = CE $= v_1(1+Gt)$.

Difference in these volumes $= v_1(1+\beta_a t) - v_1(1+Gt)$
$= v_1 t(\beta_a - G)$. .....**(1)**

The same difference may be expressed by employing the coefficient of apparent expansion ; thus :

$$\text{Difference in volumes} = v_1\beta t. \quad \ldots\ldots(2)$$

$$\therefore\ v_1\beta t = v_1 t(\beta_a - G),$$

$$\beta = \beta_a - G,$$

or

$$G = \beta_a - \beta. \quad \ldots\ldots(3)$$

Hence the coefficient of absolute expansion of the vessel is the difference between the coefficients of absolute and apparent expansion of the contained liquid.

Coefficient of absolute expansion by balancing two columns of liquid.—The principle of this method is illustrated in Fig. 351. A bent tube having both limbs open to the atmosphere contains the liquid under test. Jackets round the tubes (not shown in Fig. 351) provide the means of preserving the temperature of the column AC at t_1, and for maintaining BD at a higher temperature t_2. A and B are sections at the same level ; the liquid in the tube between AB may be assumed to be at constant temperature, and therefore to have uniform density. Hence the fluid pressures at A and B are equal.

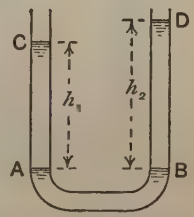

FIG. 351.—Coefficient of absolute expansion.

Let d_1 = the density of the liquid in AC.

d_2 = ,, ,, ,, ,, BD.

h_1 = the height of the column AC.

h_2 = ,, ,, ,, ,, BD.

β_a = the coefficient of absolute expansion of the liquid.

Then Pressure at A = Pressure at B,

$$h_1 d_1 g = h_2 d_2 g,$$

or

$$\frac{d_1}{d_2} = \frac{h_2}{h_1}. \quad \ldots\ldots(1)$$

Also, from equation (1), p. 333, we have

$$\frac{d_1}{d_2} = 1 + \beta_a(t_2 - t_1);$$

$$\therefore\ \beta_a(t_2 - t_1) = \frac{h_2}{h_1} - 1 = \frac{h_2 - h_1}{h_1},$$

or

$$\beta_a = \frac{h_2 - h_1}{h_1(t_2 - t_1)}. \quad \ldots\ldots(2)$$

The method used by Regnault in the determination of the coefficient of absolute expansion of mercury is shown in outline in Fig. 352.

Two vertical tubes AB and CD are connected near their tops by a tube AC of fine bore having a small hole at L. These tubes are connected at B and D to a bent tube BEFGHD ; a branch at K is connected to a pump, by means of which air may be forced into FG. The mercury occupies the tubes as shown in Fig. 352 ; the free surfaces at A and C are subjected to the pressure of the atmosphere, and the surfaces in the tubes EF and GH are subjected to an air pressure sufficient to maintain the levels as shown. Means are provided for maintaining the mercury in CDHG and in FE at a constant temperature t_1, and that in AB at a higher temperature t_2.

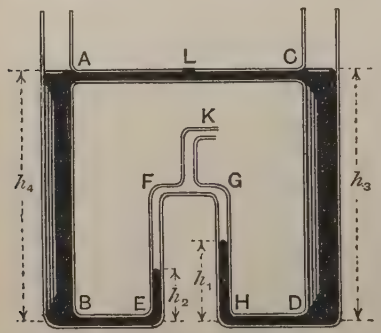

FIG. 352.—Diagram of Regnault's apparatus.

Let d_1 and d_2 be the densities of mercury at the temperatures t_1 and t_2 respectively. Let p be the pressure of the air in FG, and denote the pressures at other points by suffixes ; all these pressures are stated in excess of that of the atmosphere. The portions of the tube BE and HD are assumed to be coaxial, and the heads of mercury h_1, h_2, h_3, h_4 are measured from this axis. Then,

$$p_D = p_H = h_3 d_1 g ; \qquad p_B = p_E = h_4 d_2 g.$$

Also
$$p_H = p + h_1 d_1 g ; \qquad p_E = p + h_2 d_1 g ;$$

$$\therefore \; h_3 d_1 g = p + h_1 d_1 g, \quad \text{.....(1)}$$

and
$$h_4 d_2 g = p + h_2 d_1 g. \quad \text{.....(2)}$$

Again,
$$d_1/d_2 = 1 + \beta_a(t_2 - t_1), \text{ (p. 335), ...(3)}$$

where β_a is the coefficient of absolute expansion of the mercury.

Solving these equations, we obtain

$$\beta_a = \frac{h_4 - h_3 + h_1 - h_2}{(h_3 - h_1 + h_2)(t_2 - t_1)},$$

or, since h_3 and h_4 are equal,

$$\beta_a = \frac{h_1 - h_2}{(h_3 - h_1 + h_2)(t_2 - t_1)}. \quad \text{.....(4)}$$

Regnault's mean value for β_a may be taken as 0·000181.

Taking the density of mercury at 0° C. as 13·5955, the density at

any other temperature t may be calculated from equation (1), p. 333, using Regnault's value of $\beta_a = \cdot 000181$. Thus

$$d_t = \frac{d_0}{1+\beta_a t} = \frac{13{\cdot}5955}{1+0{\cdot}000181t}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

EXAMPLE.—Find the density of mercury at 100° C.

$$d_{100} = \frac{13{\cdot}5955}{1+0{\cdot}0181} = \underline{13{\cdot}35}.$$

EXPT. 66.—**Determination of the coefficient of absolute expansion of a glass vessel.** A small glass bottle (sometimes called a "weight thermometer") having a fine stem is employed in this experiment (Fig. 353). Determine the mass m of the bottle by weighing it when empty. Fill it with mercury by heating it slightly and dipping the mouth into the liquid, some of which will flow in as the bottle cools. Repeat the process until the bottle is quite full, taking care to get rid of air. Put the full bottle into a beaker containing some water, and let it stand for some minutes to allow the temperature to become steady. Add some more mercury if required in order to fill the bottle to the top of the stem. Take the temperature of the water, t_1 say. Remove the bottle carefully, dry its external surface, and determine the total mass by weighing; by deducting m calculate the mass m_1 of the mercury filling the bottle.

FIG. 353.

Raise the temperature of the water in the beaker and repeat the experiment, thus determining the mass m_2 of mercury which fills the bottle at the temperature t_2. Let v_1 and v_2 denote the volumes of the contained mercury at t_1 and t_2 respectively, and let G be the coefficient of absolute expansion of the glass. Then

Change in volume of the bottle $= v_2 - v_1 = \mathrm{G}v_1(t_2 - t_1)$;

$$\therefore \ \mathrm{G} = \frac{v_2 - v_1}{v_1(t_2 - t_1)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

To evaluate v_1 and v_2, we have

$$m_1 = v_1 d_1, \quad \text{and} \quad m_2 = v_2 d_2,$$

or

$$v_1 = \frac{m_1}{d_1}, \quad \text{and} \quad v_2 = \frac{m_2}{d_2},$$

where d_1 and d_2 are the densities of mercury at the temperatures t_1 and t_2 respectively, and are calculated from equation (5) above. Inserting the values in (1) above, we obtain the value of G for the material of the bottle. Also, since the volume v_1 of the bottle at t_1 is now known, together with the coefficient of absolute expansion of the glass, the volume v_2 of the bottle at any other temperature t_2 can be calculated from

$$v_2 = v_1\{1 + \mathrm{G}(t_2 - t_1)\}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

EXPT. 67.—**Coefficients of expansion of a liquid.** The bottle used in Expt. 66 should be employed. Use it in the same manner, and thus determine the masses m_1 and m_2 of the given liquid which fill the bottle at the temperatures t_1 and t_2 respectively. Find by calculation the volumes v_1 and v_2 of the bottle at these temperatures. The densities of the liquid d_1 and d_2 at the temperatures t_1 and t_2 are then found from

$$d_1 = \frac{m_1}{v_1}, \quad \text{and} \quad d_2 = \frac{m_2}{v_2}.$$

Hence, from equation (2), p. 333, we have for the coefficient of absolute expansion of the liquid:

$$\beta_a = \left(\frac{d_1}{d_2} - 1\right) \frac{1}{(t_2 - t_1)}$$

$$= \left(\frac{m_1 v_2}{m_2 v_1} - 1\right) \frac{1}{(t_2 - t_1)}. \quad \text{.....(1)}$$

The values of G and β_a being now known, the value of β, the coefficient of apparent expansion of the liquid, may be calculated from equation (3), p. 335:

$$G = \beta_a - \beta,$$

or

$$\beta = \beta_a - G.$$

In this way, determine the values of β_a and β for the liquid for ranges from 10° to 20°, 20° to 30°, etc., up to 90° C.

If the coefficient of apparent expansion β only is required in the experiment, the change in volume of the bottle may be disregarded, when v_1 and v_2 will be equal, and equation (1) becomes

$$\beta = \left(\frac{m_1}{m_2} - 1\right) \frac{1}{(t_2 - t_1)} = \frac{m_1 - m_2}{m_2(t_2 - t_1)}. \quad \text{.....(2)}$$

Maximum density of water.—When water is cooled, it is found that a contraction occurs until a temperature of 4° C. is reached. Further cooling is accompanied by an expansion until freezing temperature is reached. During the conversion into ice considerable expansion occurs. It follows that water attains its maximum density at 4° C.

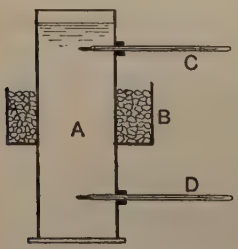

FIG. 354.—Hope's apparatus.

EXPT. 68.—**Hope's experiment on the maximum density of water.** In Fig. 354 is shown a metal vessel A having a trough B surrounding the vessel near the middle of its height. C and D are thermometers.

Pour water into the vessel, and place a freezing mixture consisting of broken ice and salt in the trough. Read the thermometers simultaneously every minute.

Suppose the water to have an initial temperature of 12° to 15° C. The

water half-way up the vessel is cooled and contracts, thus acquiring greater density, which causes it to sink to the lower part of the vessel; this fact is rendered evident by the readings of the thermometer D being lower than those of C. As cooling goes on it will be found that the thermometer D shows that the water in the lower part of the vessel has attained the temperature of 4° C. From this point onwards, the cooling below 4° C. of the water near the middle of the vessel results in an increase of volume,

FIG. 355.—Density of water.

and consequently a decrease in density, which causes the colder water to rise to the top. This is indicated by the thermometer C falling gradually to 0° C., and ultimately a layer of ice is formed on the surface of the water while the temperature shown by the thermometer D is still at or near 4° C.

These facts are of importance in the economy of nature. But for the expansion which occurs while the temperature is lowered from 4° to 0° C., the colder water would remain at the bottom of ponds and lakes, and freezing would start at the bottom and proceed

upwards. Finally the lake would be frozen into a solid mass of ice. Actually freezing occurs at the surface, and as the layer of ice conducts heat but slowly, the water underneath does not fall much below 4° C. Thus life is preserved in such waters.

Owing to the lack of uniformity in the expansion of water, it is not correct to speak of its density without also stating the temperature. The reason for taking unit mass in the C.G S. system as the quantity of matter in one cubic centimetre of water at 4° C. will be apparent. One gallon of water at 60° F. has a mass of 10 pounds. A graph showing the density of water at temperatures from 0° to 100° C. is given in Fig. 355.

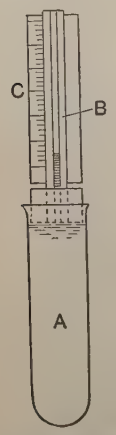

Fig. 356.—Expansion of water while freezing.

Expt. 69.—**Expansion of water while freezing.** Fit a rubber stopper to a test tube A (Fig. 356), and bore a hole in it to receive a tube B. Determine by filling the test tube from a burette the volume v_1 of water, first well boiled and then cooled nearly to freezing, which the test tube can contain up to the stopper. Measure the bore of the tube B, and hence calculate its cross-sectional area (Expt. 7, p. 20). Fit the tube to the stopper as shown in Fig. 356, and attach a scale C. Add a little water so that the level stands a short distance up the tube. Note this level on the scale.

Freeze the water from the bottom upwards by lowering the test tube very slowly into a freezing mixture. If attention be not paid to this, the upper layer will freeze first, and the tube will be burst by the expansion. When freezing is complete in A, read the level on the scale. Take the difference in level, and hence calculate the additional volume v cubic centimetres. This increase in volume has occurred in an initial volume v_1 cubic centimetres, assuming that the water in the tube B does not freeze. Evaluate the ratio $v_1/(v_1+v)$; this will give the density of the ice.

The density of ice is about 0·92 grams per cubic centimetre. Hence about 10 per cent. of the volume of a floating iceberg will be above the surface of sea-water. Melting of the submerged portion of the iceberg will proceed slowly as the iceberg travels with ocean currents into warmer waters. The melting will ultimately affect the stability of flotation of some icebergs and such have been observed occasionally in the act of capsizing.

Exercises on Chapter XXV.

1. The density of a piece of brass is 8·456 grams per c.c. at 15° C., and the coefficient of cubical expansion is 57×10^{-6}. Find the density of the brass at 125° C.

2. The density of a piece of zinc is 7·124 grams per c.c. at 12° C. and 7·122 at 42° C. Find the coefficient of cubic expansion of the zinc.

3. Describe an experiment for determining the coefficient of cubical expansion of a solid. The density of a piece of iron is 7·81 grams per c.c. at 0° C., and its coefficient of linear expansion is $11{\cdot}9 \times 10^{-6}$ per degree Centigrade. What is its density at 100° C. ?

4. A small glass vessel having a fine stem contains 4·56 c.c. of mercury which just fills the vessel at 15° C. The coefficient of linear expansion of the glass is $8{\cdot}4 \times 10^{-6}$. What volume of mercury (measured at 45° C.) will flow out of the vessel if the temperature is raised to 45° C. ? The coefficient of absolute cubical expansion of mercury is 0·00018.

5. A weight thermometer contains 24 grams of mercury at 0° C. On being heated to 100° C. it is found to contain only 23·622 grams. Calculate the coefficient of linear expansion of the envelope, the coefficient of absolute expansion of mercury being 0·00018. L.U.

6. Describe how you would determine the coefficient of apparent expansion of a given sample of kerosene in a given glass envelope.

7. A glass tube of uniform bore is closed at the lower end and is arranged vertically. Mercury partly fills the tube to a height of 76 cm. at the temperature of 15° C. If the temperature is raised to 20° C., find the height of the column of mercury. The coefficient of linear expansion of the glass is 0·0000085, and the coefficient of absolute expansion of mercury is 0·00018.

8. A Centigrade thermometer has a range from −10° to 110° C.; the length of the scale between these marks is 25 cm. The bore of the stem is 0·5 mm. Find the volume of bulb required. Take the coefficient of linear expansion of the glass as 0·000008 and the coefficient of absolute expansion of mercury 0·00018.

9. Describe clearly how to find the coefficient expansion of a liquid by using a weight thermometer. A weight thermometer contains 100 grams of mercury at 0° C.; when the temperature is raised to 100° C. it is found that 1·72 grams of mercury overflow. If the coefficient of absolute expansion of mercury is 0·000181, find the coefficient of cubical expansion of the material of the thermometer.

10. Describe an experiment for finding the coefficient of absolute expansion of a liquid by balancing two columns of the liquid at different temperatures in vertical tubes connected by a horizontal tube at their lower ends. The temperature of the cold column is 5° C., and that of the hot 95° C.; the heights of the columns are 50 cm. and 50.22 cm. respectively. Find the coefficient of expansion.

11. Describe any experimental evidence you are acquainted with which proves that the maximum density of water occurs at 4° C. What bearing has this fact on the freezing of water in lakes ?

12. Describe how to determine experimentally the change in volume which occurs when water freezes.

13. If ice weighs 57·5 lb. per cubic foot, find the volume of an iceberg weighing 10,000 tons. What volume of ice will be above the surface of sea-water weighing 64 lb. per cubic foot ?

14. Give a brief description of Regnault's method of determining the coefficient of absolute expansion of mercury.

15. The coefficient of expansion of mercury is $\frac{1}{5550}$. If the bulb of a mercurial thermometer is 1 c.c., and the section of the bore of the tube 0·001 sq. cm., find the position of the mercury at 100° C., if it just fills the bulb at 0° C. Neglect the expansion of the glass. Calcutta Univ.

CHAPTER XXVI

CALORIMETRY

Quantity of heat.—When a hot and a cold body are brought into contact, a transference of some kind is evidenced by both bodies coming ultimately to the same temperature, lying between the temperatures originally possessed by the bodies.

EXPT. 70.—**Distinction between heat and temperature.** Take two vessels, one, A, containing about one litre of water at about 15° C., the other, B, containing about 0·5 litre of water at about 80° C. Place a thermometer in each vessel, stir well, and take the temperatures. Pour the water from B into A, stir again, and read the temperature. With the quantities mentioned, the final temperature will be about 36° C. The water originally in A has been warmed about 21° C., that originally in B has been cooled about 44° C.

As the temperature of the water in A has not been increased to the same extent that the temperature of the water in B has been lowered, it is evident that what has been transferred has not been temperature, but some other quantity to which the name of **heat** is given.

The quantity of heat possessed by a body depends on several factors, of which temperature is one only. For example, a quantity of water set to boil over a bunsen burner receives a great quantity of heat, as estimated by the time taken, while its final temperature is comparatively low. A wire held in the flame comes to a very high temperature almost immediately, but evidently receives only a small quantity of heat.

Heat is not a material substance which a body may absorb like a sponge taking up water. A body when hot weighs no more than the same body when cold. We shall discuss afterwards evidence which proves that **heat is a form of energy**, and exists in a body in the form of motion of the molecules.

Units of heat.—The unit of heat in any system of units is the quantity of heat required to raise the temperature of unit mass of water through 1°.

The C.G.S. unit of heat is the **calorie**, and is the quantity of heat required to raise the temperature of one gram of water through 1° C. When a larger heat unit is desirable, the **major-calorie** is employed; this unit is equal to 1000 calories.

In Britain two heat units besides the C.G.S. units are employed. These are:

The **Centigrade unit** of heat (lb.-deg.-Cent.), being the heat required to raise the temperature of one pound of water through 1° C.

The **Fahrenheit unit** of heat (lb.-deg.-Fah.), or **British thermal unit** (written B.Th.U.); this is the quantity of heat required to raise the temperature of one pound of water through 1° F.

Since 1·8° F. are equivalent to 1° C., it follows that 1·8 B.Th.U. are required to raise the temperature of one pound of water 1° C.; *i.e.*

$$1 \text{ Centigrade heat unit} = 1{\cdot}8 \text{ B.Th.U.}$$

$$1 \text{ B.Th.U.} = \tfrac{5}{9} \text{ Centigrade heat unit.}$$

EXAMPLE.—What factor must be employed to convert a given quantity of heat stated in calories into B.Th.U. and into Centigrade heat units?

1 calorie can raise the temp. of 1 gram water through 1° C.

453·6 calories " " " 453·6 grams " " 1° C.

Since 1 pound = 453·6 grams, the latter statement may be written:

453·6 calories can raise the temp. of 1 pound water through 1° C.

$\therefore$ $(\frac{5}{9} \times 453{\cdot}6)$ " " " " " " " " 1° F.

Hence $1 \text{ B.Th.U.} = \frac{5}{9} \times 453{\cdot}6 = 252$ calories.

To convert from calories to B.Th.U., multiply the calories by $\frac{1}{252} = 0{\cdot}003968$.

To convert from B.Th.U. to calories, multiply the B.Th.U.'s by 252.

To convert from Centigrade heat units into calories, multiply the Centigrade heat units by 453·6.

To convert from calories to Centigrade heat units, multiply the Centigrade heat units by $\frac{1}{453{\cdot}6} = 0{\cdot}002205$.

The Centigrade heat unit has several important advantages over the B.Th.U., and its use in this country is extending rapidly.

Specific heat.—Experimental evidence shows that equal masses of different substances require unequal quantities of heat to raise their temperatures through the same range. The **specific heat** of a substance may be defined as the quantity of heat required to raise the temperature of unit mass of the substance through one degree. Thus the specific heat of iron is about $\frac{1}{9}$ calorie, *i.e.* $\frac{1}{9}$ calorie can

raise the temperature of one gram of iron through one degree Centigrade, or $\frac{1}{9}$ lb.-deg.-Cent. unit of heat can raise the temperature of one pound of iron through one degree Centigrade. The number expressing the specific heat of a substance is the same irrespectively of the system of units employed. The specific heat of most substances varies somewhat, depending upon the temperature. Thus, if the specific heat of water is taken as unity at 20° C., the values at 40°, 60° and 100° are 0·9982, 1·0000 and 1·0074 respectively. In many calculations the specific heat of water can be assumed to be unity at all temperatures.

Specific Heats.*

(The range in temperature is from ordinary atmospheric temperatures up to 100° C. unless otherwise stated.)

Material.	Specific heat.	Material.	Specific heat.
Aluminium	0·219	Tin	0·0552
Copper	0·0936	Zinc	0·093
Iron	0·119	Glass, Crown 10° to 50° C.	0·16
Lead	0·0305	Glass, Flint 10° to 50° C.	0·12
Nickel	0·109	Ice, −20° to −1° C.	0·502
Platinum	0·0324		

Heat capacity, or water equivalent of a body.—The heat capacity, or water equivalent of a given body is the quantity of heat which the body absorbs when its temperature is raised through one degree. The same quantity may be defined as the mass of water which requires the same quantity of heat to raise its temperature through one degree as the body itself requires. Thus the water equivalent of 9 pounds of iron is one pound, and of 9 grams of iron, one gram.

Let M = the mass of the body,

s = the specific heat of the material.

Then Heat capacity, or water equivalent of the body = Ms.

Measurements of quantities of heat are effected in vessels called calorimeters. The calorimeter generally contains water, and the heat undergoing measurement is passed into the water, thus causing a rise in temperature. The temperature of the calorimeter also rises, and the heat thus absorbed is taken into account by adding the

* For fuller tables of specific heats, see *Physical and Chemical Constants*, by Kaye and Laby (Longmans).

water equivalent of the calorimeter to the weight of the contained water.

Calculations regarding heat transference.—In making calculations of the ultimate temperature attained when heat is transferred from one body to another, it may be assumed in the first instance that no heat is wasted in raising the temperature of any body other than the colder one considered. Corrections may then be estimated and applied for any heat known to be wasted. Calling the two bodies A and B, we may state as an approximate solution :

Heat passing from A = heat entering B.

EXPT. 71.—**Final temperature in mixtures of water.** Put some cold water in a copper calorimeter A and heat another quantity of water in a vessel B. Obtain the masses of water M_A and M_B by weighing and deducting the weights of the empty vessels. Let t_A and t_B be the initial temperatures in A and B respectively. Pour the water from B into A, stir well, and observe the final steady temperature t. Calculate the final temperature as follows :

Heat passing from B = Heat entering A.

$$M_B(t_B - t) = M_A(t - t_A),$$

$$M_B t_B - M_B t = M_A t - M_A t_A,$$

$$t = \frac{M_A t_A + M_B t_B}{M_A + M_B}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Compare the result of this calculation with the experimental value of t. Setting aside errors in measuring the temperatures accurately, and the possible error due to some of the hot water being left in the vessel B, the principal source of discrepancy lies in the fact that the calorimeter A has been raised in temperature. Take account of this by adding to M_A the water equivalent of the calorimeter, which may be estimated by taking the product of the mass M of the calorimeter A and the specific heat s of its material. This now gives

$$M_B(t_B - t) = (M_A + Ms)(t - t_A),$$

from which

$$t = \frac{(M_A + Ms)t_A + M_B t_B}{M_A + Ms + M_B}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The result thus determined should be in close agreement with the experimental value.

The masses used in the above calculations should all be in grams, or all in pounds; the same scale of temperature must be used throughout. If the calorimeter is made of copper, the specific heat s may be taken as 0·1.

EXPT. 72.—**Specific heat of a solid by the method of mixtures.** In this experiment, a small piece of iron, copper, brass, or other material is first heated and then lowered into a calorimeter containing water. The arrangement for heating the sample is shown in Fig. 357. The sample A has a

thread attached to it, and is contained in a copper tube B having a plug of cotton-wool at its lower end. The upper end is closed by a stopper to which a thermometer is fitted. The tube B is enclosed in another larger tube having a branch C for the introduction of steam from a boiler; D is a discharge branch. Steam is allowed to pass into the heater for a few minutes; the temperature is then read, the plug of cotton-wool is withdrawn quickly, and the sample is lowered into the calorimeter, which is placed below the heater. The arrangement enables a dry, hot sample to be obtained, and permits the minimum duration of contact with the atmosphere between the sample leaving the heater and entering the calorimeter.

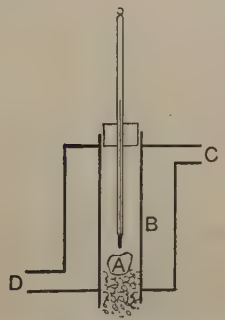

FIG. 357.—Arrangement for heating the sample.

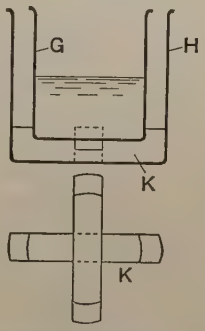

FIG. 358.—Section of a calorimeter.

The calorimeter is shown in Fig. 358, and consists of an inner vessel G and an outer vessel H, both made of copper and separated by a wooden cross K. This arrangement assists in preventing loss of heat from the calorimeter.

Weigh the sample. Determine the water equivalent of the inner vessel G. Find the mass of the water in the calorimeter by weighing. Heat the sample, and when it is ready for transference to the calorimeter, take the temperatures of the heater and of the water in the calorimeter. Lower the sample into the calorimeter, and keep it moving in the water until the temperature of the water becomes steady. Note this temperature.

Let M_s = mass of the sample.

M_w = „ „ water in G.

$M_G s_G$ = water equivalent of the calorimeter.

t_s = temperature of the hot sample.

t_1 = initial temperature of the water.

t_2 = final temperature of the water and sample.

s = specific heat of the material of the sample.

Assuming that the heat given up by the sample is equal to the heat taken up by the water and calorimeter,

$$M_s(t_s - t_2)s = (M_w + M_G s_G)(t_2 - t_1),$$

$$s = \frac{(M_w + M_G s_G)(t_2 - t_1)}{M_s(t_s - t_2)}.$$

Compare the value found from this equation with that given in the Table, p. 345.

In Expt. 72, if the temperature of the room be above or below that of the calorimeter, heat will pass from the atmosphere into or out of the calorimeter during the experiment. This interference may be avoided partially by adjusting the initial temperature of the water in the calorimeter so that the temperature of the atmosphere in the room is the mean of the initial and final temperatures of the water. Thus, if a rise of 4° C. is expected and the temperature of the room is 15° C., the initial temperature of the water should be 13° C.; the heat entering the calorimeter will then be balanced approximately by the heat leaving it.

Specific heat of a liquid.—If a sufficient quantity of the given liquid is available, the specific heat may be found by the method explained in Expt. 72. The liquid is used in the calorimeter instead of water, and a hot body of known specific heat is employed.

EXPT. 73.—**Specific heat of a liquid by the method of mixtures.** Find the specific heat of the sample of lubricating oil supplied, following the method explained in Expt. 72.

Let M_B = mass of the hot body.
M = „ „ liquid in the calorimeter.
$M_G s_G$ = water equivalent of the calorimeter.
t_B = temperature of the hot body.
t_1 = initial temperature of the liquid.
t_2 = final temperature of the liquid and hot body.
s_B = specific heat of the hot body.
s = specific heat of the liquid.

Then

Heat entering liquid and calorimeter = heat given up by hot body;

$$(Ms + M_G s_G)(t_2 - t_1) = M_B(t_B - t_2)s_B;$$

$$s = \frac{M_B s_B}{M}\left(\frac{t_B - t_2}{t_2 - t_1}\right) - \frac{M_G s_G}{M}.$$

EXPT. 74.—**Newton's law of cooling.** The apparatus employed for experiments on the cooling of a liquid is illustrated in Fig. 359. The liquid is contained in a test tube A, fitted with a cork and a thermometer B. A

wire stirrer C enables the liquid to be stirred prior to its temperature being observed. D and E are metal cans, one placed inside the other and having ice packed between; the test tube is suspended so as not to touch the inner can. The temperature of the air space in the inner can will remain practically constant, and may be observed by the thermometer F. The cans are closed at the top by means of a cover G. The object of the arrangement is to enable the surroundings of the liquid under test to be preserved as uniform as possible.

Put a measured volume of hot water in the test tube, and complete the arrangement of the apparatus. Observe the temperatures of the water and of the air space at one minute intervals during 20 or 30 minutes. Plot a

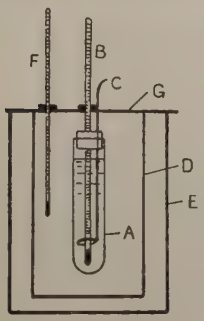

FIG. 359.—Experiment on cooling.

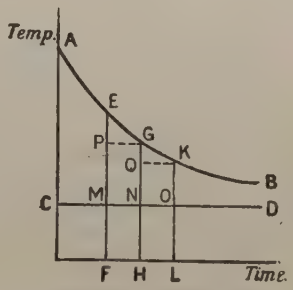

FIG. 360.—Cooling curve.

graph AB (Fig. 360), showing the temperatures of the water as ordinates and times as abscissae. Show the temperature of the air space on the graph as indicated by CD in Fig. 360.

Select equal intervals of time FH and HL. The fall in temperature of the water during the interval FH is EP, and during HL the fall is GQ. During the interval FH the mean difference in temperature of the water and the air space is $\frac{1}{2}(EM + GN)$; during the interval HL the mean difference in temperature is $\frac{1}{2}(GN + KO)$. Evaluate the ratios of fall in temperature to mean temperature difference for both intervals, viz. $EP \div \frac{1}{2}(EM + GN)$ and $GQ \div \frac{1}{2}(GN + KO)$. It will be found that these ratios are practically equal, showing that the rate of cooling of the water is proportional at any instant to the difference in temperature between the water and its surroundings.

Assuming that the fall in temperature of a liquid is proportional to the quantity of heat abstracted from it, we may infer from the above result that the quantity of heat passing from a cooling liquid per unit of time is proportional to the temperature difference between the liquid and its surroundings, a law which is known as **Newton's law of cooling.**

EXPT. 75.—**Specific heat of a liquid by cooling.** Repeat Expt. 74, using an equal volume of some liquid other than water. Plot a cooling graph for this liquid in the same manner as for the water. Obtain from the graphs the intervals of time in which the water and the liquid cool through the same range of temperature from t_1 to t_2. Ascertain, by weighing, the masses of both liquids.

Let T_1 = the time in minutes for the water to cool through the given temperature range.

T_2 = the corresponding time for the other liquid.

M_1 = the mass of the water.

M_2 = the mass of an equal volume of the liquid.

s = the specific heat of the liquid.

Then, since the temperatures are identical in the two experiments,

$$\frac{\text{Heat lost by the liquid}}{\text{Heat lost by the water}} = \frac{M_2 s(t_1 - t_2)}{M_1(t_1 - t_2)} = \frac{T_2}{T_1};$$

$$\therefore \quad s = \frac{M_1 T_2}{M_2 T_1}.$$

It will be noted that equal volumes of water and of the other liquid are used in order that the area of the wetted interior of the test tube may be the same for both. This precaution, together with the practical uniformity of the air-space temperature, ensures that the surrounding conditions are the same in both experiments.

EXERCISES ON CHAPTER XXVI.

1. Convert 1414 lb.-deg.-Centigrade units of heat into lb.-deg.-Fah. heat units and also into calories.

2. Convert 778 lb.-deg.-Fah. thermal units into lb.-deg.-Cent. units and also into calories.

3. Define "Specific heat of a substance." A copper calorimeter weighs 0·4 lb., and the specific heat of the material is 0·094. Find the quantity of heat required to raise the temperature from 15° to 55° C. What is the water equivalent of this calorimeter?

4. A steam boiler is made of mild steel and weighs 10 tons. The specific heat of the material is 0·12. The boiler contains 8 tons of water. Find the quantity of heat required to raise the temperature of the whole from 15° to 100° C., assuming no waste.

5. Fifteen gallons of water at 10° C. are mixed in a tank with 20 gallons of water at 65° C. Find the final temperature, assuming no waste.

6. A piece of zinc weighing 65 grams is at the temperature 100° C., and is lowered into a calorimeter containing 350 grams of water at 15° C. The water equivalent of the calorimeter is 8·5 grams. The final temperature is 16·5° C. Find the specific heat of the zinc.

7. A piece of iron weighing 100 grams is allowed to remain in a current of hot gases in a flue for a few minutes, and is then lowered into a calorimeter containing 500 c.c. of water at 15° C. The water equivalent of the calorimeter is 40 grams. The final steady temperature was observed to be 22·5° C. Calculate the temperature of the gases, assuming that the average specific heat of the iron is 0·11.

8. A sample of lubricating oil weighing 240 grams is contained in a calorimeter having a water equivalent of 12 grams. The initial temperature is 14° C. A piece of copper weighing 72 grams, specific heat 0·093, is raised to 100° C., and then lowered into the calorimeter. The final steady temperature was 18·2. Find the specific heat of the oil.

9. Fifteen grams of water contained in a copper calorimeter weighing 22 grams are found to cool from 80° F. to 70° F. in 3 minutes. An equal volume of another liquid weighing 14 grams cools from 80° F. to 70° F. in the same calorimeter in 110 seconds. The specific heat of the copper is 0·09. Find the specific heat of the liquid.

10. Describe any experiment by which you would find the specific heat of a sample of metal. Give sketches of the apparatus employed.

11. Write an account of the determination of specific heats by the method of cooling, and explain the theory of the method. L.U.

12. The temperatures of equal masses of three different liquids, *a*, *b*, *c*, are 15°, 25°, and 35° C. respectively. On mixing *a* and *b*, the temperature of the mixture is 21° C., and on mixing *b* and *c*, the temperature of the mixture is 32° C. Supposing *a* and *c* were mixed, what would be the temperature of the mixture ?

13. If equal quantities of heat are applied to equal volumes of copper and iron, compare the rises of temperature produced. Density of copper, 8·9 ; density of iron, 7·8. Specific heat of copper, 0·094 ; specific heat, of iron, 0·12.

14. Describe how you would determine experimentally the water equivalent of a calorimeter. If 50 grams of lead shot (specific heat = 0·031) at 97° C. are poured into 75 grams of a liquid at 31° C., contained in a calorimeter of water equivalent 4·5, and the final temperature is 33° C., what is the specific heat of the liquid ? Madras Univ.

CHAPTER XXVII

NATURE OF HEAT. NATURAL SOURCES OF HEAT

Nature of heat.—At the beginning of the nineteenth century it was believed that heat was an elastic fluid, capable of being soaked in or squeezed out, as it were, by a body. The name caloric was given to this substance. This theory has been rejected on the evidence of experiments, of which the most important were made by Rumford, Davy and Joule.

Count Rumford noticed that the metal chips removed by the boring tool in boring a cannon became hot. He arranged a cannon in a bath of water, and started boring with a blunt boring tool which removed but little material. It was found that heat sufficient to boil the water could be obtained during a comparatively short run of the apparatus. As there was evidently no limit to the quantity of heat which could be evolved from the bodies concerned in the experiment, he concluded that it was impossible that the heat produced could be a material substance contained in the bodies prior to starting boring.

Sir Humphry Davy experimented by rubbing together two pieces of ice, taking precautions to ensure that no heat could be communicated to them from outside sources. In a short time it was found that the ice melted, showing that heat had been generated by the rubbing.

Dr. Joule, of Manchester, gave the most conclusive experimental proof that heat is not a material substance. In the experiments of Rumford and Davy, the bodies experimented on changed character during the operations. In Joule's experiments heat was evolved by the stirring of water, and the conditions at the end of the experiment were exactly the same as at the start, excepting that the temperature of the water had been raised.

It is now believed that heat is energy possessed by a body by virtue of the state of motion of the molecules of which it is composed. The molecules of a solid do not alter their positions

relatively to each other, but are in a state of vibration which is increased by the addition of heat, the energy of the molecules being thus made greater. Heat imparted to a liquid may increase the motion of the molecules and at the same time may cause currents of molecules to travel from one part of the liquid to another part. In gases the molecules are in rapid motion; collisions with each other and with the walls of the vessel containing the gas are frequent. The continual bombardment produces pressure on the walls of the vessel. Heat imparted to a gas increases the speed of

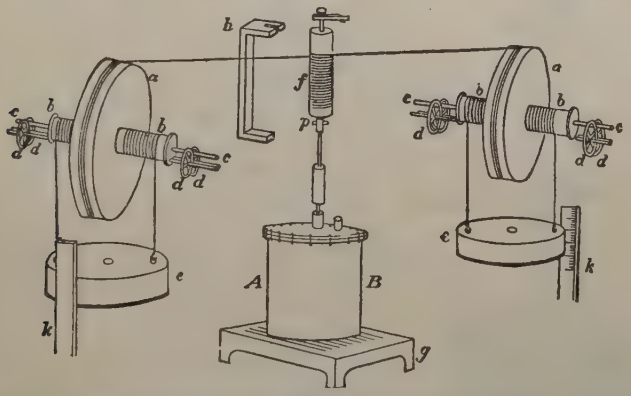

FIG. 361.—Apparatus used by Joule in his experiments on the mechanical equivalent of heat.

the molecules, thereby increasing both their kinetic energy and the pressure on the walls.

Mechanical equivalent of heat.—Heat energy is capable of being converted into other forms of energy and *vice versa*. There are many practical operations having for their object the conversion of heat into mechanical work. Since no energy can be destroyed (p. 170), it follows that a definite quantity of mechanical work is equivalent to a given quantity of heat.

Joule's experiments, already mentioned, were projected for the purpose of ascertaining the quantity of mechanical work equivalent to one thermal unit. The apparatus employed is shown in Fig. 361. Falling weights *ee* are arranged so as to drive paddles revolving in a calorimeter AB containing water. The calorimeter, shown separately in Fig. 362, was fitted with baffle plates having spaces cut so as to permit the paddles to pass, the object being thoroughly to churn the

water. The weights were allowed to descend a measured height, revolving the paddles by means of the cords wrapped round the paddle axle at f (Fig. 361); f was then disconnected from the paddles by means of the pin p, and the weights were wound up again by means of the handle at the top of f. It will be noted that the arrangement of two weights and two cords wrapped round f in opposite directions applied a couple to the paddle axle, thus producing pure rotation.

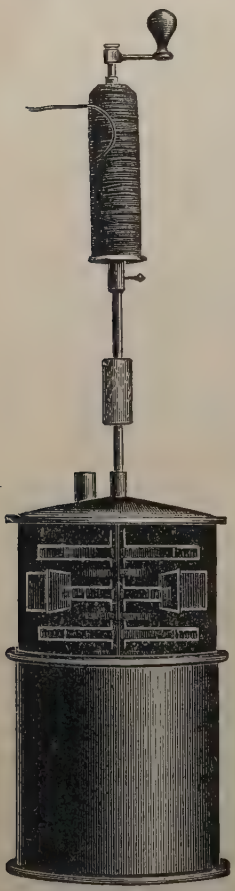

FIG. 362.—Joule's calorimeter.

Corrections of various kinds were applied, such as the kinetic energy of the weight on reaching the bottom, the frictional resistances of the various bearings, the heat capacity of the calorimeter, etc. The final result was that about 772 foot-lb. of mechanical work were equivalent to one British thermal unit.

Subsequent experiments by Rowland in America, and Osborne Reynolds and Griffiths in Britain, give 774 and 778 as more accurate results. The experiments of Osborne Reynolds are of interest on account of the large scale of the apparatus employed. The power developed by the experimental steam engines at Owen's College in Manchester was absorbed by the resistance provided by stirring water in a hydraulic brake. The rate of flow of the water passing through the brake and its rise in temperature were measured, as well as the horse-power absorbed by the brake, thus providing data for calculating the mechanical equivalent of heat. Carefully estimated corrections were applied.

At present the best authorities employ 778 foot.-lb. as equivalent to 1 B.Th.U. These numbers are called **Joule's mechanical equivalent** of heat, and are denoted by the symbol J. In the C.G.S. system J may be taken as $4{\cdot}18 \times 10^7$ ergs of mechanical work, equivalent to one calorie of heat. Another useful value for J is 1400 foot-lb. of

mechanical work equivalent to one Centigrade heat unit, *i.e.* to the heat required to raise the temperature of one pound of water through 1° C.

First law of thermodynamics.—Thermodynamics is the name given to the study of the conversion of heat into mechanical work and *vice versa*. The first law may be enunciated as follows: **Heat and mechanical work are mutually convertible, and in any operation involving such conversion $4{\cdot}18 \times 10^7$ ergs of mechanical work disappear for each calorie generated, or $4{\cdot}18 \times 10^7$ ergs of mechanical work appear for each calorie expended.** In the British system, substitute 1400 foot-lb. and one Centigrade heat unit in the latter part of the law.

The student must be prepared for very large waste in all operations involving the conversion of heat into mechanical work. It is very difficult to prevent heat being dissipated into forms which are useless for any practical purpose. The operation of converting mechanical work into heat is not accompanied by such excessive waste, and laboratory experiments on the value of Joule's equivalent usually follow this method.

EXPT. 76.—**Value of Joule's equivalent by Callendar's machine.** The Callendar apparatus provides a very convenient laboratory method of

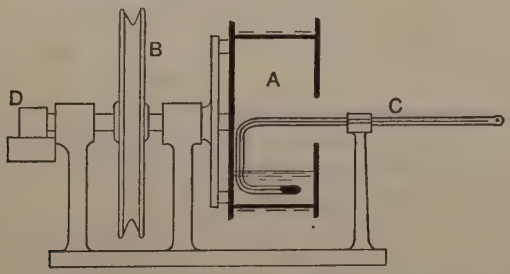

FIG. 363.—Calorimeter of Callendar's machine.

determining J, and is illustrated in Fig. 363. A brass drum A is fixed to the end of a shaft and can be rotated by means of a band passed over a pulley B; the band is driven by a small electromotor not shown in Fig. 363. A revolution counter at D enables the number of revolutions of the drum to be observed. The drum serves the purpose of a calorimeter, and contains a measured quantity of water; a bent thermometer C passes through a central hole in the end of the drum and dips into the water. A band brake, made of three strips of silk ribbon, passes round the drum and covers nearly the whole of its external cylindrical surface. The brake

carries a dead load W at one end (Fig. 364), and has a shackle E attached to the other end to which smaller weights w may be added. A light spring balance pulls the shackle E upwards and enables the brake load to be read delicately, and also helps to produce steadier running.

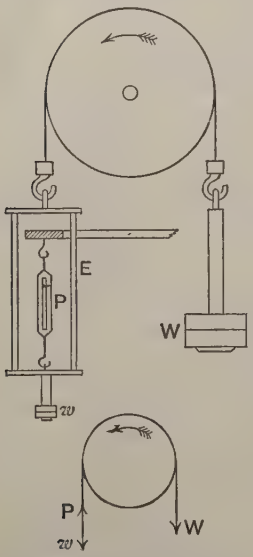

FIG. 364.—Brake of Callendar's machine.

Work is done against the frictional resistances of the silk brake rubbing on the drum; this work is converted into heat, which passes with difficulty outwards through the silk, but passes very easily through the metal drum into the water. The test is made with the initial temperature of the water equal to that of the room; the drum is revolved until the thermometer indicates a rise in temperature of about 5 or 6 degrees Centigrade.

The work done against the resistance of the brake may be calculated as follows: The load W and the pull P of the spring balance both resist the motion of the drum; w assists the rotation. Hence the net resistance is $(W+P-w)$, and this is overcome through a distance equal to the circumference of the drum during each revolution. If D is the diameter of the drum, and if it makes N revolutions, then the total work done is

$$(W+P-w)\,\pi DN.$$

The following record of a test with the Callendar machine is given in order to illustrate the method of reducing the results, especially with reference to the application of cooling corrections, which often has to be done in calorimetric measurements.

DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT BY CALLENDAR'S MACHINE.

Diameter of the drum, D - - -	15·2 cm.
Brake load, W - - - - -	4000 grams weight.
Load at spring balance end, w - -	300 grams weight.
Pull of the spring balance, P - - -	30 grams weight.
Initial counter reading - - - -	62,440.
Final „ „ - - - -	63,335.
Total revolutions during the test, N -	895.
Mass of water used, M_1 - - - -	250 grams.
Water equivalent of the calorimeter, M_2 -	382·65 grams.

Time, min.	Observed temp. C.	Mean temp. during interval. C.	Rate of cooling, from graph (Fig. 366). Deg. Cent. per min.	Correction to be added to the mean temp.	Corrected mean temp. C.
0	15·1				
1	15·8	15·45	0·015	0·015	15·465
2	16·5	16·15	0·035	0·015 + 0·035 = 0·05	16·2
3	17·1	16·8	0·06.	0·05 + 0·06 = 0·11	16·91
4	17·7	17·4	0·08	0·11 + 0·08 = 0·19	17·59
5	18·4	18·05	0·1	0·19 + 0·1 = 0·29	18·34
6	19·0	18·7	0·12	0·29 + 0·12 = 0·41	19·11
7	19·6	19·3	0·14	0·41 + 0·14 = 0·55	19·85
8	20·04	19·82	0·16	0·55 + 0·16 = 0·71	20·53
9	20·06	20·05	0·165	0·71 + 0·165 = 0·875	20·925
10	19·9	19·98	0·162	0·875 + 0·162 = 1·037	21·017
11	19·8	19·85	0·16	1·037 + 0·16 = 1·197	21·047
12	19·7	19·75	0·155	1·197 + 0·155 = 1·352	21·12
13	19·4	19·55	0·15	1·352 + 0·15 = 1·502	21·052
14	19·2	19·3	0·14	1·502 + 0·14 = 1·642	21·082

The machine was run for 8 minutes and then stopped. The temperature was read every minute for 14 minutes ; columns 1 and 2 show these readings. Column 3 gives the mean temperatures during each interval of one minute ; these numbers are obtained from column 2. Mean temperatures and time were plotted (Fig. 365), giving the lower graph. The effect of cooling is shown in this graph by the curve drooping after the machine was stopped. To obtain the cooling corrections throughout the test, we have from this graph :

Mean temperature at 11 minutes = 19·76° C.

Mean temperature at 13 minutes = 19·45° C.

Fall in 2 minutes = 0·31° C.

Fall per minute = 0·155° C.

$$\text{Mean temperature during this fall} = \frac{19{\cdot}76 + 19{\cdot}45}{2}$$

$$= 19{\cdot}6^\circ \text{ C.}$$

Plot a graph (Fig. 366) in which abscissae represent mean temperatures and ordinates represent fall in temperature per minute. The fall of 0·155° C. per minute is plotted at 19·6° C., and a straight line drawn through this point and 15·1° C. (the temperature of the room) on the OX axis ; at this temperature there would be zero rate of cooling. The cooling rate at any mean temperature shown in Fig. 365 can now be obtained from Fig. 366. Thus, at the mean temperature 15·45° C., the rate of cooling is 0 015° C. per minute ; hence the corrected mean temperature for the first interval of one minute is 15·45 + 0·015 = 15·465° C. At the mean temperature of 16·15° C., the rate of cooling is 0·035° C. per minute, and as the test has now proceeded during two minutes, the correction for

the first interval must be added, giving a total correction of 0·015 + 0·035 = 0·05° C. The corrected mean temperature for the second interval of one

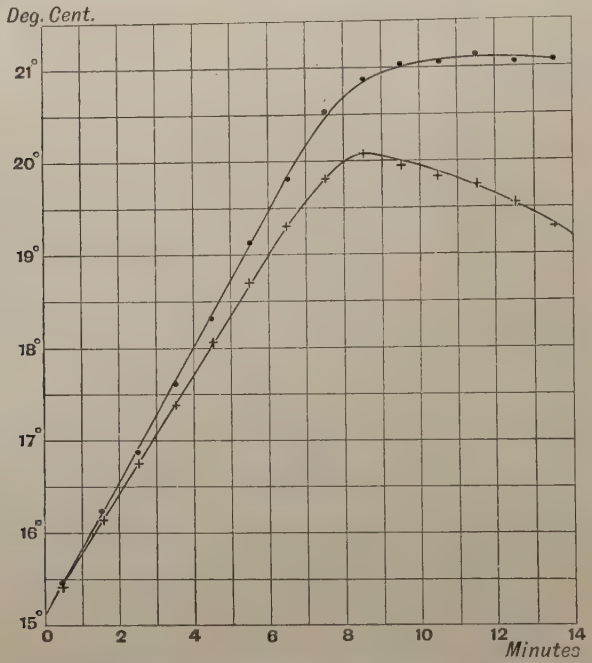

FIG. 365.—Graphs for an experiment with Callendar's machine.

minute is therefore 16·15 + 0·05 = 16·2° C. The rates of cooling obtained from Fig. 366 are shown in column 4, the corrections to be applied in

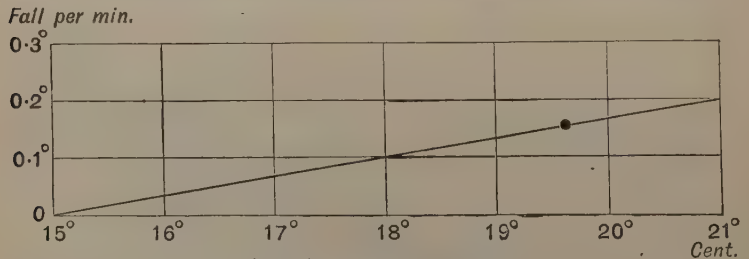

FIG. 366.—Correction graph.

column 5, and the corrected mean temperatures are shown in the last column.

The upper graph (Fig. 365) shows the corrected mean temperatures, and represents what would be obtained experimentally if the conditions had been such that no heat was dissipated from the apparatus during the test. The final corrected temperature is 21·1° C. nearly; the initial temperature is 15·1° C.; the rise in temperature is therefore 6·0° C. We now have:

Work done against friction $= (W - w + P)\pi DNg$ ergs.

Heat developed $= (M_1 + M_2)t$ calories,

where t is the rise in temperature.

$$\therefore J = \frac{(W - w + P)\pi DN}{(M_1 + M_2)t} g$$

$$= \frac{3730 \times 22 \times 15{\cdot}2 \times 895 \times 981}{7 \times 632{\cdot}65 \times 6}$$

$$= 4{\cdot}12 \times 10^7 \text{ ergs.}$$

Natural sources of heat.—Heat, being a form of energy, cannot be created; all heat is obtained from natural stores, or is produced by methods which depend for their working upon natural stores of heat. The most obvious natural source of heat is the sun. Direct heat from the sun is now being used for the production of mechanical work to a small extent in Egypt and America. The plan adopted is to concentrate the sun's rays, by use of long parabolic mirrors, on a pipe containing water. The pipe serves as a steam boiler, and supplies steam to an engine. The sun's heat is also responsible indirectly for the large stores of energy available in water collected in elevated lakes in hilly country. This water comes from rain-clouds which owe their existence to water evaporated from the sea by the heat of the sun. Winds are caused by unequal heating by the sun of large masses of air, and provide energy utilised in the driving of windmills. Volcanic heat is now being utilised for power production in Central Tuscany, where powerful jets of very hot steam are discharged from cracks in the ground. This steam is employed instead of coal for heating the water in steam boilers, and the steam generated in the boilers is used for driving steam turbines, which provide motive power for electric generators. Three large installations on this system were put into operation in the year 1916.

The principal commercial sources of heat are **fuels**. A fuel is any substance capable of chemical combination with the oxygen of the atmosphere, with the evolution of heat and light (*i.e.* **combustion**), and existing in quantities sufficient to be of commercial value.

Solid fuels.—Coal is the fuel in most extensive use at present. It consists of mineralised vegetable matter, and consequently its origin may be traced to the heat and light of the sun. The vegetation of past ages being buried in the earth undergoes compression and slow mineralisation. The first product is **lignite**—a coal of very poor quality. **Anthracite** is the most perfectly mineralised coal and consists chiefly of carbon. **Bituminous coal** is intermediate in composition, and contains volatile constituents composed of compounds of hydrogen and carbon called hydrocarbons. Over 400,000,000 tons of coal are mined annually in Europe. It is estimated that the European coal supply yet remaining is about 350,000,000,000 tons. The consumption of coal is increasing rapidly every year.

The heat which may be produced by the complete combustion of one pound of good coal is about 8000 Centigrade heat units; this number is called the **heating value** of the coal.

Other solid fuels are **coke**, produced by distilling coal in closed retorts; the volatile constituents are driven off, leaving coke, which consists of carbon and ash, the latter being the incombustible material present in the coal; **timber**; **charcoal** produced from wood by driving off the moisture and volatile matter by slow heating, leaving practically pure carbon; **peat**, which is the remains of comparatively recent vegetation found in bogs.

Liquid fuels.—Mineral oils suitable for fuels are obtained as (*a*) crude petroleum, (*b*) paraffin oil; these are both mixtures of various hydrocarbons.

Crude petroleum is obtained by drilling wells in the earth's crust; the bulk of the supply comes from the United States and Russia. The crude oil is refined by distillation, giving gasoline, burning oils, oils suitable for gas making, and other materials. Light gasoline oils are used for driving motor vehicles; the heavier burning oils are also used for operating engines. Crude oil is also used as fuel. The heating value ranges from about 10,800 for the lighter oils to about 12,500 for the heavier, both stated in Centigrade heat units per pound of oil. The extent of the world's store of petroleum is unknown; the consumption is enormous—about 60,000,000 tons per annum—and is increasing rapidly.

Paraffin oil is produced by the distillation of bituminous shales and boghead coal.

Gaseous fuels.—Ordinary **lighting gas** is produced by heating bituminous coal in closed retorts; coke is a bye-product of the process. The gases driven off are purified, and are then available for lighting and heating. About 50 per cent. by volume of the gas is hydrogen, the remainder consists of various hydrocarbons and carbon monoxide. A ton of coal yields about 10,000 cubic feet of gas. The average heating value is about 300 Centigrade units per cubic foot of gas.

Various gases are manufactured for power purposes on the large scale. Among these may be mentioned **Dowson gas**, manufactured by blowing a mixture of air and superheated steam through incandescent anthracite or coke. This gas has a composition by volume roughly as follows: Hydrogen, 19 per cent.; carbon monoxide, 25 per cent.; nitrogen, 49 per cent. The heating value is about 90 Centigrade units per cubic foot of gas. **Mond gas** is another power gas manufactured from cheap bituminous small coal by blowing air saturated with steam at 70° C. through the coal, which is kept burning at a dull red heat. The composition by volume is roughly as follows: Hydrogen, 28 per cent.; carbon monoxide, 12 per cent.; carbon dioxide, 15 per cent.; nitrogen, 43 per cent. The heating value is about the same as that of Dowson gas.

Natural gas is discharged from wells bored into the earth's crust in certain localities. The heating value of American (Pittsburg) natural gas is about 550 Centigrade units per cubic foot of gas. The supply is giving out rapidly.

Combustion of carbon.—In burning carbon completely to carbon dioxide, about 8040 Centigrade units are given out per pound of carbon. Theoretically about 12 pounds of air occupying a volume of about 155 cubic feet must be supplied; in practice from 18 to 24 pounds of air are required.

By limiting the supply of oxygen, carbon **may be incompletely** burned, producing carbon monoxide. In this process about 2470 Centigrade units of heat are given out per pound of carbon. Carbon monoxide is combustible, and when burned completely the product is carbon dioxide; it gives out about 5600 Centigrade units of heat per pound of gas.

Combustion of hydrogen.—When hydrogen is burned, the product is water vapour. About 34,500 Centigrade units of heat are given out per pound of hydrogen. About 35 pounds of air, occupying about 450 cubic feet, must be supplied per pound of hydrogen.

Hydrogen and other combustible gases and vapours may be exploded with violence when mixed with proper proportions of air. Carbon burns slowly unless it is powdered finely and mixed as a dust with oxygen, in which case an explosion may be produced.

Some of the practical methods used in the determination of the heating values of fuels may now be studied.

Heating value of coal.—In the Darling calorimeter, the heat evolved during the combustion of the sample of coal passes into a measured

quantity of water. The instrument is shown in Fig. 367. The sample of coal is first finely powdered and heated at 100° C. in an oven in order to get rid of moisture. About one gram is then weighed and introduced into a crucible C, which is held in clips at the top of a tube A. A bell-glass B encloses the crucible, and is clamped to a plate R. A gentle stream of oxygen is supplied to the bell-jar through a tube O. W, W are wires passing through the bell-glass stopper, and are connected at the lower ends by a piece of fine iron wire which dips into the coal. An electric current is used for ignition; on passing the current through the iron wire, it is heated to incandescence (some of it generally burns) and ignites the coal. The vessel S contains a measured quantity of water, the temperature of which is measured by a thermometer T.

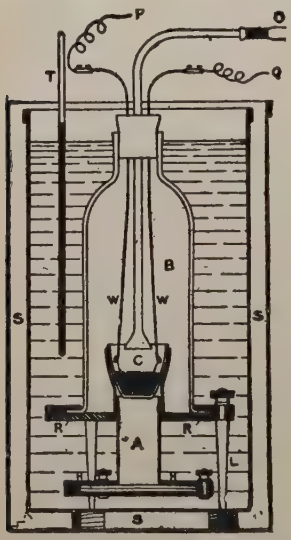

FIG. 367.—Darling calorimeter for testing the heating value of coal or other solid fuel.

The combustion is thus effected in an atmosphere of oxygen; the products of combustion pass downwards through A and escape into the water through a number of small holes. The resulting bubbles give up their heat to the water as they pass upwards.

Let Q = the heating value in calories per gram of coal.

M = the mass of water used, in grams.

M_C = the water equivalent of the instrument, in grams.

M_f = the mass of coal burned, in grams.

t_1 = the initial temperature of the water, C.

t_2 = the final temperature of the water, C.

Then $$Q = \frac{(M + M_C)(t_2 - t_1)}{M_f}.$$

Heating value of gaseous fuels.—In calorimeters of the Darling type, described above, a definite quantity of water is used, and the temperature of the calorimeter rises as the test proceeds. In testing gaseous fuels, calorimeters are used in which the heat evolved during the combustion is passed into water which circulates through the instrument. Arrangements are made so as to maintain, as steady as possible, both the flow of gas to the burner and the flow of water; hence the temperatures remain nearly constant throughout the

experiment. The calorimeter designed by Prof. C. V. Boys is illustrated in Fig. 368, and is typical of this kind of calorimeter.

The gas is burned at jets B; the products of combustion pass upwards into a bell H and then downwards through E, in which space is a pipe coil M made of motor-car radiator tube. Circulating water enters the pipe coil at O, where its temperature is measured, passes through the outer coil N, then through the inner coil M. Leaving M, the water enters a space K where it is thoroughly mixed before being discharged at P, where its temperature is again measured. The instrument is used in conjunction with an accurate gas meter. The quantity of water passed through the calorimeter during the test is discharged into graduated jars, and is so measured; from this quantity and the rise in temperature, the heat evolved by the combustion of the measured quantity of gas is determined.

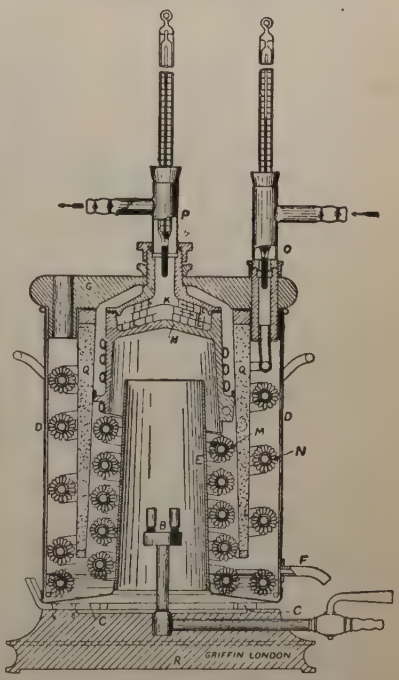

FIG. 368.—Section of Boys's calorimeter for testing the heating value of gas.

Bomb calorimeter.—The original form of the bomb calorimeter is the Berthelot-Mahler; there are now several different designs, one of which is shown in Fig. 369. Both solid and liquid fuels can be tested in this instrument, and very accurate results can be obtained on account of the completeness of the combustion. The general principles are the same as those in the Darling type, but the combustion is effected in an atmosphere of highly compressed oxygen.

A is the bomb, and consists of a strongly made metal vessel fitted with a gas-tight cover B. The measured quantity of fuel is placed in a platinum crucible C held in a stiff wire loop D. Ignition is accomplished electrically by means of a fine iron wire F, supplied with current through leads D and E. After closing the bomb, oxygen under a pressure of 20 atmospheres is passed into it through the pipe G; the valve H is then closed and the pipe G is disconnected. The

bomb is then lowered carefully into a vessel K containing a measured quantity of water and fitted with stirrers L which are rotated by hand. The temperature is observed by means of a delicate thermometer T. Another vessel M surrounds K, the space between forming

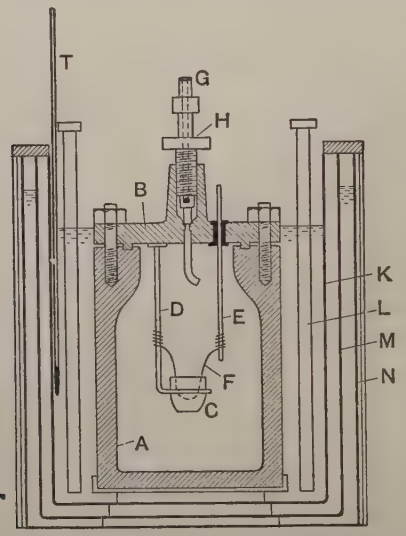

FIG. 369.—Section of a bomb calorimeter.

an air jacket; the vessel N surrounds M, and the space between is charged with water; a flannel jacket is wrapped round the outside of N.

After ignition, the stirrers are operated until the temperature ceases to rise. The heating value of the fuel is then calculated in the same manner as in the Darling calorimeter (p. 362).

EXERCISES ON CHAPTER XXVII.

1. Find the mechanical energy given out when one horse-power is maintained for one hour. Find the heat equivalent of this energy. State the result in lb.-deg.-Cent., lb.-deg.-Fah. and calories.

2. Give a brief account of the evidence that we have for the statement that heat is a form of energy.

3. A tank contains 4 gallons of water, and is fitted with a stirring arrangement which takes 0·28 horse-power to drive it. Supposing that all the work done is converted into heat and retained by the water, how long will it take to raise the temperature of the water from 15° to 25° C.?

4. A train has a mass of 200 tons, and reduces speed from 40 to 30 miles per hour by application of the brakes. Assume that the whole of the work done against the frictional resistance of the brakes is converted into heat, and find this heat. State the result in Centigrade heat units.

5. Give a brief description, with sketches, of Joule's water-stirring experiment for the determination of the mechanical equivalent of heat.

6. Give sketches and describe the Callendar machine, or any other laboratory appliance you are acquainted with for determining the value of J.

7. Explain carefully how to apply cooling corrections in a calorimetric experiment.

8. One pound of coal has a heating value of 8000 lb.-deg.-Cent. By means of suitable machinery 500 gallons of water can be raised to a height of 100 feet for each pound of coal burned. What percentage of the heat contained in the coal is converted into useful work ?

9. Name the principal solid fuels used in practice, and briefly describe each.

10. Give a list and a brief description of the principal gaseous fuels.

11. One ton of coal costs 22 shillings and has a heating value of 8000 lb.-deg.-Cent. units per pound. One gallon of petrol costs 2 shillings and has a heating value of 10,800 lb.-deg.-Cent. units per pound ; a gallon of petrol weighs 7·3 lb. Lighting gas has a heating value of 300 lb.-deg.-Cent. units per cubic foot and costs 3 shillings per 1000 cubic feet. Which of these fuels gives the best heat value ? Answer this question by finding for each fuel the heat obtained for one penny.

12. How much heat is available by the combustion of 100 cubic feet of hydrogen having a heating value of 34,500 lb.-deg.-Cent. units per pound ? Take the weight of one cubic foot of hydrogen as 0·0056 lb.

13. Describe any experiment for the determination of the heating value of a given sample of coal.

14. Answer Question 13 in relation to a given combustible gas.

15. Answer Question 13 in relation to a sample of liquid fuel. The calorimeter described must be different from that chosen in answer to Question 13.

16. Describe one method of determining the mechanical equivalent of heat.
A calorimeter of copper (specific heat 0·095) weighs 122 grams. It contains 1680 grams of aniline oil (specific heat 0·5). The liquid is stirred by a rotating paddle which requires a couple of moment 10^8 dyne cm. to drive it. The temperature of the liquid is raised 8° C. after 450 revolutions. Calculate the mechanical equivalent of heat. L.U.

17. One gram of coal was burned in a Darling calorimeter containing 1400 grams of water. The water equivalent of the calorimeter is 282 grams. The observed rise in temperature was 4° C. Find the heating value in lb.-deg.-Cent. units per pound of coal.

18. In a test with a bomb calorimeter, 0·742 gram of petroleum was burned. The calorimeter contained 2000 grams of water and had a water equivalent of 720 grams. The rise in temperature was 2·98° C. Find the heating value in lb.-deg.-Cent. units per pound of petroleum.

CHAPTER XXVIII

TRANSFERENCE OF HEAT

Conduction.—If heat be imparted to one part of a body, other parts in its neighbourhood have their temperatures raised owing to heat being passed on to them. Thus heat is passed through the body from layer to layer without alteration in the relative positions of the parts of the body. The process has not yet been investigated thoroughly, and is called **conduction.**

EXPT. 77.—**Conduction of heat along a wire.** Coat the surface of a copper wire with paraffin wax, and heat one end of the wire in a flame. The conduction of heat along the wire may be observed by the melting of the wax. If the wire is short, the end remote from the flame becomes in a short time too hot to be held in the hand.

Convection.—When heat is transferred from place to place by the actual motion of the hot body, the heat is said to be conveyed, and the process is called **convection.** In most cases, convection occurs automatically. Thus, in fluids the portions of the fluid adjacent to the source of heat become hot. Expansion takes place, and the density of the hotter fluid becomes smaller than that of the colder portions of the fluid. Motion is then set up under the action of gravity, and the hotter portions of the fluid move away from the source of heat, thus permitting colder portions to approach the source and to become heated in turn. Thus heat is transmitted in convection by reason of currents of fluid approaching and receding from the source of heat.

EXPT. 78.—**Convection currents in a liquid.** The apparatus shown in Fig. 370 is intended to illustrate the convection currents in an appliance sometimes fitted to steam boilers. A is a glass vessel to which is fitted a glass tube B closed at its lower end; the upper end of B is nearly flush with the bottom of A. Another tube C of smaller diameter, open at both ends, is suspended centrally in B, its lower end being at a small height

above the bottom of B, and its upper end being a little below the surface of water which is contained in A and fills both tubes. On applying a flame gently at the lower end of B, the water there is warmed and expands; the density decreases and an ascending current of warm water is established in the inner tube. At the same time a downward current of colder water passes from A through the space between the two tubes. Ultimately the whole of the water becomes heated by a source of heat applied at one part of the vessel only.

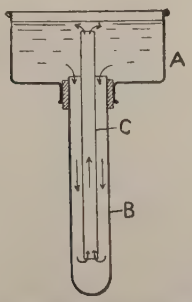

FIG. 370.—Apparatus for showing the circulation of water due to convection currents.

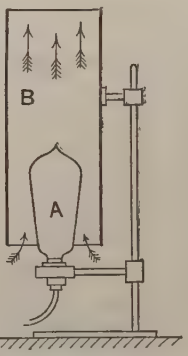

FIG. 371.—Convection currents in a gas.

EXPT. 79.—**Convection currents in a gas.** In Fig. 371, A is an ordinary incandescent electric lamp; B is a cardboard tube open at both ends. When the lamp is lighted, the air inside the tube near the lamp becomes heated and expands. Its density is thus diminished and an ascending current of air is established in the tube. The convection currents of air approaching the lower end of the tube and discharging from the upper end may be rendered visible by holding smouldering brown paper near the lower end of the tube. This experiment illustrates the action of an ordinary chimney, in which the upward draught is caused by the diminished density of the hotter air inside the chimney as compared with that of the colder air outside.

Radiation.—In **radiation**, heat is transferred from a source of heat to other bodies by a kind of wave motion in the ether, a medium which is assumed to fill all interstellar space as well as the spaces between the molecules of material bodies. The heat waves travel at very high speed; on arrival at a body they become absorbed, thus producing the ordinary effects of heat.

EXPT. 80.—**Radiation of heat distinguished from conduction and convection.** Hold the hand a few inches below a lighted incandescent electric lamp.

The sensation of warmth perceived cannot be due to either conduction or convection, since there are ascending currents of air in the neighbourhood of the heated lamp (Expt. 79). The hand is thus surrounded by colder air streaming *towards* the lamp, and hence cannot be affected thermally either by conduction or convection. Radiation alone can account for the sensation of warmth.

Thermal equilibrium.—A body is said to be in **thermal equilibrium** when it receives and gives out equal quantities of heat in the same interval of time. Any heat leaving the body is balanced immediately by the entrance of an equal quantity of heat, and the temperature of the body remains constant. It does not follow that a body at constant temperature is not exchanging heat with other bodies, but only that the exchanges are equal.

Temperature is the condition of matter which determines the direction in which the resultant flow of heat takes place. If a body A can exchange heat with another body B at a lower temperature, then the resultant heat flow always takes place from the body A at higher temperature to the body B. A certain quantity of heat passes per second from A to B, and a lesser quantity passes per second from B to A; ultimately both bodies arrive at the same temperature, when the exchanges of heat become equal.

The **theory of exchanges** may be explained by considering a body surrounded by an envelope. According to this theory, the body continually gives out heat at a rate which is independent of the temperature of the envelope, and depends solely upon the temperature of the body. Similarly the rate at which the envelope gives out heat depends upon its temperature and is independent of that of the body. Thermal equilibrium in both body and envelope is attained when the temperature of the body is equal to the temperature of the envelope.

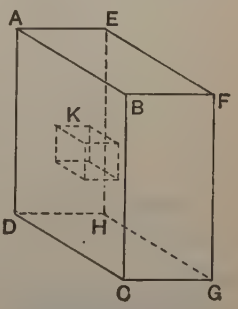

FIG. 372.—Conduction of heat through a plate.

Thermal conductivity.—Consider a large plate of a substance, having parallel faces ABCD and EFGH (Fig. 372). If the face ABCD be maintained at a temperature higher than that of EFGH, heat will flow by conduction through the plate in the sense from A towards E. The plate is supposed to be very large, so that the local effects of

heat escaping from the edges may be disregarded, and it may be assumed that sections of the plate taken parallel to the face ABCD have uniform temperature throughout.

Let all the conditions be steady, and consider a cube K (Fig. 372) of one centimetre edge, embedded in the plate and having two faces parallel to ABCD. The **coefficient of conductivity**, or the **conductivity** of the substance is defined as the quantity of heat flowing through the unit cube per second per degree difference in temperature of its opposite edges.

Let Q = the quantity of heat flowing per second through the unit cube.

t = the difference in temperature of the opposite faces of the cube.

C = the conductivity of the substance.

Then $$C = \frac{Q}{t}.$$

Heat flow along an insulated bar.—In Fig. 373 is shown a metal bar AC, one end of which is heated by a bunsen burner. The portion

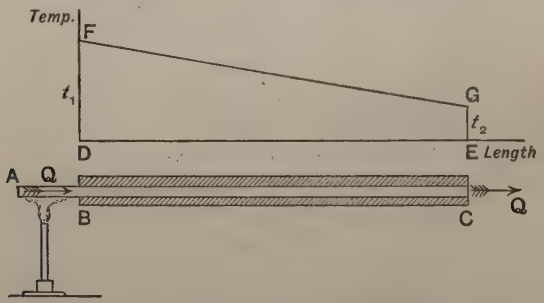

FIG. 373.—Conduction of heat along an insulated bar.

BC is surrounded by non-conducting material, so that no heat entering this portion across the section at B can leave it otherwise than through the end C. Hence, if Q units of heat enter the bar at B, an equal quantity will leave it at C. Under these ideal conditions, the temperature will fall uniformly from B to C. The graph in Fig. 373 is drawn by taking DF to be the temperature of the bar at B, and EG the temperature at C. The straight line FG is drawn, and shows the temperature of the bar at any point in its length. The fall in temperature per unit length is called the **temperature gradient.**

Let Q = the heat flowing along the bar and discharged per second, in calories.
a = the sectional area of the bar, in sq. cm.
l = the length of the bar, in cm.
t_1 = the temperature at B, deg. Cent.
t_2 = „ „ „ C, „
C = the conductivity of the material.

Then Temperature gradient $= G = \frac{t_1 - t_2}{l}$ deg. Cent. per cm. .....(1)

Also, Heat flow per second per sq. cm. of section $= \frac{Q}{a}$.

$$\therefore C = \frac{Q}{a} \div \frac{t_1 - t_2}{l} = \frac{Q}{aG}, \quad \text{.....(2)}$$

or
$$Q = \frac{Ca(t_1 - t_2)}{l} = CaG. \quad \text{.....(3)}$$

Equation (3) may be employed in calculating the heat transmitted through a plate, provided the temperatures t_1 and t_2 are known. It should be noted that there is difficulty in measuring the temperatures of the surfaces of a plate. Equation (2) indicates that experiments might be devised for determining the value of C for the material of a bar, but the difficulty of procuring a perfect heat insulator for surrounding the bar prevents the practical application of this method.

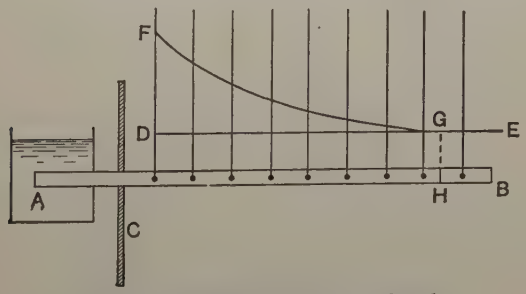

FIG. 374.—Conduction of heat along a bare bar.

Heat flow along a bare metal bar.—In Forbes's method of determining conductivity a bare metal bar AB is used (Fig. 374). The portion near the end A is maintained at constant temperature by being immersed in a bath of molten solder, and a screen C protects the remainder of the bar from being influenced by heat effects from the bath. The bar has a number of pockets, equally spaced, containing

mercury and fitted with thermometers. The temperature of the bar is raised above that of the atmosphere by the heat conducted along the bar towards the end B, and radiation takes place from the bare surfaces. The dissipation of heat which thus takes place is assisted by convection currents of air. The fall in temperature along measured lengths of the bar is thus greater than in the insulated bar discussed above. Ultimately, when conditions have become steady, and if the bar is long enough, there will be a section H at which the whole of the heat entering the bar at A has been dissipated into the surrounding atmosphere. The temperature of the bar between H and B will then be equal to that of the atmosphere.

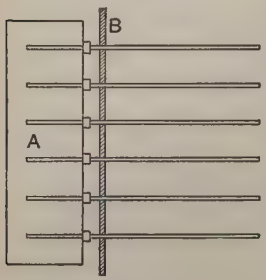

FIG. 375.—Comparative conductivities.

The temperature gradient may be shown by plotting the temperatures indicated by the thermometers (Fig. 374); the horizontal line DE represents the temperature of the atmosphere. The fall is more rapid than in Fig. 373, and atmospheric temperature is attained at G (Fig. 374).

A separate experiment is made on the rate of cooling of a short bar made of the same kind of material, thus determining the heat dissipated at any temperature per square centimetre of exposed surface of the longer bar. From this information and from the temperature gradient graph in Fig. 374, the conductivity of the material is determined.

Comparative conductivities.—The following experiment enables a comparison to be made between the conductivities of various metals:

EXPT. 81.—**Comparative conductivities by Ingen-Hausz's method.** Several rods of different metals are supplied. The rods are all of equal diameters and lengths, and their surfaces should be in the same state as regards polish. A trough A, shown in plan in Fig. 375, is also supplied; one end of each rod can be inserted through a hole in the side of the trough; the trough contains water which is brought to boiling, and a screen B protects the rods from the action of the bunsen flames.

Coat each rod by dipping it into a bath of melted paraffin wax; when the coating has solidified, insert the rods in the trough. Bring the water to boiling; melting of the wax will take place over lengths of the rods which will depend upon their conductivities. When the water has been boiling for 15 minutes, measure the length of each rod from which the

wax has melted; the conductivities are proportional to the squares of these lengths. Thus

$$C_1 : C_2 : C_3, \text{ etc.} = l_1^2 : l_2^2 : l_3^2, \text{ etc.}$$

Express the conductivities of the rods in terms of the conductivity of the copper rod.

The relative value of different heat insulators may be examined by means of the following experiment:

EXPT. 82.—**Comparative value of heat insulators.** A number of copper or aluminium vessels about 500 c.c. in capacity are prepared by fixing short pieces of metal tube about 2 cm. in bore to the centres of the top covers. This permits of thermometers being inserted. The vessels are completely covered, one with flannel, another with cotton-wool, another with felt, a fourth with asbestos, or any other heat insulator available. It is an advantage to have two other similar vessels, one having its bare surface brightly polished, and the other having its surface coated with lampblack. Arrange all these vessels on the bench in a place free from draughts, pour equal quantities of hot water into each through the top tube, using a funnel and being careful that no water is spilled over the insulating material. Insert thermometers, and read the temperatures at intervals of 5 minutes.

Plot temperatures and time for each vessel on a single sheet of squared paper. The resulting graphs will indicate the relative values of the various heat insulators employed, those which have the steeper curves being the poorer heat insulators. Make a list of the substances arranged in order of merit.

The vessels having polished and blackened surfaces should be specially noted; these are cases of radiation, and the results for them indicate that a polished surface provides a better heat insulator than a blackened one.

Conductivity of liquids.—The conductivity of liquids has been determined by a modification of the method of Ingen-Hausz. Experiments on liquids present some difficulty on account of the convection currents which may be set up, thus preventing conduction alone from being examined. That water is a poor conductor of heat is rendered evident by the following experiment:

EXPT. 83.—**Illustration of the poor thermal conductivity of water.** Tie a weight to a small piece of ice and sink it to the bottom of water contained in a test tube. Incline the tube, and apply a small bunsen flame near the surface of the water, thus preventing to a large extent the setting up of convection currents. It will be found possible to have the water boiling near the top of the tube whilst the ice is still unmelted at the bottom, thus showing that but little heat is conducted through the water.

Conduction of heat through a plate.—Consider the heating of water in a kettle or other metal vessel. One side of the bottom of the vessel is exposed to the flame, and the other side is in contact with the water. No part of the metal bottom, however, has a temperature anywhere approaching that of the flame. This may be shown by sticking a strip of paper on the outside of the bottom, when it will be found that the water may be boiled without charring the paper. The experiment indicates the existence of a thin film of comparatively cold gas in contact with the metal plate and practically at rest. It has been shown that the thickness of this film is approximately $\frac{1}{40}$ inch.* Its existence may be further confirmed by the experiment of boiling water in a paper bag.

We may therefore conclude that the temperature of the plate nowhere greatly exceeds the temperature of the water. A large drop in temperature occurs in the film of gas in contact with the plate. Gases are very poor thermal conductors, and this drop in temperature is necessary in order to cause the heat to be conducted through the film.

On the water side of the plate there is a similar film of water in contact with the plate and adhering thereto. The bulk of the water is heated by convection currents, but no such currents exist in this film. Heat is transmitted across it by conduction, but, as water is a better conductor of heat than gases, the drop in temperature is much less than in the gas film.

The transmission of heat from the flame into the water therefore involves a very large fall in temperature in the gas film, and comparatively trifling falls in the plate and the water film. In fact, the transmission is affected to a small degree only by the lack of perfect conductivity in the metal plate.

Methods of increasing the transmission of heat through a plate.—It will be evident—and practical experience and experiment confirm the impression—that a much larger quantity of heat may be transmitted if the mass of hot gases be projected forcibly as a strong current against the plate. The effect is partially to remove the film of gas adhering to the plate. Thus this surface of the plate becomes raised to a higher temperature, and more heat is conducted. Rapid circulation of the water by artificial means on the water side will also assist the heat transmission by partially removing the adhering film of water. Thus we infer that in steam boilers the quantity of heat

* "Heat Transmission," Prof. W. E. Dalby, *Proc. Inst. Mech. Eng.* 1909.

transmitted per square unit of plate surface will be increased very largely by scrubbing both the hot gases and the water vigorously against the surfaces of the plate.

The student will now understand the impossibility of arranging experiments on the determination of the conductivity of a metal by having a plate with, say, boiling water on one side and ice on the other. It is quite impossible to state the real temperatures of the plate surfaces, and without this information the conductivity cannot be estimated.

Effects of oil and scale on heat transmission.—If one side of a thin plate be exposed to a flame and the other side coated with a poor thermal conductor, the temperature of the plate may approach more nearly to the temperature of the flame. An illustration of this occurs in an ordinary frying-pan. The oil used for frying is a very poor conductor, and the bottom of the pan is raised to a much higher temperature than would be the case if water were in the pan. The fact is evidenced by the comparatively rapid burning of the metal and the consequent formation of holes in the pan. For this reason oil must on no account enter a steam boiler.

Many waters contain solids in solution, and when the water is evaporated the solids remain in the vessel and adhere to the plates, forming scale. Such scales are often very hard and are very poor thermal conductors, leading to burning of the plates. Steam boilers require periodic cleaning in order to remove the scale.

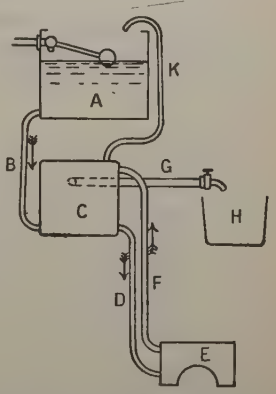

FIG. 376.—Domestic hot water supply.

Hot water supply.—In Fig. 376 is illustrated the method in general use for supplying hot water to lavatory taps. A is an open cold water tank from which the general supply of cold water is obtained, and is connected to a closed hot water storage tank C by a pipe B, which enters C near the bottom. The tank C is connected by a pipe D to a boiler E, which is usually placed at the back of the kitchen fireplace. The pipe D is connected to both C and E as low down as possible. Another pipe F is connected to the boiler near the top, and leads to the top of the storage tank C. The pipes D and F are called circulating pipes. Another pipe G leads from the upper part of C to the bath tap at H, and may have branches leading to other

taps at different parts of the house. K is a pipe for returning to A any water which may be thrown upwards from C by reason of ebullition or other causes, and also to get rid of air from the system.

In working, the water in the boiler becomes heated, and its density is lowered. Convection currents are set up and warmed water leaves the boiler, ascends the pipe F and enters C; meanwhile a further supply of cold water travels downwards from C through D, and enters the boiler to be heated in turn. After a time it will be found that the water in the upper part of C has become hot. The colder, heavier water accumulates at the bottom of C; hence the reason for the cold water supply pipe B being connected to the lower part of C, and also for the tap supply pipe being connected near the top of C. If the tap at H be opened, hot water will be drawn from the upper part of C, and an equal quantity of cold water will flow from A through B into the lower part of C.

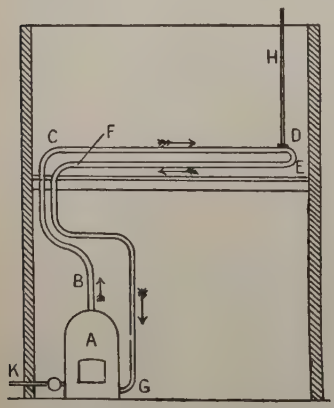

FIG. 377.—Arrangement for heating a building by hot water.

Heating buildings by hot water circulation.—The arrangement is shown in outline in Fig. 377. A is a boiler completely filled with water and generally situated in the basement of the building. A pipe B leads the heated water into the room or rooms to be heated, where it travels along the pipe CD, then back along EF, giving up part of its heat to the air in the room. The transference of heat from the hot water to the room is effected by conduction of heat through the metal of the pipes, then principally by convection currents induced in the air in the neighbourhood of the pipes; radiation of heat from the surfaces of the pipes plays a comparatively small part. The water is returned to the lower colder part of the boiler at G. H is an air pipe leading up to the roof, and allows air to escape from the system; a small automatic air valve is often substituted for this pipe, and closes when the pipe CD becomes hot. Further supplies of cold water may be admitted as required into the boiler by means of the valve at K. Cold air may be brought into the room through openings in the walls behind the pipes CD and EF, and is carried by convection currents throughout the room.

The heating surface may be made greater by connecting to CD, at intervals, groups of short vertical pipes; these are called **radiators**, although their action is principally due to the increase in surface from which convection currents arise.

The system described above is employed also for heating greenhouses. Since it is possible to maintain the temperature of a glass house in the neighbourhood of 15° C., it may be inferred that glass is not a very good conductor of heat, nor does it permit radiant heat to pass easily. This fact is further confirmed by the possibility of holding in the hand an ordinary drinking glass containing liquid hot enough to scald the hand.

Atmospheric circulation.—Circulation of the atmosphere—evidenced by winds—is caused by the air at one place being at a higher temperature, and thus having a lower density, than the air at adjacent places. Convection currents are set up by the warmed air ascending and colder air taking its place. The circulation of the atmosphere on the large scale is produced by reason of the higher temperature in equatorial regions. Lower currents of colder air flow from the temperate regions towards the equator, and upper currents of heated air travel away from the equator. When very large bodies of air are in motion, the directions of the currents are modified by the rotation of the earth. A current of air which would otherwise flow from the north to the equator along a meridian will reach a point on the equator westwards of the place over which it would have passed had the earth been at rest. This is owing to the motion of the earth's surface from the west towards the east, and also to the fact that the whirling velocity of the air accompanying the earth is lower at higher latitudes than the velocity at the equator. The result is a wind coming from the north-east instead of from due north in the northern hemisphere, and from the south-east in the southern hemisphere instead of from due south. These motions of the air towards hot equatorial regions are known as the **trade winds**.

Land and **sea breezes** occur with great regularity in hot countries, and are caused by the greater specific heat of the sea compared with that of the land. During a hot day the land acquires a temperature considerably higher than that of the sea, but falls in temperature much more rapidly at night. The effect is to cause ascending currents of hot air from the land during the day, while cooler air comes from the sea to take its place, producing a **sea breeze**. The

effect is reversed at night, when the land cools quickly to a temperature lower than the sea, and a **land breeze** blows. Winds, such as the **monsoons** in the Indian Ocean, may be produced in this way, and extend in one direction or the other for definite seasons. The conditions required are a hot continent and a cooler ocean for sea breezes, produced in summer; and a cool continent and a hotter ocean for land breezes, which predominate in winter.

Owing to heating of small portions of the earth's surface, local ascending currents are often set up. It is difficult to make observations of these currents; in India the presence of such currents may be detected at certain times during the day by the circling and soaring flight of birds. The bird finds an ascending current and keeps circling in it without flapping its wings, with the result that it gradually ascends without any apparent expenditure of energy on its part.

Exercises on Chapter XXVIII.

1. State the three ways in which heat may be transferred from one point to another, and give examples of each.

2. Give a sketch and explain the working of the ordinary household system for the supply of hot water to lavatory taps.

3. Describe briefly a method of heating a building by means of hot water. Give a sketch of the arrangement.

4. Give a brief account of the cause of winds. Explain the phenomena of trade winds, land and sea breezes and monsoons.

5. Define "thermal equilibrium"; state clearly what is meant by "the temperature of a body." One end of a long bare copper bar is maintained at a temperature 10° C. higher than that of the surrounding atmosphere. Describe clearly what is occurring at various points along the length of the bar.

6. Mention two cases in which it is advantageous to employ good thermal conductors, and other two cases in which it is advisable to employ poor thermal conductors. Give reasons for the fitting of a copper bottom to an ordinary kettle. The bottom of a steel frying-pan burns very readily; explain the reason for this.

7. Calculate the quantity of heat which will flow per hour through an iron plate 1·25 cm. thick. State the result in calories per square metre of plate. The coefficient of conductivity is 0·14, and one face of the plate is at a temperature of 10° C. higher than that of the opposite face.

8. Answer Question 7 in the case of a copper plate of the same thickness. The coefficient of conductivity is 0·91.

9. A current of gases at a temperature of 500° C. flows along one face of a wrought-iron plate 0·4 inch thick, and there is water at 100° C. in

contact with the opposite face. It is found that 5000 Centigrade heat units pass through each square foot of the plate per hour. If the coefficient of conductivity is 0 14, find the temperature of the face of the plate which is exposed to the hot gases. Account for this temperature being much lower than that of the hot gases.

10. Describe how the conductivity of a metal bar may be determined by Forbes's method.

11. Describe experiments by means of which the comparative conductivities of a number of metal rods may be found.

12. Describe briefiy the passage of heat through the plates of a boiler; what must be done in order to obtain and maintain the best efficiency of transmission ?

13. A room has a glass window 2 metres high, 1 metre wide and 7 mm. thick. The room is kept at a temperature of 15° C., and the temperature outside is 0° C. Assume that the temperatures of the two surfaces of the glass are 10° and 2° C. respectively, and calculate the quantity of heat which will pass through the glass per hour. The thermal conductivity of the glass is 0·0005.

14. Define thermal conductivity, and explain how its value may be determined in the case of copper. L.U.

15. Why is it difficult to make accurate measurements of the conduction of heat in liquids ? Describe two experiments to illustrate the low thermal conductivity of water. Sen. Cam. Loc.

16. Heat is conducted through a slab composed of parallel layers of two different materials, of conductivities 0·32 and 0·14, and of thicknesses 3·6 cm. and 4·2 cm. respectively. The temperatures of the outer faces of the slab are 96° C. and 8° C. Find the temperature gradient in each portion. L.U.

CHAPTER XXIX

TRANSFERENCE OF HEAT—*CONTINUED*

Thermal radiations.—Radiation waves are given out by all bodies at all temperatures. If a piece of metal be heated in a darkened room, the waves at first are comparatively long and the periods of vibration great; the eye is unable to detect such waves and the body is invisible. As the temperature of the body is raised, the waves become shorter and the vibrations more rapid, and a point is reached at which the eye is able to detect the effects; the body is then luminous. The only difference between the phenomena of light and radiant heat is that the eye is unable to perceive any effect until the temperature of the body has been raised sufficiently; the laws of transmission are the same.

The laws of transmission of light will be found in a later section of this book. A brief statement of some of the laws of radiant heat is given here in order that reference may be made to the special methods of examining thermal radiations which may or may not be luminous.

No material substance is necessary for the transmission of thermal radiations.—This fact is evident from the consideration that heat is transmitted from the sun to the earth through space containing no ponderable matter. On a smaller scale, it has been illustrated by Sir Humphry Davy, who found that a platinum wire heated in a vessel from which the air had been extracted was capable of influencing thermometers placed outside the vessel. Another illustration is provided by the heating of the glass bulb of an incandescent electric lamp. An ordinary mercurial thermometer is not a very delicate instrument for the detection of thermal radiations, but may be made more sensitive to them by coating the bulb with dull black paint. As we shall see presently, a blackened body absorbs thermal radiations better than one having a polished surface.

Thermal radiations are transmitted with the same velocity as light.—This fact has been proved by observations taken during total eclipses of the sun. Both the light and heat radiations from the sun are cut off simultaneously; hence the velocity of propagation of heat radiations is the same as that of light, viz. about 186,000 miles per second. That the heat radiations from the sun are arrested during an eclipse also affords evidence that **thermal radiations take place in straight lines.** The heat rays are unable to bend round the moon, and a heat shadow is produced, corresponding to and coinciding with the light shadow (Chap. XLI).

FIG. 378.—Ether thermoscope.

Ether thermoscope.—The ether thermoscope (Fig. 378) affords a simple means of detecting thermal radiations. Two glass bulbs, A and B, are connected by means of a bent glass tube. Air is withdrawn entirely and some ether is introduced; the apparatus thus contains ether and ether vapour only. The bulb A is coated with dull black paint. Ether is a very volatile liquid, and any thermal radiations falling on and being absorbed by the bulb A will be rendered evident by an increase in the pressure of the vapour contained in this bulb. Consequently the level of the liquid ether at C will fall and that at D will rise.

Thermopile.—This instrument affords a sufficiently delicate means of indicating thermal radiations in many laboratory experiments. It consists of a number of bismuth and antimony bars arranged as shown in Fig. 379, in which bismuth and antimony bars are marked B and A respectively. The bars B_1 and A_1 are soldered together at their front ends, and are otherwise insulated by mica sheets. A_1 and B_2 are soldered together at their back ends, and are insulated elsewhere.

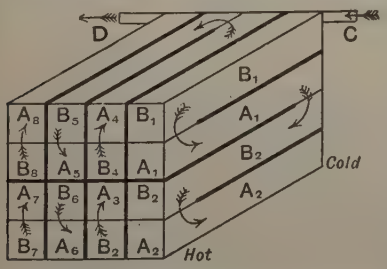

FIG. 379.—Arrangement of bars in a thermopile.

The other bars are connected in similar fashion, thick lines representing insulation. A_2 and B_3 are soldered together at their back ends, as are also A_4 and B_5, also A_6 and B_7. An external electric circuit containing a galvanometer is completed by connecting wires to C and D (bars B_1 and A_8). If the front ends of the bars be

heated, an electric current will flow at the front junctions from each bismuth bar to each antimony bar, and at the colder junctions at the back end the current will flow from the antimony bars to the bismuth bars. Thus a current flows from C down the first pile of bars; then from A_2 to B_3 and up the second pile; passing from A_4 to B_5 the current passes downwards through the third pile, then upwards through the left-hand pile and makes its exit at D. The arrangement gives a cumulative effect to the electromotive force, and makes it much greater than could be secured by the use of a single pair of bismuth and antimony bars. Thus for a given difference in temperature between the two ends of the thermopile, the current is greater than could be obtained by use of a single pair of bars. The ends of the bars are blackened so as to absorb radiant heat more readily. For a fuller discussion of the theory of the thermopile, the student is referred to the Part of the volume devoted to Electricity.

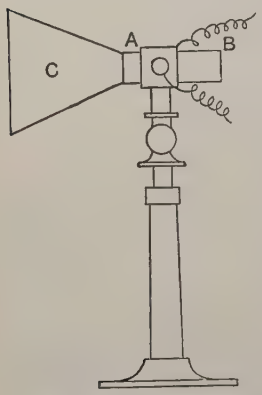

FIG. 380.—Thermopile.

The pile is mounted as shown in Fig. 380. AB is the pile clamped to a stand which can be adjusted in height. Each end of the pile can be covered by a brass cap so as to prevent radiant heat reaching it. A metal cone C can be attached to one end of the pile; this cone screens the end of the pile from radiant heat coming from bodies other than that being tested, and also shields it from air currents.

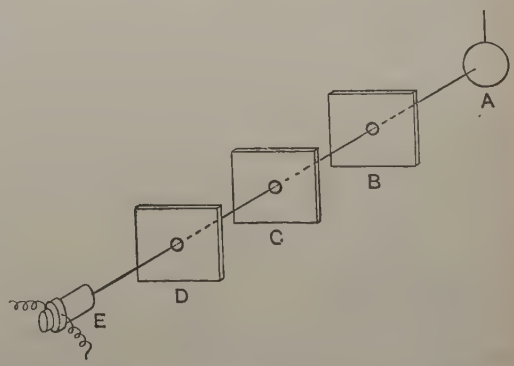

FIG. 381.—Rectilinear transmission of thermal radiations.

EXPT. 84.—**Thermal radiations are transmitted in straight lines.** In Fig. 381, A is a source of radiant heat, such as an iron ball heated to whiteness, or an electric lamp. B, C and D are bright tin screens, each pierced with a hole. E is a thermopile. Arrange

the screens so that the holes are in a straight line. Thermal radiations can then pass from A to the thermopile, and are indicated by the galvanometer. If any one of the screens be displaced somewhat so as to put the holes out of line, it will be found that the galvanometer no longer indicates the reception of radiant heat by the thermopile.

Reflection of radiant heat.—The law of reflection of radiant heat from a plane surface is the same as the law of reflection of light, viz. the angle of incidence is equal to the angle of reflection (Chap. XLIII). This may be proved by the following experiment:

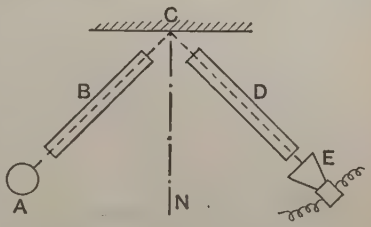

FIG. 382.—Reflection of radiant heat.

EXPT. 85.—**Reflection of radiant heat.** In Fig. 382, A is a source of heat and B and D are tin tubes arranged in plan as shown. E is a thermopile, and C is a polished tin reflector. Remove the reflector, when it will be found that no indication of thermal radiation is given by the thermopile. Replace the reflector, and adjust it so that the thermopile gives maximum indication. It will then be found that the reflector is so situated that the angle of incidence ACN (CN is normal to the reflector) is equal to the angle of reflection ECN.

Refraction of radiant heat.—Radiant heat rays passing from one medium to another, *e.g.* from air to glass, change direction unless the incident ray is normal to the surface of the medium which the ray is about to enter. The law followed is the same as that of light rays, viz. the ray after entering the medium is inclined at a smaller angle to the normal (Chap. XLV). This action is called **refraction**, and becomes evident in the case of heat rays by the well-known application of the burning glass. An ordinary lens refracts sun rays to a point called the focus, and a piece of paper placed at the focus becomes scorched, showing that heat rays have been refracted. Window curtains have sometimes been ignited by refraction of radiant heat rays passing through a globe of water placed near the window.

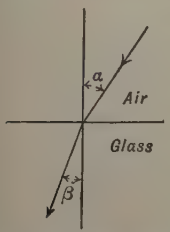

FIG. 383.—Refraction of heat rays.

If the solar spectrum, obtained by the refraction and dispersion of light through a prism, be examined by means of the thermopile,

it is found that the heating effect increases greatly as the red end of the spectrum is approached. But little heat is indicated at the violet end. Beyond the red end of the spectrum, where no light is visible, the greatest heating effect is obtained (Chap. XLIX). Since red rays have a much longer wave-length than violet rays, it may be inferred that non-luminous thermal radiations are of very long wave-length.

The inverse square law.—The quantity of heat transmitted by thermal radiations from a source of radiant energy and received by a surface of given area is inversely proportional to the square of the distance between the source and the surface.

EXPT. 86.—**Proof of the inverse square law.** In Fig. 384, A is a large Leslie's cube, consisting of a copper box containing water which may be

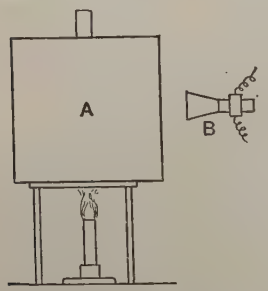

FIG. 384.—Leslie's cube.

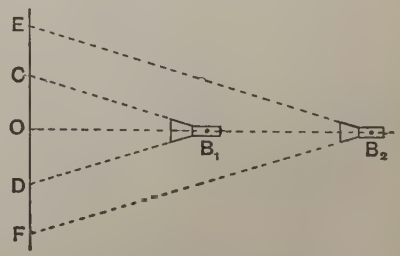

FIG. 385.—The inverse square law.

maintained at boiling point by means of a bunsen burner. B is a thermopile arranged to receive heat radiated from one of the sides of the cube, and fitted with a conical hood. There is thus a cone of heat rays having its apex at the thermopile, and its base falls entirely on the side of the cube provided that the distance of the thermopile from this side is not too great. On varying the distance of the thermopile from the side of the cube, it will be found that the galvanometer readings remain constant, showing that the quantity of heat received by the thermopile does not vary.

With the thermopile at B_1 (Fig. 385), the diameter of the base of the cone of rays is CD ; on moving the thermopile to B_2, the diameter is EF. The action of the conical hood is to make the cones B_1CD and B_2EF similar ; hence the areas of their bases are in the ratio of OB_1^2/OB_2^2, and therefore the heating surface emitting rays which reach the thermopile varies as the square of the distance ; hence we infer that the quantity of heat reaching the thermopile from any unit area of the side of the cube varies inversely as the square of the distance.

Radiating power of a source of heat.—It is found that different surfaces maintained at the same temperature have varying powers of radiation. The **radiating power** of a given surface depends upon the nature of the surface and also upon the temperature, and is generally expressed in terms of a surface coated with lampblack, which is taken as 100.

EXPT. 87.—**Radiating powers of different surfaces.** Use a Leslie's cube having one side lampblacked, another side polished, another side dull copper and the last side papered. Water is kept boiling gently in the cube A, and each side is presented in turn at the same distance from a thermopile B (Fig. 386). A delicate reflecting galvanometer C should be used, and a resistance box D should be inserted in the circuit.

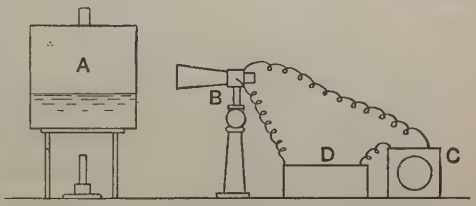

FIG. 386.—Determination of the radiating powers of different surfaces.

With the cube removed or screened from the thermopile, adjust the galvanometer and scale so that the reading is zero. Then present the lampblacked surface to the thermopile, and adjust the resistance so as to obtain as large a galvanometer deflection as possible. Wait until the scale reading is steady and note it. Make similar observations with the other surfaces of the cube, taking care in each case that the base of the cone of rays falls entirely on the surface, and to preserve unaltered the resistance of the circuit. The deflections of the galvanometer so obtained are proportional to the radiating power of the surfaces.

The following experimental record indicates how the observations may be entered and reduced:

EXPERIMENT ON RADIATING POWERS.

Surface.	Galvanometer deflections.	Radiating power.
Lampblacked -	442	100
Polished - -	212	$\frac{212}{442} \times 100 = 47{\cdot}9$
Dull copper - -	348	$\frac{348}{442} \times 100 = 78{\cdot}7$
Papered - -	395	$\frac{395}{442} \times 100 = 89{\cdot}2$

The **emissivity** of a given surface is generally defined as the quantity of heat given out per unit area in one second per unit temperature excess between the surface and its surroundings. It will be noted

from the above experiment that polished surfaces emit much less radiant heat than blackened surfaces. Advantage is taken of this fact in the construction of the vacuum vessel devised by Dewar for storing liquid air and other very cold liquids, and in **thermos flasks.** A double-walled glass vessel has the space between the walls exhausted of air, and the walls are silvered. The vacuum minimises conduction effects, and the mirror surface reduces radiation to a minimum; hence such vessels keep practically constant the temperature of hot or cold liquids for a considerable period of time.

Diathermancy.—When thermal radiations pass into a substance, some of the heat may be absorbed, as is evidenced by the temperature of the substance rising; the remainder of the heat passes through the substance and emerges in the form of thermal radiations. The quantities of heat respectively absorbed and transmitted depend on the nature of the substance and on the wave-length of the thermal radiations. Substances which can transmit thermal radiations of a given wave-length are said to be **diathermanous** for those radiations. Substances which are unable to transmit thermal radiations of a given wave-length, but absorb them, are said to be **athermanous** for these thermal radiations.

Glass affords an example of a substance which is diathermanous to one kind of thermal radiations and athermanous to another kind. Radiations from the sun pass through glass with but little reduction, and the glass remains cool; hence the high temperature inside a green-house exposed to the sun's rays. On the other hand, thermal radiations from an ordinary fire are not well transmitted by glass, a fact which makes glass a valuable material for fire-screens. A glass fire-screen placed in front of a fire absorbs most of the thermal radiations and becomes very hot. Rock-salt is diathermanous to most non-luminous thermal radiations.

In making experiments on diathermancy, care must be taken to state the character of the thermal radiations; in particular, the wave-length should be known, otherwise it is impossible to interpret the results.

A substance which shows considerable absorbing powers for a given kind of thermal radiations also exhibits correspondingly great powers of radiating the same kind of radiations. Thus a blackened surface both emits and absorbs radiant heat freely. A china plate having a white ground and a dark pattern has this appearance to the eye because the darker portions absorb light radiations and the white portions scatter them. If the plate be heated to a high temperature, it will be found that the initially dark portions have

become bright, and the formerly white portions are now dark, showing that those portions which are good absorbers are also good radiators.

The **transmitting power** for thermal radiations of a given substance is the ratio of the thermal radiations transmitted through a unit cube of the substance to the thermal radiations incident normally on one face of the cube. Thus :

$$\text{Transmitting power} = \mathsf{T} = \frac{\text{radiation transmitted}}{\text{radiation incident}}.$$

Expt. 88.—Transmitting powers of different substances. Use the Leslie's cube and a thermopile arranged as in Expt. 87 (p. 385). Observe the galvanometer deflection and let this be θ_1. Clamp one of the substances to be tested between the cube and the thermopile so as to intercept the thermal radiations. Again read the galvanometer deflection, θ_2, say. Measure the thickness t cm. of the substance. Then

$$\text{Transmitting power} = \mathsf{T} = \left(\frac{\theta_2}{\theta_1}\right)^{\frac{1}{t}}.$$

In this manner determine the transmitting powers of colourless, blue and red glass.

The above equation may be obtained as follows: Suppose that the galvanometer deflection is θ when no substance is interposed between the source of thermal radiation and the thermopile, and that the deflection is θ_1 when a plate 1 cm. thick is interposed. Then

$$\mathsf{T} = \frac{\theta_1}{\theta}.$$

If the quantity of heat incident on the plate is Q, then the quantity transmitted will be

$$\mathsf{QT} = \mathsf{Q}\,\frac{\theta_1}{\theta}.$$

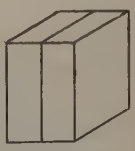

Fig. 387.

Suppose that the heat emerging from this plate is received by a second plate of unit thickness in contact with the first plate (Fig. 387), then the heat transmitted by the second plate will be

$$\mathsf{QT} \times \mathsf{T} = \mathsf{QT}^2.$$

If the galvanometer now gives a deflection of θ_2, the heat transmitted will be $\mathsf{Q}\frac{\theta_2}{\theta}$. Hence

$$\mathsf{QT}^2 = \mathsf{Q}\frac{\theta_2}{\theta},$$

or

$$\mathsf{T} = \sqrt[2]{\frac{\theta_2}{\theta}}.$$

In the same way, if three plates of unit thickness be employed, and if the galvanometer deflection is θ_3, then

$$T = \sqrt[3]{\frac{\theta_3}{\theta}}.$$

Hence, in general, if the thickness of the plate be t cm., and the galvanometer deflection θ_t,

$$T = \left(\frac{\theta_t}{\theta}\right)^{\frac{1}{t}}.$$

Diathermancy of liquids and gases.—By substituting a cell charged with a given liquid for the plate in Expt. 88, the absorbing properties of the liquid may be examined. Experiments show that pure water is very opaque to radiant heat, and salt water almost equally so.

Tyndall, in his experiments on the diathermancy of gases and vapours, used a tube which could be charged with the gases to be examined; the ends of the tube were closed with plates of rock salt, which absorbed but little of the thermal radiations. The source of heat was placed at one end of the tube and a thermopile at the other end. The apparatus was rendered more delicate by having the thermopile fitted with a funnel at each end, one directed towards the experimental tube and the other towards a second source of heat which could be adjusted so as to nullify the thermal effects on the thermopile of the rays emerging from the experimental tube. The tube was first exhausted and the second source of heat arranged so as to bring the galvanometer needle to zero. The tube was then charged with gas and any deflection noted.

In this way it was shown that pure dry air, oxygen, nitrogen and hydrogen absorb but little radiant heat; olefiant gas and ammonia absorb a large quantity. If the heat absorbed by air is denoted by unity, that absorbed by ammonia is nearly 1200.

Exercises on Chapter XXIX.

1. Give a brief explanation of the transmission of heat by radiation. Give any evidence you know of for the statement that no material substance is required in this method of heat transmission.

2. What evidence have we for stating that heat is radiated with the same speed as light?

3. Give sketches and describe the construction and action of a thermopile.

4. Describe an experiment showing that thermal radiations are transmitted in straight lines.

5. How would you show that the law of reflection of thermal radiations is the same as that of the reflection of light?

6. Explain how you would show that solar heat rays can be refracted.

7. Show how to prove experimentally that the radiant heat received by a given surface is inversely proportional to the square of the distance of the surface from the source of heat.

8. Describe how you would compare the thermal radiating powers of different surfaces.

9. Describe the construction and explain the principles involved in a Dewar flask.

10. Explain briefly the meaning of the term diathermancy. Give some illustrations. Describe how to compare the absorbing powers of two partially transparent substances for radiant heat.

11. What evidence have we for stating that good radiators are also good absorbers of heat ?

12. Describe how you would determine the transmitting power for thermal radiations of a given substance.

13. State the ways in which a hot body loses heat. Give some practical examples showing how these losses are made as small as possible.

14. Explain the heating of a green-house by the sun. Why are fire screens sometimes made of glass ?

15. A beam of radiant heat of a definite wave-length loses 0·12 of its energy in passing through 1 mm. of glass. What is the intensity of the beam after it has passed through 2·5 mm. of glass ?

16. Mention three facts bearing upon the similarity in character between light and radiant heat.

Describe generally the changes in character of the radiation from a body as it is raised from the ordinary temperature to white heat. L.U.

CHAPTER XXX

PROPERTIES OF GASES

Distinction between vapours and gases.—Substances in the gaseous state may exist either as **vapours**, or as **permanent gases**. The permanent gas was supposed at one time to remain in the gaseous state under all conditions of pressure and temperature; it is now known that all gases may be liquefied by means of increasing the pressure and diminishing the temperature. A vapour may be condensed by increase of pressure without reduction of temperature. The same substance, when far removed from the conditions of easy liquefaction, may be described as a permanent gas. Oxygen, hydrogen, nitrogen, air and several other gases are examples of permanent gases when under ordinary atmospheric conditions of pressure and temperature. Steam in an ordinary kettle containing some boiling water can be condensed by pressure alone, and is an example of a vapour.

Pressure of a gas.—As has been noted already (p. 353), the molecules of a substance in the gaseous state are in rapid motion and move about in all directions inside the vessel containing the gas. The continual bombardment by the molecules produces forces distributed over the walls, and the total force exerted on unit area is called the pressure of the gas. The pressure of the atmosphere is rendered evident by the barometer, an experiment on which is described on p. 259.

The height of the mercury column in a barometer at any given time depends on the atmospheric conditions then existing; 76 cm. at 0° C. is taken as a standard. The weight of mercury is about 13·59 grams per cubic centimetre; hence the standard height corresponds to a pressure of $76 \times 13{\cdot}59 = 1032{\cdot}8$ grams wt. per square centimetre, or to 1·0328 kilograms wt. per square centimetre. In reading the barometer it is customary to state the height of the mercury column only; the pressure is proportional to this height.

One atmosphere is often taken as a unit of gaseous pressure; it is the pressure produced by a mercury column 76 cm. in height at 0° C. From the above calculation it will be seen that a pressure of one atmosphere is equivalent roughly to one kilogram per square centimetre. In the British system one atmosphere of gaseous pressure is generally taken as equivalent to 30 inches of mercury (76·2 cm.), which is equivalent to a pressure of nearly 14·7 lb. per square inch.

In meteorology, the unit of pressure is the **megabar**, and is equivalent to a mercury column 750 mm. in height, at 0° C., at sea level in latitude 45°. One **bar** = 1 dyne per sq. cm.

As has been explained on p. 268, the pressure of a gas may be stated as so much above or below the pressure of the atmosphere as shown by a barometer at the time of observation; pressures so stated are called **gauge pressures.** The **absolute pressure** of a gas is measured from perfect vacuum, *i.e.* a space containing no gas and therefore having absolute zero of pressure. The absolute pressure may be calculated by adding the pressure of the atmosphere to the observed gauge pressure.

Forms of barometer.—In **Fortin's standard** barometer (Fig. 388) a screw A is fitted to the mercury cup so as to enable the level of the mercury in the cup R to be brought into coincidence with a fixed point P before taking the reading. The upper part of the case is furnished with a scale, the zero of which is at the same level as P, and a sliding vernier operated by means of a thumb-screw B. Mirrors are fitted behind the cup R and the scale S. To read the instrument, first adjust the level of the mercury in the cup R; bring the eye to the level of the top of the mercury column (the mirror aids this), and operate the screw B until the top of the vernier coincides with the top of the mercury column. Readings of the scale and vernier are then taken. The thermometer fitted in the case should also be read: a knowledge of the temperature permits of a correction being applied for the altered density of the mercury due to expansion

FIG. 388.—Fortin's barometer.

In the **aneroid barometer** (Fig. 389) the action depends on the movements of the flexible top and bottom of a circular closed box A under the changes of atmospheric pressure. A is fixed to the base-plate of the instrument, and is exhausted of air as thoroughly as possible; its top side is connected at B to a powerful spring C. C is held in a bracket which is secured to the base-plate at D, and has two bearing screws at E and F. A rod GH, fixed to the spring at G, communicates any rise or fall of the spring through the lever system HK, KL, LM to a fine chain MN, which is wrapped round the spindle carrying a pointer P. P moves over a graduated dial divided

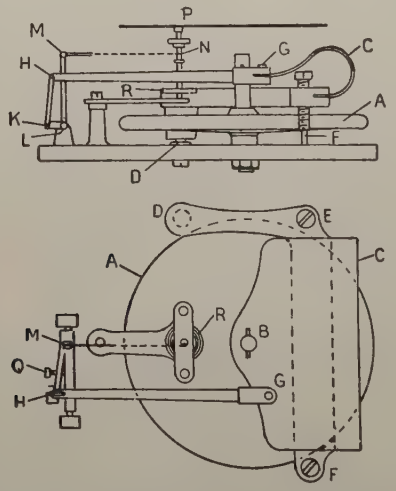

FIG. 389.—Arrangement in an aneroid barometer

to show centimetres, or inches, of barometric height corresponding to the readings of a mercurial barometer. A hair-spring at R keeps the fine chain taut. Q is an adjusting screw whereby the effective length of the lever arm KL may be altered. E and F also serve as adjusting screws.

Aneroid barometers are liable to changes by reason of alterations in the elastic qualities of the metal of which the box A and the spring C are made. It is necessary to check them at frequent intervals by comparison with a mercurial barometer. A recording barometer or **barograph** may be constructed by connecting a lever carrying a pen to the rod GH; the pen draws a curve on a piece of paper wrapped round a drum which is driven by a clock. Ordinates on the resulting chart show barometric heights and the abscissae show time.

Errors in a standard mercury barometer.—The standard barometric height with which other barometric readings are compared is 76 cm. of mercury at 0° C., the instrument being at sea-level at latitude 45°. The observed barometric height is corrected as follows :

1. **Correction for the altered density of the mercury due to expansion** (Fig. 390).

Let h = the observed height in cm. at t° C.

h_0 = the height in cm. which would produce the same pressure at 0° C.

d = the density of the mercury at t° C.

d_0 = the density of the mercury at 0° C.

β_a = the coefficient of cubical expansion of mercury = 0·000181.

Then $hd = h_0 d_0$.

Also $d_0 = d(1 + \beta_a t)$, (p. 333) ;

$\therefore\ h_0 = h(1 - \beta_a t)$. .....(1)

The correction for the altered density of the mercury is therefore effected by multiplying the observed height by $(1 - \beta_a t)$.

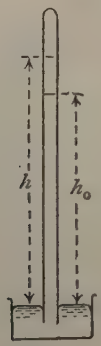

FIG. 390.

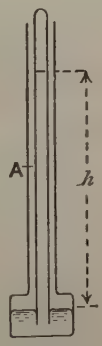

FIG. 391.

2. **Correction for the expansion of the scale.** In the Fortin barometer the scale is carried by a brass tube A (Fig. 391). On the temperature being raised, the length of this tube increases ; hence the height as shown by the scale will be too small at all temperatures higher than 0° C.

Let h = the observed height in cm. at t° C.

h_0 = the height in cm. which would be observed if the scale were at 0° C.

α = the coefficient of linear expansion of brass = 0·000019.

The change in length of the tube A due to a fall in temperature from $t°$ to 0° C. is hat; hence

$$h_0 = h + hat = h(1 + at). \quad \text{.....(2)}$$

Therefore the observed height must be multiplied by $(1 + at)$ in order to obtain the height free from errors due to the expansion of the scale.

The height h_0 corrected for the expansion of both mercury and scale may be obtained in one calculation. Let h be the observed height, then

$$h_0 = h(1 - \beta_a t)(1 + at)$$
$$= h(1 + at - \beta_a t - \beta_a a t^2).$$

The term involving $\beta_a a$ may be neglected, as both β_a and a are very small quantities. Hence we may write

$$h_0 = h\{1 + t(a - \beta_a)\}$$
$$= h\{1 + t(0{\cdot}000019 - 0{\cdot}000181)\}$$
$$= h(1 - 0{\cdot}000162t). \quad \text{.....(3)}$$

3. **Correction for the variations of gravitational effort on the mercury.** The value of the acceleration g due to gravity depends upon the latitude and upon the elevation above sea-level (p. 35); therefore the weight of the mercury per cubic centimetre is not constant. If the barometer is situated at a height H metres above sea-level at a place in latitude λ, then the observed height may be corrected for latitude 45° by multiplying it by

$$1 - 0{\cdot}0026 \cos 2\lambda - 0{\cdot}0000002\text{H}. \quad \text{.....(4)}$$

This correction amounts to 0·13 mm. per 1000 metres above sea-level, to be deducted from the observed height.

4. **Correction for the vapour pressure of the mercury.** The pressure of the vapour of mercury in the tube above the column tends to depress the column. The correction is very small and amounts to an addition of $0{\cdot}0002t$ cm., where t is the temperature Centigrade.

5. **Correction for capillarity.** Surface tension tends to depress the mercury column; the effect is more marked in a tube of narrow bore. The method of cleaning the inside of the tube also influences the amount of depression. The correction is constant for a given instrument, and is best obtained by comparison with a standard barometer; it is usually of the order of 0·002 cm., to be added to the observed reading.

Types of pressure gauges.—A pressure gauge is an appliance for measuring the gaseous pressure inside a closed vessel. In general, pressure gauges indicate the difference between the gaseous pressure inside the vessel and the pressure of the atmosphere. If the difference is small, such as would be the case in a pipe conveying illuminating

gas, or in a boiler chimney, a simple form of U gauge suffices. A chimney gauge is shown in Fig. 392, and consists of a glass U tube A containing some water and having a scale attached by means of which differences in water level in the two limbs may be read. The tube is connected to an iron pipe BC, which passes into the interior of the chimney; the other limb of the tube is open. In action the superior pressure of the atmosphere causes a change in the surface levels of the water as shown. The difference in levels h is called the **chimney draught**, and is stated usually in inches of water. If the difference in levels is considerable, mercury may be used. The absolute pressure inside the vessel may be calculated by adding algebraically the barometer reading to the reading of the pressure gauge, first dividing the latter by 13·59 if water has been used in the gauge.

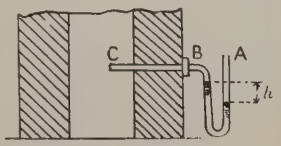

FIG. 392.—U pressure gauge.

Pressure gauges for indicating high pressures, such as that in a steam boiler, are usually of the Bourdon type. The action in these gauges depends on the tendency which a curved, partially flattened tube has to become straight when subjected to internal pressure.

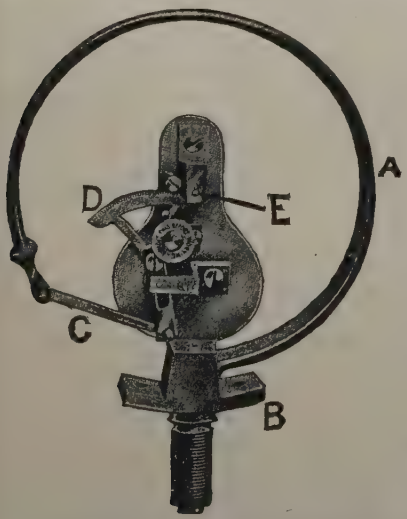

FIG. 393.—Interior parts of a steam pressure gauge.

EXPT. 89.—**Bourdon action.** Attach a piece of rubber tube about a yard long to a water tap; close the outer end by a clip, or a piece of glass rod; bend the tube into a curve lying on the table. Quickly open the water tap, when it will be found that the rubber tube, which has been slightly flattened by the bending, will distinctly show movement in the attempt to become straight.

The interior parts of a Bourdon pressure gauge are shown in Fig. 393. A is a flattened tube of hard, solid-drawn phosphor bronze

secured to a bracket B, which has passages in it forming the steam inlet to the tube. The free end of the tube is closed, and is connected by means of a short link C to a small-toothed sector D. The sector gears with a pinion on the spindle E, which carries a pointer travelling over a scale of pressures marked on the outside of the case.

Bourdon gauges show the difference between the pressure inside the vessel and the pressure of the atmosphere. If the pressure inside the vessel is lower than that of the atmosphere, the gauge is called a **vacuum gauge**, and the dial is graduated in inches or centimetres of mercury so as to facilitate the process of calculating the absolute pressure. The absolute pressure is obtained by deducting the vacuum gauge reading from the observed barometric height. For example, if a vacuum gauge reads 72 cm. at a time when the barometer stands at 75·8 cm., the absolute pressure inside the vessel is 3·8 cm. of mercury.

Boyle's law.—This law has already been enunciated (p. 269), and is stated again for convenience of reference. **The absolute pressure of a given mass of any gas varies inversely as the volume, provided the temperature remains constant.** Taking a given mass of gas under conditions of absolute pressure and volume, p_1 and v_1, let these be changed, at constant temperature, to any other conditions, p_2 and v_2; then

$$p_1 : p_2 = v_2 : v_1,$$

or

$$p_1 v_1 = p_2 v_2.$$

The law may be written pv = a constant.

Boyle's law is not followed by vapours, but is very closely obeyed by the permanent gases. A **perfect** gas is an ideal gas which is imagined to obey Boyle's law.

EXPT. 90.—**Verification of Boyle's law.** Reference is made to Fig. 394. A graduated burette A, of 100 c.c. capacity, is closed by a well-fitted tap at the top, and is connected to a reservoir of mercury B by means of a long piece of thick rubber tubing C. The burette scale is assumed to be so constructed as to show the volume of the gas enclosed between the tap and the surface of the mercury. The 100 c.c. scale division is near the lower end of the burette. If the scale is arranged otherwise, a preliminary experiment must be made in order to determine the volume between the highest scale division and the tap. The reservoir B may be clamped to a support at any convenient height, and has a small side branch D which travels vertically in close proximity to a scale E when B is raised or

lowered. The device enables the level of the mercury in the reservoir to be read on the scale. The level of the mercury in the burette may be read on the scale by employing a U tube containing some mercury and having both limbs open. The mercury levels F and G (Fig. 394) are in the same horizontal plane; the tube is held so that one limb is close to the burette and raised or lowered so as to obtain coincidence of mercury levels in this limb and the burette; the other limb is held close to the scale, and the scale reading at the mercury level in this limb gives the level of the mercury in the burette.

The burette is charged with dry air at atmospheric pressure in the following manner. Open the tap and raise B; the mercury level will rise in A, expelling the contents of the burette. Stop raising B when the mercury level in A is just below the tap. Now lower B slowly, when the mercury level in A will fall again, and air will be drawn through calcium chloride contained in the tube above the tap; the calcium chloride abstracts moisture from the entering air. Adjust the height of B and clamp it so that the level of the mercury in A is at the 100 c.c. mark; since the mercury is at the same level in both A and B, it follows that the air in A is at the same pressure as that of the atmosphere. Close the tap, thus isolating the contents of the burette.

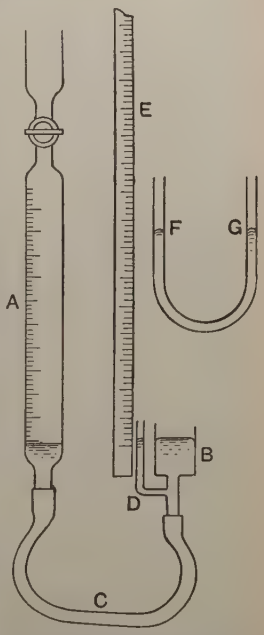

FIG. 394.—Apparatus for verifying Boyle's law.

The initial volume of the inclosed air is 100 c.c. and its absolute pressure may be obtained by reading the barometer; let this be h_1 cm. of mercury. The volume of the air in the burette may be diminished by raising B. The compression of the air may be accompanied by a rise in temperature, and the volume should not be read until two or three minutes have elapsed; the interval permits of the temperature of the air in the burette to fall again to the temperature of the room. It is advisable to have a thermometer attached to the burette; this will enable any alteration in the atmospheric temperature in the neighbourhood of the burette to be observed. Let v be the observed volume of the enclosed air in cubic centimetres.

The level of the mercury in B is now higher than that in A. Read each level on the scale and take the difference; let this be l cm. Since the pressure on the surface of the mercury in B is h_1 cm. of mercury, the pressure on the surface of that in B will be $(h_1 + l)$ cm. of mercury.

If Boyle's law is being followed by the enclosed air, the product $(h_1 + l)v$ will be equal to the product of the initial pressure and volume, viz. $h_1 \times 100$. Make eight or ten experiments, increasing the height of B progressively. Take readings also when the height of B is decreased through the same steps. Tabulate the readings and results as follows :

EXPERIMENT ON BOYLE'S LAW FOR DRY AIR.

Height of barometer = — cm. (h_1).

PRESSURE INCREASING.				
Temp. of room Cent.	Vol. of air v c.c.	Difference in levels, l cm.	Pressure $(l+h_1)$ cm. of mercury.	Products $(l+h_1)v$.

A similar table should be made for diminishing pressures. The products in the last columns will be found to be equal, or nearly so.

Plot a graph in which the pressures $(l + h_1)$ are plotted as ordinates, and the volumes v as abscissae.

Graphs for illustrating Boyle's law.—The graph of the equation $pv = a$ constant, represents Boyle's law, and is a rectangular hyperbola. It may be plotted from experimental data as in Expt. 90, or initial conditions of pressure and volume may be assumed, and values of p corresponding to a sufficient number of other values of v may be calculated. The following method is useful.

Let $pv = c.$

Take logarithms of both sides, giving

$$\log p + \log v = \log c,$$

or

$$\log p = -\log v + \log c.$$

The graph for this equation is a straight line. Ascertain if this is so by plotting the logarithms of $(l + h_1)$ and of v from the experimental data obtained in carrying out Expt. 90.

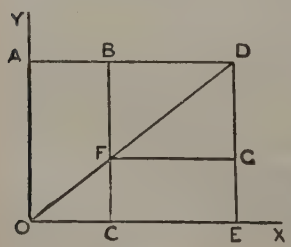

FIG. 395.

The following geometrical method of drawing a rectangular hyperbola is useful. In Fig. 395 set off OA along OY to represent p_1, and OC along OX to represent v_1 to convenient scales of pressure and volume. Make $OE = v_2$, and complete the rectangles OABC and OADE. Join OD cutting CB in F, and draw FG parallel to OX. Then EG is

equal to p_2, and G is a point on the rectangular hyperbola. The proof is as follows: Since the triangles OCF and OED are similar, we have

$$FC : OC = DE : OE,$$
$$EG : OC = BC : OE.$$

But $OC = v_1$,

$BC = p_1$,

and $OE = v_2$;

$\therefore EG : v_1 = p_1 : v_2$,

or $$EG = \frac{p_1 v_1}{v_2} = p_2.$$

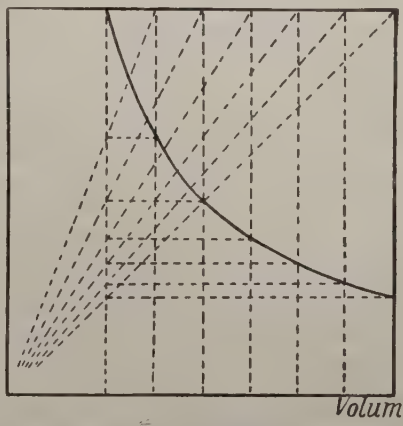

FIG. 396.—Boyle's law curve drawn by means of a geometrical construction.

Other points may be found in a similar manner giving the complete curve shown in Fig. 396.

Such curves, representing operations carried out at constant temperature, are called **isothermal curves**; the operations of expansion or compression are called **isothermal operations.**

EXERCISES ON CHAPTER XXX.

1. Distinguish a vapour from a perfect gas. What is the pressure of a substance in the gaseous state due to ?

2. If the barometer reads 74·32 cm. of mercury, what is the pressure of the atmosphere in grams wt. per square centimetre ?

3. An oxygen cylinder is charged to a pressure of 120 atmospheres. What is the pressure in lb. wt. per square inch ?

4. Give a sketch and description of a standard mercury barometer.

5. Describe, with sketches, the construction of an aneroid barometer. Criticize the accuracy of this type of barometer.

6. A standard mercury barometer reads 76·21 cm. at 15° C. Correct this reading for expansion of the mercury and scale. This barometer is situated at a height of 500 metres above sea-level in latitude 50°. Obtain the constant gravitational correction for this station.

7. A U gauge attached to a boiler chimney reads 0·8 inch of water. The barometer reads 29·45 inches of mercury. What is the absolute pressure inside the chimney ?

8. Describe the construction and action of a Bourdon pressure gauge. Give a sketch.

9. State Boyle's law for gases. How would you prove it experimentally ?

10. A volume of 1 cubic foot of gas at an absolute pressure of 150 lb. wt. per square inch expands at constant temperature. Find the pressure when the volume is 2, 3, 4, 5, 6 cubic feet. Plot a graph showing the relation of pressure and volume.

11. Take the numbers obtained in answer to Question 10 and draw a graph by plotting the logarithms of the pressures and volumes.

12. Use a graphical method for drawing an isothermal curve for a gas expanding from a volume of 2000 c.c. at an absolute pressure of 15 atmospheres down to a pressure of 3 atmospheres.

13. An aneroid barometer reads 30·15 inches at ground level. When taken to the top of a tower, the barometer reads 30·06 inches. Calculate the height of the tower if the density of the air at the time was 0·00125 gram per c.c. and that of mercury 13·6.

14. Describe carefully an experiment to determine the relation between the pressure and the volume of a given mass of gas at constant temperature. Tasmania Univ.

15. The cross-sectional area of the mercury column in a barometer is 1·2 sq. cm. ; the length of the vacuum at the top is 8 cm. when the barometer reads 764 mm. Calculate the volume of external air which must be introduced into the tube in order to cause the height of the column of mercury to become 382 mm.

16. Describe the principle of the mercury barometer.

A mercury barometer is in the receiver of an ordinary air pump, and at first its height is 76 cm. After two strokes the height is 72 cm. What will it be after 10 strokes ? (Volume of barometer is insignificant compared with volume of receiver.) Tasmania Univ.

CHAPTER XXXI

PROPERTIES OF GASES—*CONTINUED*

Charles's law.—This law, first enunciated by Charles and Gay Lussac, states that **a given mass of any gas expands by a constant fraction of its volume at 0° C. when its temperature is raised one degree, provided its pressure is kept constant.** Experimental evidence shows that the value of the fraction is $\frac{1}{273}$ for a rise in temperature of 1° C. This fraction may be termed the **coefficient of expansion of a gas at constant pressure.**

Let v_0 = the volume of the given mass of gas at 0° C.

v = „ „ „ „ „ „ t° C.

Then Increase in volume for a rise in temp. of 1° C. $= \frac{v_0}{273}$.

Hence
$$v = v_0 + \frac{v_0}{273}t$$
$$= v_0\left(1 + \frac{t}{273}\right). \quad \text{.....(1)}$$

This equation is not suitable for use in calculations, since the given initial volume would usually be at some temperature other than 0° C., and it would be necessary to evaluate v_0 prior to making any attempt to solve the problem.

Absolute scale of temperature.—Fig. 397 shows a graph (not drawn to scale) illustrating Charles's law in which volumes are plotted as ordinates and temperatures as abscissae. OA represents the volume

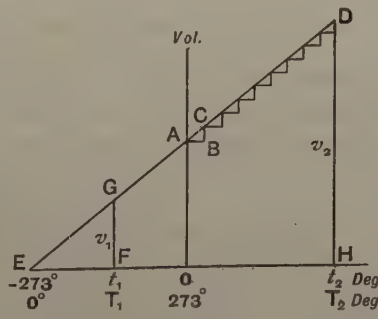

FIG. 397.—Graph illustrating Charles's law.

at 0° C.; abscissae measured to the right of O represent temperatures above freezing point, those measured to the left of O are negative and represent temperature below freezing point. AB represents a rise in temperature from 0° to 1° C., and BC represents the corresponding increase in volume. Other similar steps are shown in Fig. 397, and since the rises in temperature are all 1° C. and the increments in volume are all equal, it follows that a straight line CD may be drawn through the tops of all the steps. CD is therefore a graph illustrating the expansion of a gas obeying Charles's law.

Each of the increments, such as BC, is equal to OA/273. Produce DC until it cuts the temperature axis at E. Then, from the similar triangles CBA and AOE,

$$\frac{CB}{BA}=\frac{AO}{OE},$$

or, since BA = 1,

$$\frac{AO}{273}=\frac{AO}{OE};$$

$$\therefore\ OE=273.$$

Therefore DC cuts the temperature axis at −273° C. At E the volume of the gas as shown in the diagram is zero. This would not actually be the case, but we may say that the volume and temperature of a gas at constant pressure behave in such a manner that, if the law connecting them did not change, the volume would disappear at −273° C.

A scale of temperatures having very useful properties may be illustrated in Fig. 397 by placing 0° at E and 273° at O; other temperatures may be marked along the temperature axis to correspond with the new marking of E and O. This scale is called **the gas thermometer scale of temperatures**, and is practically the same as a scale derived from considerations of energy, called the **absolute scale of temperature** (Chap. XXXVIII.). Temperatures on the absolute scale will be denoted by the letter T.

Another statement of Charles's law.—In Fig. 397 take any two points F and H on the temperature axis, and draw the ordinates FG and HD. $FG=v_1$ is the volume of the gas at an absolute temperature $EF=T_1$; $HD=v_2$ is the volume when the absolute temperature is equal to $EH=T_2$. From the similar triangles GFE and DHE, we have

$$\frac{GF}{FE}=\frac{DH}{HE},$$

or $$\frac{v_1}{T_1}=\frac{v_2}{T_2};$$

$$\therefore \frac{v_1}{v_2}=\frac{T_1}{T_2}. \quad \text{.....(2)}$$

Hence Charles's law leads to the following statement: **The volumes of a given mass of a gas at constant pressure are proportional to the absolute temperatures.**

Temperatures stated in the Centigrade scale may be converted to the absolute scale by adding 273. Thus:

$$T=t^\circ \text{ C.}+273.$$

Absolute zero on the Fahrenheit scale is $(273\times\frac{9}{5})=491^\circ$ F. below freezing point, or $(491-32)=459^\circ$ F. below zero Fahrenheit. Hence temperatures stated in the Fahrenheit scale may be converted to the appropriate absolute scale by adding 459. Thus

$$T=t^\circ \text{ F.}+459.$$

EXAMPLE.—A room measures 50 feet × 30 feet × 25 feet. If the temperature of the air in it be raised from 10° C. to 15° C., what percentage of the initial volume of air will be expelled? The pressure is assumed to remain constant.

$v_1=50\times30\times25=37{,}500$ cubic feet.

$T_1=273+10=283^\circ$ absolute (C.).

$T_2=273+15=288^\circ$ absolute (C.).

Let v_2 be the volume of the air originally in the room if heated to 15° C., then

$$v_2=\frac{v_1T_2}{T_1}=\frac{37500\times288}{283}$$

$$=38{,}160 \text{ cubic feet.}$$

$$\text{Volume of air expelled}=38160-37500$$

$$=660 \text{ cubic feet.}$$

This volume of 660 cubic feet has been measured at 15° C., hence

$$\text{Percentage of air expelled}=\frac{660}{38160}\times100$$

$$=\underline{1{\cdot}73}.$$

Isothermal lines of a gas.—Suppose we have a given mass of gas under initial conditions of pressure and volume, p_1 and v_1, and at a temperature of 273° absolute (C.). In Fig. 398 ordinates and abscissae represent pressure and volume respectively, and the initial conditions are plotted at A. If the gas expands following Boyle's law, the isothermal line AB results. The temperature

everywhere on AB is 273° absolute. Suppose, when the gas is under the conditions represented by the point A, that its temperature be raised to T° absolute (C.) at constant pressure. Applying Charles's law, we have for the new volume v_2,

$$\frac{v_2}{v_1}=\frac{T}{273}; \quad \therefore\ v_2=\frac{T}{273}v_1.$$

Plotting the new conditions, viz. pressure p_1 and volume v_2, we obtain the point C. Now let the gas expand at constant temperature

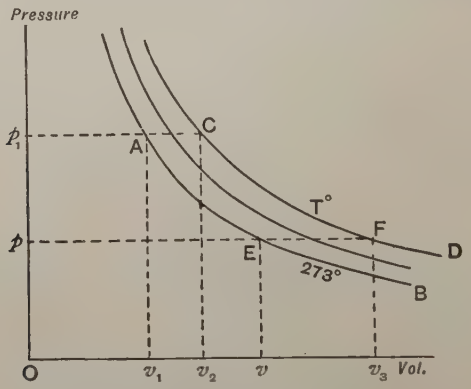

FIG. 398.—Isothermal lines of a gas.

T. Boyle's law will be followed, and the isothermal line for the temperature T will be CD. As many points as may be necessary for plotting CD may be found from points such as E on AB. Thus, at E the pressure is p and the volume is v; the temperature at E is 273° and at F is T degrees; the volume at F is v_3.

$$\frac{v_3}{v}=\frac{T}{273}; \quad \therefore\ v_3=\frac{T}{273}v.$$

The factor T/273 for converting horizontally any point on AB to the corresponding point on CD is a constant. In this way a series of isothermal lines may be drawn for 273, 274, 275, etc., degrees absolute; some of these are shown in Fig. 398.

In Fig. 399 ordinates and abscissae represent volumes and absolute temperatures respectively. The point A represents a given mass of gas at 273° absolute (C.), and having a volume of, say, 4 cubic units and a pressure of, say, 76 cm. of mercury. AOC represents changes going on in obedience to Charles's law, *i.e.* the pressure at any point in this line is 76 cm. of mercury, and the line may be called a line of constant pressure.

Let the conditions indicated at A be changed at constant temperature 273° absolute to 3 cubic units, say. The pressure p_2 will be obtained by applying Boyle's law, thus

$$p_1v_1=p_2v_2\,;\quad \therefore\ 76\times 4=p_2\times 3\,;$$
$$\therefore\ p_2=\tfrac{4}{3}\times 76=101{\cdot}33 \text{ cm. of mercury.}$$

The point E will represent these new conditions. OEH now represents changes in obedience to Charles's law under a constant pressure of 101·33 cm. of mercury. Similarly, by reducing the initial volume to 2 and 1 cubic unit, the constant pressure lines OFK and OGL are obtained.

The change in volume taking place in the given mass of gas when raised in temperature from 273° to 373° at constant pressure 76 cm. of mercury is represented by (DC – BA). The corresponding change in volume at a pressure of 304 cm. of mercury is represented by (DL – BG).

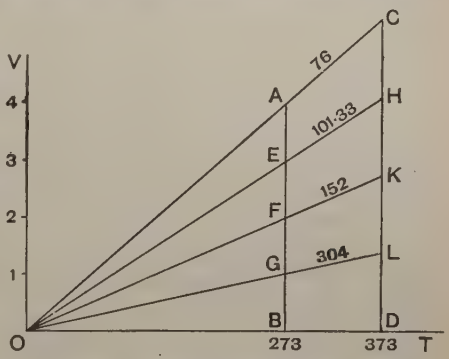

FIG. 399.—Constant pressure lines of a gas.

It is clear, by inspection of Fig. 399, that the change of volume for the given rise in temperature is much greater if the pressure is low than if the pressure is high.

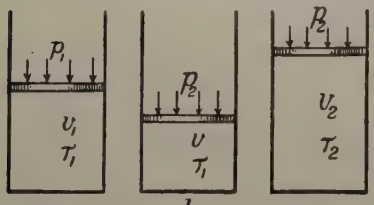

FIG. 400.—Diagram showing changes occurring in the pressure, volume and temperature of a gas.

Combination of Boyle's and Charles's laws.—If the given mass of gas is undergoing simultaneous changes in pressure, volume and temperature, the law followed may be found by the following process. In Fig. 400 (*a*) a given mass of gas is enclosed in a cylinder fitted with a piston, and is under initial conditions p_1, v_1 and T_1. Suppose, first, that p_1 and v_1 are changed at constant temperature T_1 until a pressure p_2 and a volume v are obtained. Applying Boyle's law, we have

$$p_1v_1=p_2v\,;\quad \therefore\ v=\frac{p_1v_1}{p_2}. \qquad \text{.....(1)}$$

The gas will now be as shown in Fig. 400 (*b*).

Now change the temperature from T_1 to T_2, keeping the pressure constant at p_2. The volume will change from v to v_2 as shown in Fig. 400 (*c*). Applying Charles's law,

$$\frac{v}{v_2}=\frac{T_1}{T_2}; \quad \therefore v=\frac{v_2 T}{T_2}. \qquad \text{.....(2)}$$

Hence, from (1) and (2), we have

$$\frac{p_1 v_1}{p_2}=\frac{v_2 T}{T_2},$$

or

$$\frac{p_1 v_1}{p_2 v_2}=\frac{T_1}{T_2}. \qquad \text{.....(3)}$$

We should have obtained the same result had the conditions varied simultaneously instead of by the step by step process adopted above. The result obtained in equation (3) indicates that when a given mass of a perfect gas is undergoing changes in pressure, volume and temperature, **the product of the absolute pressure and volume is proportional to the absolute temperature,** or

$$pv \propto T. \qquad \text{.....(4)}$$

The **characteristic equation for a perfect gas** is obtained from (4) by taking v as the volume occupied by unit mass of the given gas at freezing temperature ($T=273°$ absolute) and under standard pressure of 76 cm. of mercury. By introducing a coefficient R, we have

$$pv=RT, \qquad \text{.....(5)}$$

in which R has a value which depends upon the kind of gas considered.

Relation of pressure and temperature at constant volume in a perfect gas.—Take equation (3) above, and put $v_1=v_2$, thus causing the pressure and temperature of the gas to vary at constant volume. Thus

$$\frac{p_1 v_1}{p_2 v_1}=\frac{T_1}{T_2},$$

or

$$\frac{p_1}{p_2}=\frac{T_1}{T_2}. \qquad \text{.....(1)}$$

Hence, if we have a closed vessel containing a given mass of perfect gas at constant volume, **the absolute pressure is proportional to the absolute temperature.**

Suppose that p_1 is the pressure of the given mass of gas at 0° C.

or 273° absolute, and that the temperature be raised at constant volume to 274° absolute. We have

$$\frac{p_1}{p_2}=\frac{273}{274};$$

$$\therefore\ p_2=\frac{274}{273}p_1=p_1+\frac{1}{273}p_1.$$

The increase in pressure per degree Centigrade is therefore $\frac{1}{273}$ of the pressure at 0° C. Similarly, if the temperature be raised to t° C. above freezing point, the pressure p is given by

$$\frac{p_1}{p}=\frac{273}{273+t},$$

or

$$p=\left(\frac{273+t}{273}\right)p_1=p_1+\frac{t}{273}p_1.$$

The fraction $\frac{1}{273}$ may be called **the coefficient of increase of pressure of a gas at constant volume.** It is evident that the value of this coefficient is the same as the coefficient of expansion of a gas at constant pressure.

Verification of Charles's law.—Direct experiments for the purpose of verifying Charles's law are difficult to carry out accurately. The relation of pressure and temperature at constant volume enables Charles's law to be proved by means of an experiment which may be carried out easily.

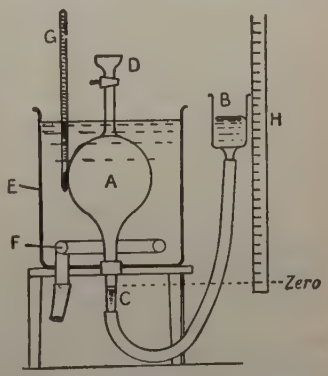

FIG. 401.—Apparatus for determining the relation of p and T.

EXPT. 91.—**Relation of pressure and temperature of air at constant volume; indirect proof of Charles's law.** Arrange apparatus as shown in Fig. 401. A is a large bulb containing dry air and connected by a long rubber tube to an open cistern of mercury at B. The bulb is immersed in water contained in a vessel E; the water may be heated by a coiled pipe F through which steam can be passed; the temperature is measured by a thermometer G placed close to the bulb. The mercury cistern may be moved vertically along a scale H and may be clamped at any height in the same way as in the Boyle's law apparatus (p. 397). The zero of the scale H is arranged so as to be at the same level as a mark C

on the neck of the bulb; this may be checked by use of the U tube mercury level described on p. 397.

Begin with cold water in E. Adjust the height of B so that the mercury level in the neck of the bulb coincides with the mark C. Wait a minute or two to make sure that the temperature of the air in the bulb is the same as that of the water; readjust the level if necessary. Note the height of the mercury level in B, say h cm., and the temperature of the water, say $t°$ C. Read the barometer; let this be h_0 cm. of mercury.

Raise the temperature of the water, say 5° C.; this will cause the mercury level at C to be depressed. Restore the level by raising B, taking the same precautions as before prior to reading the scale H and the thermometer G. Repeat the experiment several times, increasing the temperature each time by about 5° C.

If the law $p \propto T$ is being complied with, we have

$$p = cT, \quad \text{or} \quad c = \frac{p}{T},$$

where c is a constant. Try if this is so by dividing the experimental values of the absolute pressure $p = (h + h_0)$ by $T = t + 273$. Plot a graph showing the relation of p and T.

The results may be tabulated as follows:

EXPERIMENT ON THE RELATION OF p AND T FOR AIR AT CONSTANT VOLUME.

Barometer reading $= (h_0)$ cm.

Temperature.		Height of mercury. h cm.	Pressure in bulb. $p = (h_0 + h)$ cm.	$c = \frac{h+h}{T}$.
$t°$ C.	T° abs.			

Deduce from the graph the pressures at 0° and 100° C., and calculate the coefficient of increase of pressure.

Air thermometer.—In Fig. 402 is shown a modified form of the apparatus used in the above experiment. The bulb A contains dry air, and is connected by a tube of fine bore to tubes BD and EC, which may be raised or lowered along a scale. This instrument constitutes an **air thermometer**. The bulb A is introduced into the space in which the temperature has to be measured, and the sliding tubes are adjusted until the mercury level is restored to a fixed mark at B. The absolute pressure in the bulb is determined in the manner explained in Expt. 91, and the temperature is calculated from the

pressure. The instrument may be calibrated by immersing the bulb A first in melting ice and then in the steam given off by water boiling under standard pressure, taking the same precautions as in the calibration of mercurial thermometers (p. 316). The pressures corresponding to 0° and 100° C. are thus found, and intermediate graduations may be determined by permitting the bath of hot water to cool slowly and taking readings of the pressure at intervals. Used in the manner described, the instrument is a **constant volume** air thermometer.

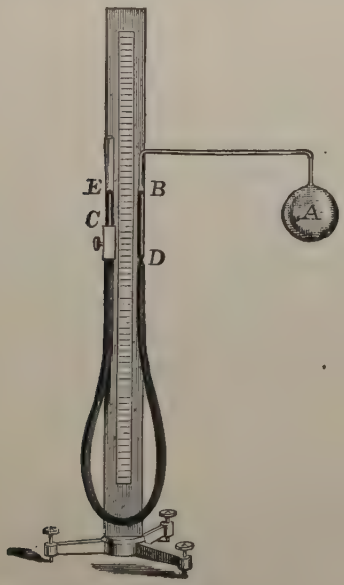

FIG. 402.—Constant volume air thermometer.

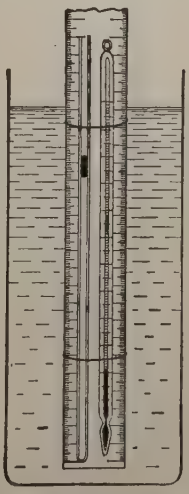

FIG. 403.—Constant pressure air thermometer.

A simple form of **constant pressure** air thermometer is shown in Fig. 403, and consists of a piece of thermometer tubing about 1 mm. in bore and 20 cm. long. The tube is carefully cleaned and dried, and one end is sealed. By slightly warming the tube, some air is expelled, and a pellet of mercury is sucked in as the tube cools. When placed vertically with the closed end at the bottom, the mercury pellet should be about 14 cm. from the lower end at ordinary atmospheric temperature. Since the upper surface of the pellet of mercury is exposed to the pressure of the atmosphere, the pressure of the enclosed air remains practically constant during an experiment. The tube is tied to a thermometer, and the scale of the thermometer

serves to indicate the position of the pellet. Assuming that the bore of the air thermometer tube is uniform, the length occupied by the enclosed air is proportional to the volume. The scale readings corresponding to 0° and 100° C. may be found as before. These scale readings correspond to 273° and 373° absolute (C.) respectively, and intermediate scale readings, obtained when the instrument is at other temperatures, are converted into absolute degrees by simple proportion. This type of constant pressure air thermometer is not very satisfactory on account of the sticking of the mercury piston.

Mixture of two different gases.—If a closed vessel contains a mixture of two different gases which have no chemical action on each other, then **the total pressure is the sum of the pressures which the quantity present of each gas would exert if it alone occupied the vessel.** The proof is as follows :

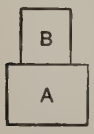

FIG. 404.

Suppose at first that the two gases occupy separate vessels A and B (Fig. 404). Let the pressure and volume of the gas in A be p_1 and v_1, and of that in B p_2 and v_2. The whole is supposed to be at the same temperature, which is preserved constant throughout the following operations.

Let the capacity of B be changed until the pressure in this vessel is p_1, and let the volume be now v. Applying Boyle's law, we have

$$p_2v_2=p_1v, \quad \text{or} \quad v=\frac{p_2v_2}{p_1}. \qquad \text{.....(1)}$$

Now let a hole be made in the partition separating A and B ; both gases being at the same pressure, quiet mingling will ensue, on the assumption that no chemical action occurs, and the mixed gases will exert a pressure p_1, and will occupy a total volume (v_1+v).

Let the portion B now resume its original volume, when the total volume occupied by the mixed gases will be (v_1+v_2), and the pressure will be p, say. Again applying Boyle's law, we have

$$p_1(v_1+v)=p(v_1+v_2) ;$$

$$\therefore\ p=\left(\frac{v_1+v}{v_1+v_2}\right)p_1=\frac{p_1v_1}{v_1+v_2}+\frac{p_1v}{v_1+v_2}.$$

Substituting for v from (1), we obtain :

$$p=\left(\frac{p_1v_1}{v_1+v_2}\right)+\left(\frac{p_1}{v_1+v_2}\times\frac{p_2v_2}{p_1}\right)$$

$$=\frac{p_1v_1}{v_1+v_2}+\frac{p_2v_2}{v_1+v_2}. \qquad \text{.....(2)}$$

Had the gas originally in A alone filled the volume (v_1+v_2), the pressure would have been p', say, and

$$p'(v_1+v_2)=p_1v_1,$$

or
$$p'=\frac{p_1v_1}{v_1+v_2}. \quad \text{.....(3)}$$

Similarly, had the gas originally in B occupied the final volume (v_1+v_2), the pressure would have been p'', say, and

$$p''(v_1+v_2)=p_2v_2,$$

or
$$p''=\frac{p_2v_2}{v_1+v_2}. \quad \text{.....(4)}$$

Hence, from (2), (3) and (4),

$$p=p'+p'', \quad \text{.....(5)}$$

thus proving the proposition.

Density of a gas.—The density of a gas at given temperature and pressure is the mass in grams per cubic centimetre. Let the initial conditions of a given mass of gas be p_1, v_1 and T_1; let the initial density be d_1, and let the final conditions be p_2, v_2, T_2 and d_2. Let m be the constant mass of the gas, then

$$m=v_1d_1=v_2d_2;$$

$$\therefore \frac{d_1}{d_2}=\frac{v_2}{v_1}. \quad \text{.....(1)}$$

Also
$$\frac{p_1v_1}{T_1}=\frac{p_2v_2}{T_2};$$

$$\therefore \frac{v_2}{v_1}=\frac{p_1T_2}{p_2T_1}.$$

Hence, from (1),
$$\frac{d_1}{d_2}=\frac{p_1T_2}{p_2T_1}. \quad \text{.....(2)}$$

If the temperature is constant, this reduces to

$$\frac{d_1}{d_2}=\frac{p_1}{p_2}. \quad \text{.....(3)}$$

If the pressure is constant, the result becomes

$$\frac{d_1}{d_2}=\frac{T_2}{T_1}. \quad \text{.....(4)}$$

If d_0 is the density of the gas at normal pressure and temperature of 76 cm. of mercury and 0° Centigrade, the density d at any other

pressure p cm. of mercury and temperature t degrees Centigrade is obtained from (2), giving

$$\frac{d_0}{d} = \frac{76(t+273)}{273p},$$

or

$$d = \frac{273p\,d_0}{76(t+273)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Expt. 92.—**Density of air.** Reference is made to Fig. 405, in which A is a glass globe furnished with a tap. The globe is connected by means of a piece of thick rubber tube to a manometer BC. The manometer contains mercury, and the closed space above C is a Torricellian vacuum; hence the gaseous pressure acting on the mercury surface at B is given by the difference in levels of the mercury in the two limbs of the manometer,

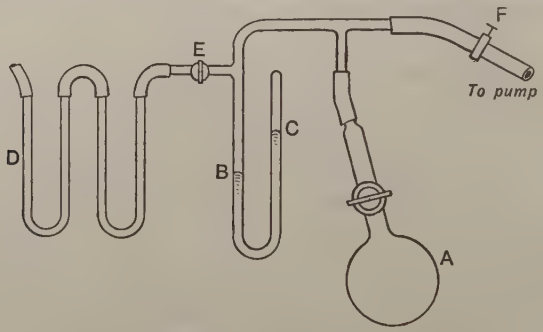

Fig. 405.—Apparatus for determining the density of air.

provided that the closed limb at C is not completely filled with mercury. There is another connection, having a tap at E, and leading to two drying tubes containing phosphorus pentoxide; this substance removes any moisture from the air before it enters the globe. The tube at D is open to the atmosphere. An exhausting air pump is connected to the apparatus, the connecting rubber tube being furnished with a clip at F; air may thus be withdrawn from the globe A.

Close the tap E and exhaust the globe; close the clip F and open E, thus permitting air to flow through the drying tubes and thence into the globe. Repeat the operation several times so as to ensure that the globe contains dry air only. Since the globe is in communication with the atmosphere, the pressure and temperature of the air it contains may be obtained by reading the barometer and a thermometer placed near the globe A. Close the tap on the globe; remove the globe and weigh it, thus obtaining the mass of the globe when full of dry air.

Connect the globe again to the apparatus and open its tap. Close the

tap E and exhaust the globe as thoroughly as possible. The pressure of any residual air is now shown by the manometer reading; it is best to read the levels at B and C by means of a cathetometer. Close the tap on the globe; remove the globe and weigh it, thus obtaining the mass of the globe when filled with residual air. Place the neck of the globe under water at the temperature of the room and open the tap, thus filling the globe with water; see that the water level reaches the tap, adding some water if necessary. Weigh again, thus obtaining the weight of the globe when full of water.

Let w_1 = the weight of the globe in grams when full of dry air at h_a and t.

w_2 = the weight of the globe in grams when full of residual air at h_m and t.

w_3 = the weight of the globe in grams when full of water.

h_a = the barometric pressure, cm. of mercury.

h_m = the pressure of the residual air, as shown by the manometer, cm. of mercury.

t = the constant temperature of the room, deg. Cent.

Since the temperature of the room has remained constant, the fraction $h_m\ h_a$ of the air originally in the globe remains after exhaustion. The difference in weights $(w_1 - w_2)$ therefore represents the weight of the air withdrawn, viz. $(1 - h_m/h_a)$ of the original quantity of air. Therefore the weight w of dry air originally in the globe is given by

$$\frac{w}{w_1 - w_2} = \frac{1}{1 - \frac{h_m}{h_a}} = \frac{h_a}{h_a - h_m};$$

$$\therefore\ w = \left(\frac{w_1 - w_2}{h_a - h_m}\right) h_a \text{ grams.} \quad \text{.....(1)}$$

Further, $(w_3 - w_1 + w)$ gives the weight in grams of the water which fills the globe, and the same quantity gives the volume of the globe in cubic centimetres. Hence, at the pressure h_a and temperature t,

$$\text{Density of air} = d = \frac{(w_1 - w_2)h_a}{(w_3 - w_1 + w)(h_a - h_m)}$$

$$= \frac{(w_1 - w_2)h_a}{(w_3 - w_2)h_a - (w_3 - w_1)h_m} \text{ grams per c.c. ...(2)}$$

The density at normal pressure and temperature may now be obtained from (5) (p. 412).

$$d_0 = \frac{76(t + 273)}{273 h_a} d$$

$$= \frac{76(t + 273)}{273} \cdot \frac{(w_1 - w_2)}{(w_3 - w_2)h_a - (w_3 - w_1)h_m} \text{ grams per c.c. .....(3)}$$

Influence of height on the pressure and density of the atmosphere.—It is well known that the pressure of the atmosphere decreases as

the height above sea-level is increased. If a barometer be taken up a mountain, the level of the mercury falls gradually during the journey. Water boils at a lower temperature at the top of a mountain than at sea-level, owing to the diminished pressure of the atmosphere. The following approximate method enables the law governing these changes to be understood.

Consider a column of air (Fig. 406) having a cross-sectional area of one square centimetre and reaching upwards to the limit of the atmosphere. Let the temperature throughout the column be 0° C., and let the pressure at sea-level A be 76 cm. of mercury. This pressure is produced by the total weight of the column, and is equivalent to 1033 grams weight per square centimetre nearly. The density of air at this pressure and 0° C. is 1·2928 grams per 1000 c.c.; assuming that the density does not change appreciably on ascending a few metres, the height of a column AB weighing one gram will be $10 \div 1{\cdot}2928 = 7{\cdot}735$ metres. The pressure at B is less than that at A by the weight of AB, and is therefore 1032 grams weight per square centimetre.

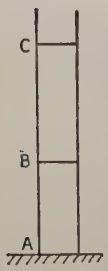

FIG. 406.

The density at this pressure and at 0° C. is given by

$$\frac{d}{1{\cdot}2928} = \frac{1032}{1033} \quad \text{(p. 411)},$$

$$d = 1{\cdot}2915 \text{ grams per 1000 c.c.}$$

Assuming that this density remains constant for another few metres above B, the height of a column BC weighing one gram is $10 \div 1{\cdot}2915 = 7{\cdot}743$ metres nearly. The pressure at C will then be 1031 grams weight per square centimetre. BC is greater than AB, and we conclude that the intervals to produce a constant pressure difference of one gram weight per square centimetre increase as we ascend.

Increase in temperature increases the heights AB and BC in Fig. 406; the effect of the change in temperature is to increase the volume, and thus to diminish the density.

Heights may be measured approximately by means of the barometer. An elevation of 900 feet corresponds nearly to one inch fall in the mercury column. The aneroid barometer (p. 392) is used generally by surveyors, and has two scales on its dial, one indicating the height of the mercury column and the other showing elevations in feet. The method is approximate only, since, for stability in the atmosphere, the temperature must fall as the altitude increases.

Flotation of balloons.—If a balloon is floating at rest in a still atmosphere, the law governing its equilibrium is the same as that for a body floating at rest in a still liquid (p. 274). Thus, in Fig. 407, W_1 is the weight of the balloon, including all material and the car; W_2 is the weight of the gas contained in the envelope; W_3 is the weight of air displaced by the balloon, and is equal to the buoyancy, *i.e.* the air surrounding the balloon exerts a resultant force equal to W_3. For equilibrium

$$W_1 + W_2 = W_3,$$

or

$$W_1 = W_3 - W_2.$$

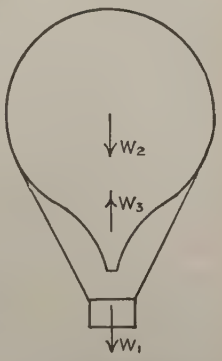

FIG. 407.—Flotation of a balloon.

It is evident that the total weight which can be raised, viz. W_1, can be increased by increasing the difference between W_3 and W_2. For a given volume of envelope, and at normal pressure and temperature, W_3 is a definite quantity, and the difference $(W_3 - W_2)$ can be increased only by choosing as light a gas as possible for filling the envelope. Hydrogen is the gas generally employed. At 0° C. and 1 atmosphere pressure, hydrogen weighs 0·00559 lb. per cubic foot, and air under like conditions weighs 0·0807 lb. Hence, at normal pressure and temperature, the weight which can be raised per cubic foot of balloon is (0·0807 – 0·00559), or 0·07511 lb. weight.

The best lifting effect would, of course, be obtained by having a vacuum inside the envelope, but this is impossible owing to the collapsing pressure of the atmosphere; an envelope strong enough to withstand this pressure would be much too heavy to be lifted. It is best to have the pressure of the internal gas equal, or nearly equal, to that of the external air, thus calling for but little strength in the material of the envelope, and enabling a very light envelope to be employed.

A balloon will ascend if $(W_1 + W_2)$ is less than W_3 (Fig. 407); since the density of the air diminishes with height of ascent, a height will be attained at which W_3 becomes equal to $(W_1 + W_2)$, and the balloon will remain at this elevation. But the pressure of the air has also decreased (p. 414); hence, if the pressure of the contained gas has not altered during the ascent, there will be a tendency to burst the envelope. This tendency may be reduced by permitting some of the gas to escape through a valve at the top of the balloon.

In modern air-ships the internal pressure can be adjusted by means of **ballonets**. These are small balloons placed inside the envelope and containing air. Air can be pumped into or withdrawn from the ballonets, which will then occupy a greater or lesser volume, and will cause the light gas in the balloon to occupy a lesser or greater volume, and thus to exert a greater or smaller pressure. The device enables the envelope to be maintained at practically constant tension without the necessity for permitting any of the light gas to escape from the balloon.

Ordinary balloons carry ballast in the form of bags of sand. If greater elevation is required, some of the sand is scattered overboard, thus diminishing the total weight to be carried. Air-ships having engines and propellers for propulsion may secure greater elevation by the use of rudders pivoted on horizontal axes. The use of these introduces greater resistance to the passage of the vessel through the air, and leads to a decrease in speed if the greater elevation is maintained by use of the rudders.

Exercises on Chapter XXXI.

1. State Charles's law for perfect gases. Define absolute zero of temperature by reference to the contraction of a perfect gas. What temperature on the absolute scale (Cent.) corresponds to 20° on the ordinary Fahrenheit scale ?

2. A chimney is 120 feet high and is 3 feet in internal diameter. The average temperature of the gases inside the chimney is 280° C. What volume would the contents of the chimney occupy if the temperature were reduced to 15° C. without change in pressure ?

3. A glass bulb having a fine stem weighs 19·24 grams when empty and 74·85 grams when full of water. When full of air, the bulb was placed for a few minutes in a hot oven at atmospheric pressure and the stem was sealed. The bulb was then immersed in a bath of water at 15° C., stem downwards and the stem was unsealed. It was found, on adjusting the pressure to that of the atmosphere, that 35·68 grams of water had entered the bulb. Find the temperature of the oven.

4. Draw the isothermal line for one cubic foot of air at 15° C. and absolute pressure 10 atmospheres when it expands to 5 cubic feet. On the same drawing, show the isothermal line for the same mass of gas at 50° C.

5. Assuming the truth of the laws of Boyle and Charles for perfect gases, prove the law $pv = \mathrm{R}T$.

6. Find the values of R in the equation $pv = \mathrm{R}T$ for air and hydrogen, given that the mass of one cubic foot of air at 0° C. and 14 7 lb. wt. per square inch is 0·0807 pound, and one cubic foot of hydrogen under the same conditions has a mass of 0·00559 pound.

7. In the c.g.s. system, 1000 c.c. of air at 0° C. has a mass of 1·2928 grams, under a pressure of $1{\cdot}0132 \times 10^6$ dynes per sq. cm. Find the value of R in the equation $pv = \mathrm{R}T$.

8. A cylinder fitted with a piston contains at a certain instant 6 cubic feet of gas at 15 lb. wt. per square inch absolute and 20° C. The weight of the gas in the cylinder is kept constant, and the piston is pushed in. At another instant, the pressure is found to be 150 lb. wt. per square inch absolute and the volume 2·5 cubic feet. Calculate the temperature at this instant.

9. Given that one gram of hydrogen at a temperature of 0° C. and a pressure of 76 cm. of mercury occupied a volume of 11·16 litres, what volume will be occupied by 3·85 grams at a temperature of 23° C. and a pressure of 74·6 cm. ? Adelaide Univ.

10. How would you determine the coefficient of expansion of a gas at constant pressure ? What is meant by the absolute zero of the air thermometer, and how is it calculated ? Sen. Cam. Loc.

11. A chimney is 50 metres in height ; a water pressure gauge (p. 395) connected to the base of the chimney indicates 2 cm. of water. The temperature of the atmosphere is 0° C., and the barometer reads 76 cm. (density of air under these conditions = 1·293 grams per litre). Find the average temperature of the gases in the chimney. The density of mercury = 13·6 grams per c.c.

12. State the two fundamental laws of change of pressure, volume, and temperature of gases, and show that they may be expressed in the form of a single equation containing one constant. What is the value of the constant in the case of hydrogen if the mass of one litre at 0° C. and 760 mm. pressure is 0·0896 gram ? (Density of mercury = 13·6.) L.U.

13. The density of oxygen at 0° C. under a pressure of 760 mm. of mercury is 1·429 grams per litre. A certain mass of the gas is enclosed in a cylinder whose volume is 2·5 litres, under a pressure of 780 mm. at a temperature of 12° C. What is the mass of gas in the cylinder ? L.U.

14. Calculate the weight of dry air in a room 30 × 18 × 15 feet in dimensions, the barometer reading 752 mm., the thermometer 30° C. (The weight of one cubic foot of air at 760 mm. pressure and 0° C. is 0·0801 lb.) Explain also the theory of your calculation. Adelaide Univ.

15. Two different gases which have n chemical action on each other are mixed in the same vessel. State and prove the law for the total pressure.

16. Describe how you would find the density of dry air. The results of an experiment are as follows : The weight of the globe when full of dry air was 35·375 grams, and when full of residual air, 35·198 grams, both at 14° C. ; the globe, when full of water, weighed 179·95 grams. The barometer read 77·19 cm., and the pressure of the residual air as shown by the manometer was 1·19 cm. of mercury. Find the density of the air in grams per c.c. at 0° C. and 76 cm. of mercury.

17. An airship 500 feet long has an average diameter of 50 feet and is charged with hydrogen. If the total weight of the structure, envelope, engines, etc., is 26 tons, find the total weight of stores (petrol, explosives, etc.) and crew which may be carried.

18. Two vessels, A and B, are connected by a pipe furnished with a tap. The tap is closed, and A and B are charged with air at pressures of 360

and 240 cm. of mercury respectively. The volume of A is 800 c.c., and that of B 600 c.c. If the tap is opened, find the final pressure in each vessel. The temperature is constant throughout.

19. A steel vessel for storing compressed air has a volume of 6 cubic feet, and has a safety valve which opens and permits air to escape when the pressure is 100 lb. wt. per square inch above that of the atmosphere, which may be taken as 15 lb. wt. per square inch absolute. The vessel contains air at a pressure of 110 lb. wt. per square inch absolute and at 15° C. If the vessel be heated, at what temperature will the safety valve open ?

CHAPTER XXXII

KINETIC THEORY OF GASES. WORK DONE BY A GAS

Pressure of gaseous molecules moving in parallel directions.—Let a hollow cube of one centimetre edge, internal dimensions, contain one molecule only. Let the mass of the molecule be m gram, and let it travel constantly in a line ab perpendicular to two opposite faces of the cube (Fig. 408). It is assumed that the velocity u is simply reversed each time the molecule strikes a face of the cube. The change in momentum during each impact is $2mu$ (p. 68), and the time taken to travel from a to b is $1/u$ second. There will therefore be u impacts per second, and the change of momentum per second is $2mu^2$. Half of the total number of impacts take place at one face, and the other half at the opposite face; hence the change of momentum per second at one face is mu^2. This is a measure of the force exerted on the face (p. 68), therefore

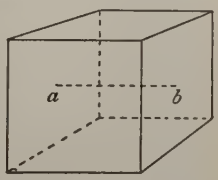

FIG. 408.

$$\text{Force} = mu^2 \text{ dynes.}$$

If the centimetre cube contains n molecules, all moving in paths parallel to ab and having the same velocity u, the total force exerted on one face is nmu^2; this force is distributed over an area of one square centimetre, and is therefore the pressure on one face of the cube. $\therefore$ Pressure $= p = nmu^2$ dynes per sq. cm.

If the molecules have velocities differing in magnitude, and if $\bar{u}^2$ is the mean of the squares of all the speeds, then

$$p = nm\bar{u}^2 \text{ dynes per sq. cm.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Pressure of a gas.—If the cube contains a gas, the molecules are moving in every conceivable direction. Let V (Fig. 409) be the velocity of one molecule, and let OX, OY and OZ be axes parallel to

the edges of the cube in Fig. 408. Resolve V into components along these axes; this may be done by first resolving V into components OM and OK, the latter being in the plane containing OX and OZ. OK is then resolved into velocities OL and ON along OX and OZ respectively. The components are thus u along OX, u_1 along OY and u_2 along OZ. From the geometry of the figure,

$$OA^2 = OK^2 + KA^2 = OL^2 + LK^2 + OM^2$$
$$= OL^2 + ON^2 + OM^2 ;$$
$$\therefore V^2 = u^2 + u_1^2 + u_2^2.$$

The velocities of the other molecules have different directions, but all can be resolved into components parallel to the edges of the cube. If $\bar{u}_1^2$ and $\bar{u}_2^2$ have the same relation to u_1^2 and u_2^2 respectively that $\bar{u}^2$ has to u^2, as explained above, and if $\bar{V}^2$ has the same relation to V^2, *i.e.* $\bar{V}^2$ is the mean of the squares of the actual velocities, then we may write

$$\bar{V}^2 = \bar{u}^2 + \bar{u}_1^2 + \bar{u}_2^2.$$

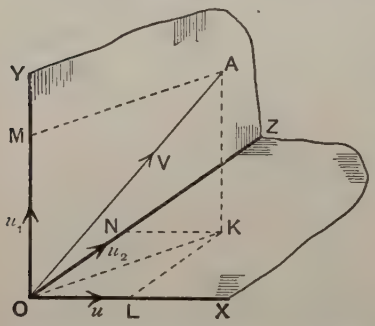

FIG. 409.—Component velocities of a molecule.

Since there is no tendency for molecules to accumulate in any part of the cube, it is reasonable to suppose that the velocities $\bar{u}$, $\bar{u}_1$ and $\bar{u}_2$ are equal; hence

$$\bar{u}^2 = \bar{u}_1^2 = u_2^2 = \tfrac{1}{3}\bar{V}^2. \quad \text{.....(2)}$$

Hence, in the case of a centimetre cube containing n molecules of a gas moving in all directions, we have from (1) and (2)

$$p = \tfrac{1}{3}nm\bar{V}^2 \text{ dynes per sq. cm.} \quad \text{.....(3)}$$

If the cube has a volume v, each cubic centimetre containing n molecules, then

$$pv = \tfrac{1}{3}nmv\bar{V}^2. \quad \text{.....(4)}$$

Now nm is the total mass of the molecules in one cubic centimetre, *i.e.* nm is the density d of the gas, and $d \times v$ is the total mass M in a volume v;

$$\therefore pv = \tfrac{1}{3}M\bar{V}^2. \quad \text{.....(5)}$$

If the cube contains unit mass of gas, M is unity, and

$$pv = \tfrac{1}{3}\bar{V}^2. \quad \text{.....(6)}$$

Some important relations.—It is known from Boyle's law that the product of the pressure and volume of a given mass of gas is constant, provided the temperature is constant, and it has just been shown that this product depends upon $\bar{V}^2$, since M in (5) above is constant. Hence it may be inferred that $\bar{V}^2$ is constant if the temperature of the gas does not alter.

Again, at constant volume, the pressure of a given mass of gas is proportional to the absolute temperature (p. 406). Writing v constant in equation (5) gives the result that p varies as $\bar{V}^2$; hence we may infer that the absolute temperature T and the mean of the squares of the velocities of the molecules are proportional, and any increase in T is accompanied by a corresponding increase in $\bar{V}^2$. Both $\bar{V}^2$ and T should become zero simultaneously, and we may define absolute zero of temperature as the temperature at which the molecules of a gas have been reduced to rest.

If p be constant in (5) above, then v varies as $\bar{V}^2$. From Charles's law we know that in a given mass of gas the volume v is proportional to the absolute temperature T, provided the pressure is constant. Hence again we infer that $\bar{V}^2$ is proportional to T.

Further, the kinetic energy of one molecule having a mass m and velocity V is $\frac{1}{2}mV^2$, and the total kinetic energy of a mass M of gas, in which the mean of the squares of the velocities is $\bar{V}^2$, is $\frac{1}{2}M\bar{V}^2$. It therefore follows, since $\bar{V}^2$ varies as T, that the total kinetic energy of the molecules is proportional to T. Suppose that a given mass of gas is maintained at constant volume, a definite quantity of heat must be added in order to produce a stated rise in temperature, and the result is a definite increase in the total kinetic energy of the molecules. We may therefore infer that the heat added has been converted into kinetic energy, and exists in the gas in the form of molecular motions. Abstraction of heat from the gas would produce a corresponding reduction in the molecular kinetic energy, and the energy would become zero at absolute zero of temperature.

Avogadro's law.—Consider two vessels of equal capacities, one containing a gas A and the other containing a different gas B. Both gases are supposed to be at the same pressure and temperature. It is assumed that single molecules of each gas have equal kinetic energies, on the average, when the temperatures are the same, *i.e.*

$$\tfrac{1}{2}m_A\bar{V}_A^2 = \tfrac{1}{2}m_B\bar{V}_B^2.$$

Since the product of pressure and volume is the same for both vessels, we have, from equation (4), p. 420,

$$\tfrac{1}{3}n_A m_A v\bar{V}_A^2 = \tfrac{1}{3}n_B m_B v\bar{V}_B^2;$$

$$\therefore\ n_A = n_B.$$

This result indicates that **in all perfect gases under the same conditions of pressure and temperature, there is the same number of molecules per cubic centimetre.** This is known as Avogadro's law.

If the gas A has a density greater than that of B, it follows that a molecule of A possesses a mass greater than that of a molecule of B. Hence, if the average kinetic energies of molecules of each gas are equal when the temperatures are the same, it follows that $\bar{v}_A^2$ must be less than $\bar{v}_B^2$. Air has a density 14 times that of hydrogen, and the mean speed of the molecules in hydrogen at normal temperature is 1800 metres per second, while the mean speed of the molecules of air at normal temperature is 450 metres per second.

Internal energy of a gas.—The **internal energy of a gas is the total heat energy stored in unit mass of the gas** by virtue of the motion and position of the molecules. Internal energy should be measured from absolute zero of temperature, but, generally speaking, there are no means available for making this estimation. It is convenient to select some arbitrary temperature—generally 0° C.—and to estimate the internal energy in excess of that possessed by the substance when at this temperature. The motion of the molecules depends upon the temperature only; hence if there is no change in the temperature of a gas undergoing changes of pressure and volume, there is no change in the internal energy of molecular motion.

Joule's experiment.—Joule made an experiment in which two vessels were connected by a pipe furnished with a tap. The tap was closed, and one of the vessels was exhausted as completely as possible, while the other vessel was charged with air. Both vessels were immersed in water and the temperature of the water was noted. On opening the tap, thus permitting the air to expand freely and fill both vessels, the temperature of the water after stirring was found to be the same as at first. It was thus inferred that no drop in temperature occurs when a gas is undergoing unresisted expansion, and hence there is no change in internal energy. This law is very nearly, but not quite true, as will be explained later.

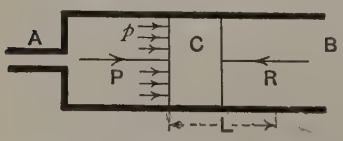

FIG. 410.—Work done by a gas.

Work done by a gas at constant pressure.—In Fig. 410 is shown a cylinder fitted with a piston C. Gas under constant pressure p is admitted at A and pushes the piston through a distance L against a

resistance R. P is the total force exerted by the gas on the piston, and is equal to the product of p and the area of the piston, a say; since P is constant, the work done is given by

$$\text{Work done by P} = \text{PL} = pa\text{L}.$$

Now aL is the volume of gas which must be admitted in order to maintain constant the pressure p, and this is also the volume through which the piston sweeps in travelling a distance L. Writing v for this volume, we have

$$\text{Work done by P} = pv. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This result will be in foot-lb. if, as is customary in engineering practice, p is in lb. wt. per square foot and v in cubic feet, and in centimetre-kilograms if p is in kilograms wt. per square centimetre and v is in cubic centimetres. In C.G.S. units, p is in dynes per square cm. and v in c.c.; the result is then in ergs.

We have obtained pv units of work by admitting v units of volume of gas; had only one unit of volume been admitted, the work done would be given by

$$\text{Work done per unit volume of gas} = \frac{pv}{v} = p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The diagram of work (see Chapter XIII.) done under the above conditions is shown in Fig. 411. AB represents the volume v swept by the piston; $v_1 = \text{OA}$ is the volume of gas present in the cylinder at the instant the piston begins to move, and $v_2 = \text{OB}$ is the volume of gas at the end of the movement. AC = BD represents the constant pressure p of the gas. The area ABDC represents the product pv, and therefore represents the work done by the gas.

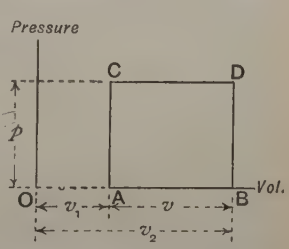

FIG. 411.—Work done by a gas at constant pressure.

Work done during the expansion of a gas.—If the supply of gas to a cylinder is cut off after the piston has moved through any given distance, and if the piston continues to move in the same direction, the pressure exerted by the gas will fall as the volume increases. The constantly diminishing pressure will continue to do work on the piston, but to a less amount in each successive centimetre of movement of the piston. In Fig. 412, $\text{OB} = v_1$ is the volume of gas in the cylinder at the instant that the supply is cut off, and $\text{BA} = p_1$ is the pressure at this instant. The diminishing pressure of the gas during the further movement of the piston is shown by the curve AC.

OD $= v_2$ is the volume to which the gas has expanded when the piston reaches D, the pressure being then p_2, represented by DC.

Consider the instant at which the pressure is EF $= p$ and the volume is OE $= v$. If the piston moves a further very small distance, the additional volume will be EG. Let this additional volume be written δv. The pressure during this small movement will remain sensibly equal to p, and the work done may be calculated from equation (1), p. 423, Work done during the small movement $= p \times \delta v$.

This work is represented by the area of the shaded strip EFHG (Fig. 412). The area of similar strips will represent the work done during other small movements of the piston; hence the total work done during the expansion from the volume v_1 to the volume v_2 is represented by the area of the diagram BACD. If the gas is following Boyle's law by expanding at constant temperature, the curve AC may be plotted, and its area found by means of a planimeter, or by application of any convenient rule of mensuration. If the area is expressed in square centimetres, the result must be multiplied by the scale of pressure, say p kilograms wt. per square centimetre to a centimetre height of the diagram, and also by the scale of volume, say v cubic centimetres to a centimetre length of the diagram. The final result will then give the work done in centimetre-kilograms.

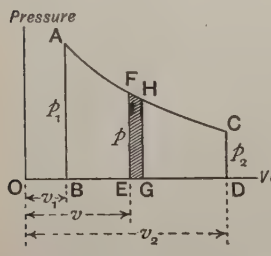

FIG. 412.—Work done by an expanding gas.

Specific heat of a gas at constant volume.—If a given mass of gas be contained in a closed vessel of constant volume, the addition of heat will produce a rise in temperature, and the heat energy supplied will be stored completely by the molecules in the form of additional kinetic energy. No expansion has taken place, and therefore no work has been done against any external resistance. Under these conditions the heat which must be supplied per unit mass of gas in order to raise the temperature one degree is described as **the specific heat of the gas at constant volume**, and is written C_v.

FIG. 413.

Specific heat of a gas at constant pressure.—In Fig. 413 is shown a cylinder fitted with a piston carrying a constant load and capable of travelling freely in the cylinder. The cylinder contains unit mass of gas under the piston, and the gas will be subjected to a constant pressure p_1 Let the initial temperature be

T_1 degrees absolute, and let the volume of the gas under these conditions be v_1.

Let the gas be raised in temperature through one degree by the addition of heat, when two effects will occur: (*a*) The volume will increase to v_2 in accordance with the law

$$\frac{v_1}{v_2}=\frac{T_1}{T_2}=\frac{T_1}{T_1+1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

(*b*) The temperature of the gas has been raised, and therefore its store of internal energy has been increased.

Suppose the operation to take place in the following manner: Imagine a thin partition to be fitted to the cylinder, in contact with the lower side of the piston. This partition will prevent expansion taking place when the piston is raised. Let the piston be raised by application of an external force to such an extent that the total volume under it is v_2. Work will have to be done on the piston to an amount given by the product of p_1, and the volume swept by the piston (p. 423), *i.e.*

$$\text{External work done}=p_1(v_2-v_1). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The space below the partition B (Fig. 414) now contains the gas; that between B and the piston A is a perfect vacuum. Let a hole be pierced in the partition so as to permit free expansion of the gas, which will then fill the whole space under the piston. This expansion will take place without change in temperature (p. 422), and we have now unit mass of gas at volume v_2 and temperature T_1.

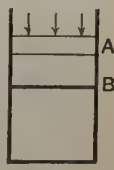

FIG. 414.

Keeping the piston fixed so as to maintain the volume constant, add sufficient heat to raise the temperature one degree. As the volume is constant, the heat required will be C_v, the specific heat at constant volume. Had the entire operation taken place without external assistance, we should have required to supply heat energy sufficient to perform the external work (given in (2) above) in addition to C_v. This additional quantity of heat may be calculated from (2) by dividing by J, the mechanical equivalent of heat. Hence

$$\text{Total heat required}=C_v+\frac{p_1(v_2-v_1)}{J}.$$

This total heat represents the **specific heat of the gas at constant pressure**, C_p say. Hence

$$C_p=C_v+\frac{p_1(v_2-v_1)}{J}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

From (1), $$v_2 = \frac{T_1+1}{T_1} v_1 ;$$

$$\therefore\ v_2 - v_1 = \left(\frac{T_1+1}{T_1}\right) v_1 - v_1 = v_1 \left(\frac{T_1+1}{T_1} - 1\right) = \frac{v_1}{T_1} ;$$

$$\therefore\ C_p = C_v + \frac{p_1 v_1}{T_1} \cdot \frac{1}{J}.$$

Now, $$p_1 v_1 = RT_1 \quad \text{(p. 406)} ;$$

$$\therefore\ C_p = C_v + \frac{RT_1}{T_1} \cdot \frac{1}{J}$$

$$= C_v + \frac{R}{J}. \qquad \text{.....(4)}$$

This equation enables the value of C_p for a given gas to be calculated, provided the other quantities are known. It is of interest to note that Mayer used known values of C_p, C_v and R in order to estimate the value of the mechanical equivalent of heat. He assumed, however, that the internal energy of a gas does not change during free or unresisted expansion, an assumption which was not valid until it had been confirmed by Joule's experiment (p. 422).

Exercises on Chapter XXXII.

1. Give a brief explanation of the reasons for stating that the absolute temperature and the mean of the squares of the velocities of the molecules in a gas are proportional.

2. The total kinetic energy of the molecules of a gas and the absolute temperature are related. State the relation and explain it briefly.

3. Make use of the kinetic theory of gases to explain what is meant by absolute zero of temperature.

4. State Avogadro's law. What deduction can be made regarding the relation of the mean of the squares of the speeds of the molecules of different gases under like conditions of pressure and temperature.

5. What is meant by the "internal energy" of a gas, and how is it measured in practice?

6. Describe Joule's experiment on the free expansion of a gas. State the result and the inference which may be made.

7. Work is done on a piston by air at a constant absolute pressure of 14·7 lb. per sq. inch. How much work is done (*a*) per cubic foot of air, (*b*) per pound of air admitted to the cylinder? Take one pound of air to occupy a volume of 12·5 cubic feet.

8. Draw a diagram of work done during the expansion of 500 c.c. of air at an absolute pressure of 6 kilograms wt. per square centimetre. The final volume is 3000 c.c., and Boyle's law is followed. Scale of pressure,

1 cm. height to 1 kilogram wt. per square cm. ; scale of volume, 1 cm. to 500 c.c. Find the area of the diagram, and hence calculate the work done.

9. In heating a building, 300,000 cubic feet of air at the temperature of 10° C. and 1 atmosphere pressure enter the heating appliance per hour ; the temperature is raised to 16° C., and the pressure remains constant. Calculate the quantity of heat required per hour, given that the specific heat of air at constant pressure is 0·237 and that one cubic foot of air at 0° C. and 1 atmosphere pressure has a mass of 0·0807 pound.

10. In Question 9, calculate the quantity of heat which is used in doing external work while the air is being heated.

11. The specific heat of air at constant pressure is 0·237 and is 1·4 times that at constant volume. Find the heat required to raise the temperature of 100 kilograms of air from 0° to 100° C. at constant volume.

12. Explain why the specific heat of a gas at constant pressure is greater than its specific heat at constant volume. Hence show that $C_p - C_v = p/dJT$, assuming that no internal work is done when a gas expands. (d = the density of the gas at pressure p and absolute temperature T.) Calculate the value of J, taking $C_p = 0{\cdot}238$, $C_p/C_v = 1{\cdot}41$, and the density of air at normal temperature and pressure = 0·001293 gram per c.c. Bombay Univ.

13. The specific heat of hydrogen at constant pressure is 3·402 calories per gram, and the density at 0° C. and 76 cm. of mercury is 0·08987 gram per 1000 c.c. Take J = 42,000,000 ergs, and calculate the specific heat at constant volume. Find also the ratio of the specific heats.

14. Explain the meaning of the term "mechanical equivalent of heat." Which is the greater, the specific heat of a gas at constant pressure, or the specific heat at constant volume ; and why ? L.U.

15. Explain briefly the relation which is supposed to exist, according to the kinetic theory of matter, between the velocity of the molecules of a gas, its temperature, and the pressure it exerts against the walls of a containing vessel. How is it deduced from this theory that two different gases contain the same number of molecules per unit volume at the same temperature and pressure ? Madras Univ.

CHAPTER XXXIII

THE EXPANSION AND COMPRESSION OF GASES IN PRACTICE

Isothermal and adiabatic expansion.—In **isothermal expansion** and **compression** of a gas, the operation takes place without change in the temperature. In **adiabatic expansion** and **compression**, no heat is allowed to enter or leave the gas during the operation. Strictly speaking, both methods of expansion and compression must be **reversible**, *e.g.* if expansion is going on, there may be certain alterations in the conditions of pressure, volume, temperature and internal energy; these conditions must be capable of exact reproduction in the reverse order if expansion be stopped and compression substituted.

Neither isothermal nor adiabatic operations in a gas can be realised perfectly in practice on account of reasons explained below, but they serve as useful standards for comparison with practical cases.

Heat must be supplied during isothermal expansion.—It has been seen that no appreciable change in temperature occurs when a gas is allowed to expand freely (p. 422). In this method of expansion no external work is done, and the internal energy of the gas remains unaltered. External work is done, however, if a gas expands in a cylinder driving a piston which offers resistance. This external work may be done at the expense of some of the internal heat energy of the gas, in which case a fall in the temperature must occur; hence such an operation cannot be isothermal. If isothermal operations are to be secured, there must be no change in the temperature, and therefore no change in the internal energy. Hence, as none of the internal stock of energy is available for doing the external work, it follows that heat must be supplied continuously to the gas in quantity sufficient to perform the external work. The total heat thus supplied during the expansion must be equivalent to the total external work done.

Let W = the total external work done.

Then $\frac{W}{J}$ = the heat which must be supplied.

Practical difficulties in isothermal operations.—The supplying of the requisite quantity of heat during isothermal expansion constitutes a practical difficulty which cannot be overcome perfectly. There is no material known of which the cylinder might be constructed that will permit heat to pass rapidly enough into the gas as expansion proceeds. Further, heat will not flow from the walls of the cylinder into the gas unless the walls are at a temperature higher than that of the gas, and this heat, supplied at the boundary, has to be distributed through the entire volume of gas. A fairly close approximation to isothermal expansion may be obtained by causing the piston to move at a very slow rate, thus giving ample time for heat to enter through the cylinder walls and to distribute itself throughout the gas.

In isothermal compression external work is done upon the gas, and the internal energy will be increased unless heat is abstracted from the gas. Anyone who has used an ordinary tyre inflator has noticed the rise in temperature of the discharge end of the inflator. Isothermal compression is the converse of isothermal expansion, and heat must be removed from the gas in quantity equivalent to the external work done on the gas during compression, *i.e.* to W/J.

Practical difficulties in adiabatic operations.—The principal difficulty which prevents the realisation of adiabatic operations are due to the non-existence of any material which will absolutely prevent any leakage of heat, either inwards or outwards. The cylinder would require to be made of a material having perfect non-conducting qualities, and also having no capacity for heat.

From what has been said regarding isothermal operations, it will be understood that the store of internal energy will diminish during adiabatic expansion, which will therefore be accompanied by a fall in temperature. The converse takes place during adiabatic compression. Internal heat energy to an amount equal to the external work done by the gas disappears from the gas during adiabatic expansion ; during adiabatic compression the internal energy is increased by an amount equal to the external work done on the gas.

In a cylinder made of any ordinary metal, the quantity of heat which can flow into, or through the walls, depends upon the time allowed for the flow to take place. An approximation to adiabatic expansion or compression can be obtained by conducting the operation very quickly ; but little heat will then enter or escape. The compressions and expansions which take place in sound waves occur so rapidly that the changes are adiabatic.

Laws of expansion.—In isothermal operations performed on a perfect gas, Boyle's law is followed, viz.

$$pv = \text{a constant.}$$

In adiabatic operations it may be shown that the law followed is

$$pv^{\gamma} = \text{a constant,}$$

where γ is the ratio of the two specific heats of the gas, viz. $C_p \div C_v$.

In practice the law followed is of the form

$$pv^{n} = \text{a constant,}$$

where the index n generally falls between the values 1 (the index for isothermal operations) and γ. The value of γ varies from 1·67 for the monatomic gases such as argon, mercury vapour, etc., to nearly 1 for gases having highly complex molecules.

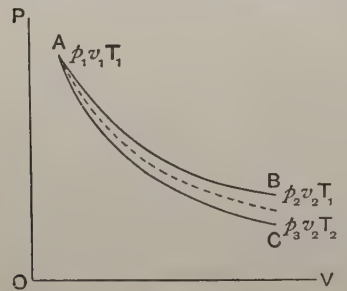

FIG. 415.—Expansion curves for a gas.

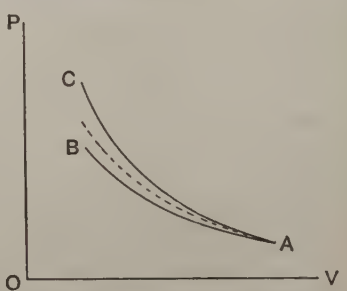

FIG. 416.—Compression curves for a gas.

In the pressure-volume diagram shown in Fig. 415 a given mass of gas has been taken under initial conditions p_1, v_1, T_1, and the point A has been plotted to represent these conditions. The curve AB represents isothermal expansion at constant temperature T_1, ending at B where the conditions are p_2, v_2 and T_1. AC represents adiabatic expansion ; this curve falls below AB, since the temperature is falling continuously, and therefore the pressure will be lower at any volume corresponding to a selected point on AB. The terminal conditions at C are p_3, v_2 and T_2. The actual expansion curve obtained in any practical cylinder would generally fall between AB and AC, as is shown dotted in Fig. 415.

In compression operations (Fig. 416) and starting at A with given conditions of pressure, volume and temperature, isothermal compression is represented by AB and adiabatic compression by AC.

Since the temperature is rising along AC, it follows that the pressure at corresponding volumes will be higher than in isothermal compression; hence AC lies above AB. The practical compression curve would generally fall between AB and AC, as is shown by the dotted line in Fig. 416.

Air-exhausting pumps.—Pumps of this type are used for withdrawing air or other gases from closed vessels. A laboratory pump is shown in Fig. 417. The cylinder A is fitted with a piston B, which is rendered tight against air leakage by means of a leather ring. The piston has a valve which opens upwards, permitting the gas to pass from the lower to the upper side of the piston, but not *vice versa*. A similar valve is fitted at C. The piston rod D is worked by a lever (not shown) attached to the top end. The vessel to be exhausted is connected by rubber tubing to E. The gases enter at E, pass through F, and may enter the cylinder through a main passage G or through a by-pass H. The exit orifice to the atmosphere is at K.

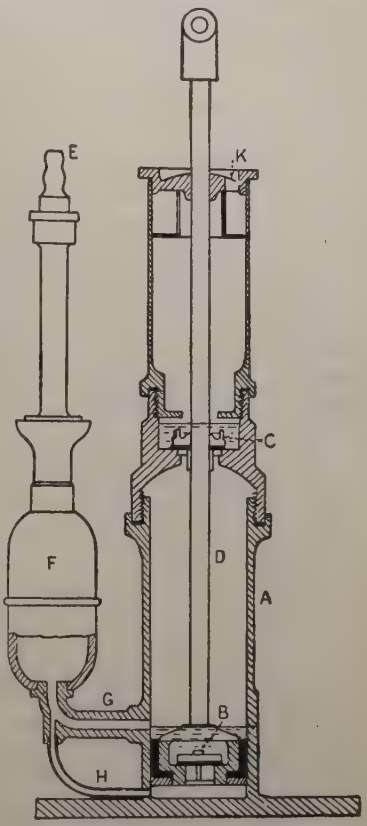

FIG. 417.—Air-exhausting pump.

Starting with the piston at the bottom of the stroke, the vessel to be exhausted is in communication with both sides of the piston through G and H; the pressures on the top and bottom of the piston are therefore equal, and the piston may be moved easily. Immediately the piston passes the opening of G, it begins to compress the gas in the space between the valve C and the piston. Compression goes on until the pressure of the enclosed gas is equal to that of the atmosphere (neglecting the weight of C), when the valve C lifts, and the gas is discharged through C and K during the remainder of the stroke.

The piston cannot quite arrive at the top of the cylinder, and hence all the gas will not be discharged. Complete discharge can be obtained by introducing some oil, which lies on the top of the piston. As the piston approaches the top of the stroke, the oil fills ultimately the whole of the upper part of the cylinder, thus driving the entire gaseous contents through C. Some oil will find its way through C, and will remain on the top of this valve during the next downward stroke.

During the upward stroke just described, more gas has been flowing into the lower part of A through G and H. The piston now descends, C closes and the space above B becomes more rarefied than that below; hence the valve in the piston opens, and the descent is completed with equal pressures on both top and bottom. On the piston passing G, the function of the by-pass H is to permit the escape of any gas or liquid which may be trapped between the piston and the bottom of the cylinder; the piston may thus be pushed to the cylinder bottom, and is then ready to repeat the action.

During each upward stroke the volume of air removed is equal to that of the portion of the cylinder lying above G; this volume is at the pressure existing in the vessel under exhaustion at the beginning of the stroke considered.

Let V = the volume of the vessel up to the opening of G into the cylinder (Fig. 417).
v = the volume of the cylinder between B and C.
p_1 = the initial pressure in the vessel, taken as equal to that of the atmosphere.

Assuming that the temperature remains constant, we may find the pressure after any number of strokes by applying Boyle's law. Thus, the piston being at the bottom, the total volume is $(V+v)$; during the first upward stroke, the volume removed is v at pressure p_1. While this stroke is proceeding, the air in the receiver expands from V at p_1 to $(V+v)$ at p_2.

$$\therefore\ p_1V = p_2(V+v),$$

$$p_2 = \left(\frac{V}{V+v}\right)p_1. \qquad \text{.....(1)}$$

This gives the pressure at the end of the first upward stroke. There is no change in pressure during the next downward stroke; hence we have the piston again at the bottom, and the volume is $(V+v)$ at p_2. During the second upward stroke a volume v is removed

at pressure p_2, and the air in the vessel expands from V at p_2 to $(V+v)$ at p_3; $\therefore\ p_2V=p_3(V+v)$,

$$p_3=\left(\frac{V}{V+v}\right)p_2=\left(\frac{V}{V+v}\right)^2 p_1. \quad \text{.....(2)}$$

Similarly, at the end of the nth upward stroke the pressure is given by

$$p=\left(\frac{V}{V+v}\right)^n p_1. \quad \text{.....(3)}$$

Mercurial air pump.—The pump shown in Fig. 418 is used for exhausting the bulb A. A is connected at B to an inverted U tube CBD, having a bore of about 1 mm. Another tube EF has a funnel at its top end, and is connected by rubber tube FC to the U tube. Mercury is poured into the funnel, and the flow through the tubes is regulated by means of a pinch-cock G. The limb BD should be about one metre in length. In action the mercury passing the branch at B separates into drops, and after descending BD is discharged into a beaker. Air from the bulb A fills the spaces between the drops, and is swept down the tube by the descending mercury. As exhaustion proceeds, the air in A becomes rarefied, the spaces between the drops become smaller, and finally the column of mercury in BD is equal to the height of the barometer. The conditions in A are then similar to those in a Torricellian vacuum.

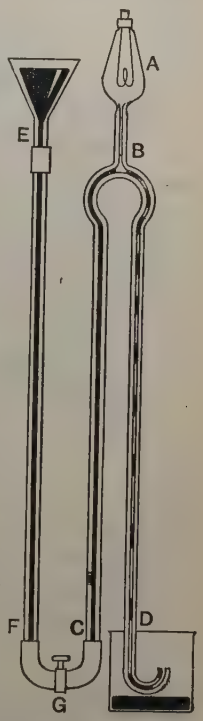

FIG. 418.—Mercurial air pump.

Gaede's molecular air pump.* — However smooth the surface of any solid may appear to be, there are two kinds of irregularities, viz., mechanical and molecular. The mechanical irregularities may be reduced by skilful workmanship, but the molecular irregularities cannot be so reduced. The formation of a film of gas, adhering to the surface of a solid in contact with the gas, is probably due to these molecular irregularities. If the surface be in motion, the adhering film moves with it, and in turn drags the adjacent layers of gas.

* See *Nature*, Vol. 90, January 23, 1913, p. 574.

The principle of the **Gaede molecular air pump** may be understood by reference to Fig. 419. A cylinder A rotates clockwise in a case B, and B has two openings n and m connected by a slot. The gas is dragged by the cylinder from n to m, and consequently a difference of pressure is established between n and m. This pressure difference is proportional to the speed of rotation and also to the internal friction of the gas. The latter is independent of the pressure, and hence the difference in pressure produced should also be independent of the pressure. This is true for relatively high pressures, but if it continued to be true down to the lowest pressures, we should be able to create an absolute vacuum by exhausting initially with another pump at n (Fig. 419) to a pressure lower than the constant difference of pressure between n and m. This is no longer the case for pressures lower than 0·001 mm. of mercury. If the surface of the cylinder A had a velocity greater than the molecular velocity, we would obtain an absolute vacuum, but such speeds are impossible in practice. However, at these low pressures the ratio of the pressures at m and n remains constant independently of the pressure, and it has been found that

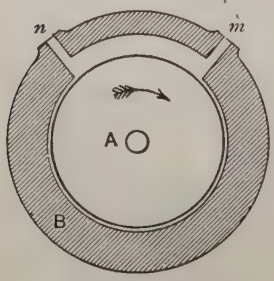

FIG. 419.—Principle of the Gaede molecular air pump.

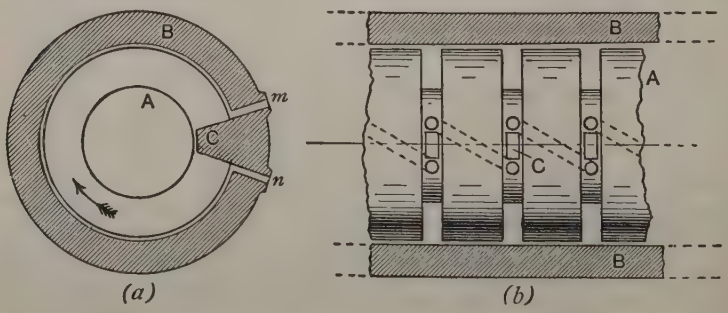

FIG. 420.—Construction of the Gaede molecular air pump.

speeds of 8000 to 12,000 revolutions per minute are sufficient to give a vacuum better than that produced by any other type of pump.

In practice the pump is constructed as shown in Fig. 420. The cylinder A is grooved, and a tongue C projects from the case into the groove; this is equivalent to a very long slot in the case. In

order to increase the efficiency, several grooves are cut in A, and are connected with one another so that the low-pressure side of one is the high-pressure side of the next (Fig. 420 *b*); there is thus a number of pumps in series. A preliminary pump is used in order to reduce the initial pressure to a few millimetres of mercury. The Gaede pump deals effectively with vapours as well as gases, since there is no compression during the removal and therefore no condensation of the vapours. An external view of the pump is given in Fig. 421.

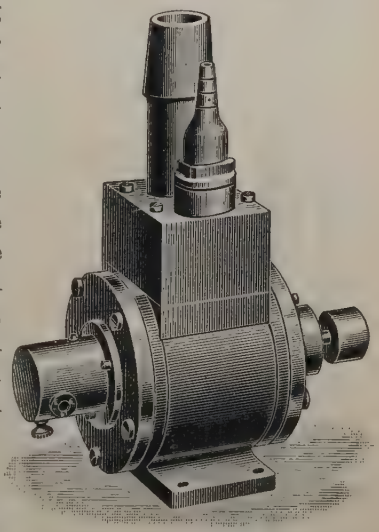

FIG. 421.—Gaede molecular air pump.

Very low pressures can be obtained by first exhausting the vessel as thoroughly as possible by means of a pump. A tube containing coconut charcoal, and in communication with the vessel, is immersed in a bath of liquid air. The remainder of the gas in the apparatus condenses in, and is absorbed by, the charcoal, thus producing a very low pressure.

M'Leod's pressure gauge.—The pressure gauge illustrated in Fig. 422 is suitable for measuring low pressures. A vertical tube ABCD is sealed at its top end; the portion AB is of small bore and there is a large bulb between B and C. The lower end is connected by flexible tubing to a mercury cistern E furnished with a tap. A branch at C leads to another tube FG, of the same bore as AB in order to avoid capillary effects, and thence to the vessel in which the pressure is to be measured.

With the mercury level adjusted so that the branch at C is just closed, the volume V contained between A and C is known from a previous calibration, as are also the volumes between A and various levels of the mercury in AB. A scale of volumes is attached to AB, zero of the scale being at A.

In using the gauge, the level of the mercury is first adjusted so as to be lower than C; the whole of the space above the mercury surface is filled with gas at the same pressure as that in the vessel. The cistern E is then raised slowly, and the mercury level in CD rises. On reaching C, the mercury seals the gas in ABC. Further

raising of the cistern ultimately produces the result shown in Fig. 422, in which the mercury surface is at K, and the entrapped gas has been compressed to a volume V′ as shown by the scale of volumes. The mercury level in FG is now at L, and is still subjected to the pressure which it is intended to measure; let this pressure be p mm. of mercury, and let h mm. be the difference in levels at K and L. Then the pressure of the volume V′ of gas in AK is $(p+h)$ mm. of mercury.

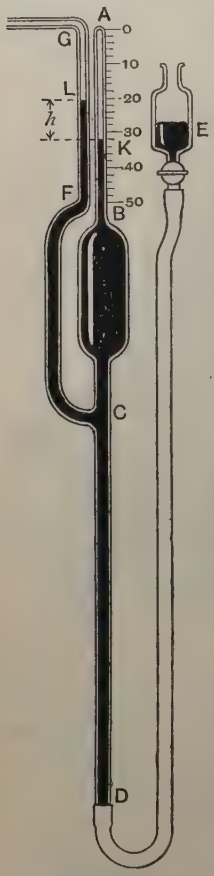

FIG. 422.—M'Leod's pressure gauge.

Assuming that there has been no change in the temperature of the room while the observations have been made, and that sufficient time has been given to permit the compressed gas in AK to return to its original temperature, we may apply Boyle's law. Thus:

$$pV=(p+h)\,V'=pV'+hV',$$

$$\therefore\ p\,(V-V')=hV',$$

or,
$$p=\frac{V'h}{V-V'}.$$

EXAMPLE.—In a M'Leod pressure gauge, V=50 c.c.; in measuring the pressure in an exhausted vessel, h was 8 mm. and V′ was 0·2 c.c. Find the pressure.

$$p=\frac{0\cdot2\times8}{50-0\cdot2}=\frac{1\cdot6}{49\cdot8}$$

$$=0\cdot0321 \text{ mm. of mercury.}$$

Action in an air compressor.—The use of compressed air for operating certain machines has become of great importance. The principal parts of an air compressor may be understood by reference to Fig. 423. A cylinder A is fitted with a piston B, which is driven up and down in the cylinder by means of a connecting rod CD, which is connected to the piston by a pin at C, and to a revolving crank ED. The crank is fixed to a shaft E, which is driven by some outside source of power, such as a steam engine or an electro motor.

At the top of the cylinder is a suction valve F, which opens when the piston is moving downwards, and this permits air from the atmosphere to flow into the cylinder. During the upward move-

ment of the piston the suction valve F is closed, and the air in the cylinder is compressed and delivered through a discharge valve G into a receiver not shown in Fig. 423. The discharge valve opens at the instant when the rising pressure in the cylinder has reached the pressure in the receiver, or a pressure slightly higher. Pipes connect the receiver to the machines to be driven.

The compression is kept as nearly isothermal as possible by means of water circulating in a water jacket H, which surrounds the cylinder. Cold water enters the jacket at K and is discharged at L. The practical objects of the cold-water jacket are twofold: (*a*) The working parts of the cylinder would otherwise become excessively hot, and damage would probably result. (*b*) Air discharged hot into the receiver will cool there by conduction of heat through the receiver walls into the atmosphere; this heat is wasted, and represents mechanical work done by the source of power on the piston of the compressor. Less power will be required to drive the machine if the temperature in the cylinder is prevented from rising considerably during compression.

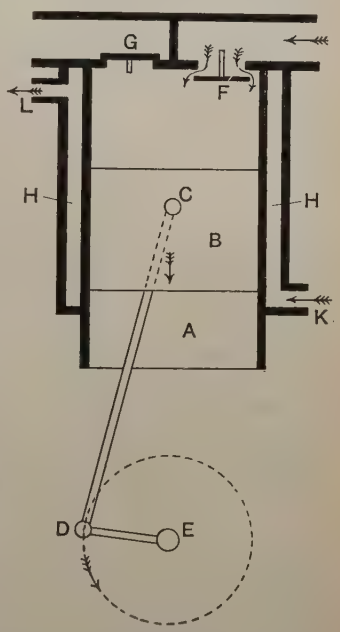

FIG. 423.—Section of an air compressor.

Of course heat is carried away by the water circulating in the jacket and is thus wasted, but it is more economical of power to abstract this heat from the air while still in the cylinder rather than from the air after it has passed into the receiver.

Diagram of work done in an air compressor.—A pressure-volume diagram for the air compressor is shown in Fig. 424. Starting with the piston at the bottom of the stroke, the cylinder is full of air at atmospheric pressure p_1, the volume being v_1; these conditions are plotted at A. As the piston moves upwards, compression (approximately isothermal) of the air takes place along the curve AB. Both

valves F and G (Fig. 423) are closed during the compression. When the pressure reaches the receiver pressure, p_2, the discharge valve G opens, and delivery into the receiver occurs at constant pressure p_2; this is shown in Fig. 424 by the horizontal line BC.

Delivery of air stops when the piston reaches the top end of the stroke. Had the whole of the contents of the cylinder been expelled into the receiver, the pressure in the cylinder would drop instantly to that of the atmosphere when the piston starts to move downwards. Total expulsion is not possible, since practical conditions require that there should be a small clearance volume, v_c, between the piston when at the top of the stroke, and the cover. Hence, at the beginning of the downward stroke there is a volume of air v_c at a pressure p_2. Expansion of this air takes place along the curve CD, and at D the pressure has fallen to p_1. The suction valve now opens, permitting a fresh supply of air to enter the cylinder, as shown by the horizontal line DA.

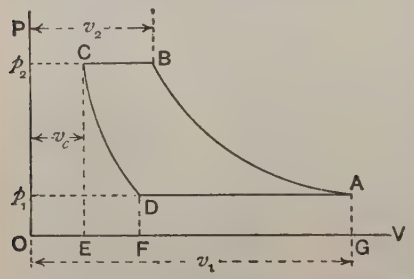

FIG. 424.—Diagram of work done in compressing air.

Assuming isothermal compression, and applying Boyle's law, we have

$$p_1v_1=p_2v_2; \quad \therefore \ v_2=\frac{p_1}{p_2}v_1.$$

Also, Air delivered into the receiver $=(v_2-v_c)$, at pressure p_2.

Let v be the volume of this air when reduced to atmospheric pressure p_1, then

$$p_1v=p_2(v_2-v_c),$$

$$v=\frac{p_2}{p_1}(v_2-v_c)=\frac{p_2}{p_1}\left(\frac{p_1}{p_2}v_1-v_c\right)$$

$$=v_1-\frac{p_2}{p_1}v_c.$$

Thus a portion only of the total volume v_1 reaches the receiver.

The total work done on the air during the compression and delivery is represented by the area ABCEGA (Fig. 424). During the downward stroke of the piston the air in the cylinder does work on the piston to an amount represented by the area CDAGEC. Therefore the net work which must be supplied to the compressor during the two strokes of the piston (one upwards and one downwards) is represented by the difference in these areas, viz. ABCDA.

Action in charging an air receiver.—Reference is made to Fig. 425, in which A is an air compressor in which the piston travels

between B and G. Air enters the cylinder from the atmosphere through a valve C, and is discharged through a valve D and pipe DE into the receiver F, which initially contains air at atmospheric pressure p_1. At the beginning of the first compression stroke, the piston is at B and the cylinder is full of air at atmospheric pressure. In the pressure-volume diagram, *al* is a horizontal line representing pressures equal to p_1, and *b* is the point corresponding to the initial conditions in the cylinder. On pushing the piston inwards, the discharge valve D opens practically at once, since the pressures are equal on both sides of it, and air will be delivered into the receiver throughout the whole stroke; this first compression stroke is represented by the curve *bc*, and the piston comes to rest at G. In this stroke the initial volume v_1 is the total volume of air in the receiver, pipe and cylinder up to the piston at B. The final volume v_2 is the total volume in the receiver, pipe and cylinder up to the piston at G. Assuming isothermal compression, and applying Boyle's law, we have

$$p_1v_1 = p_2v_2; \quad \therefore\ p_2 = \frac{p_1v_1}{v_2}.$$

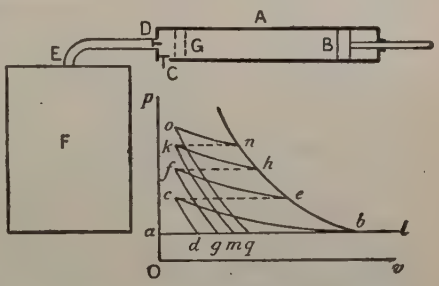

FIG. 425.—Action in charging an air receiver.

During the early part of the first return stroke, the air in the clearance volume of the cylinder, under initial conditions represented by the point *c*, expands down to atmospheric pressure along the curve *cd*. The remainder of the return stroke then takes place along the line *db*.

During the early part of the second compression stroke of the piston, both valves C and D are closed, since the pressure is rising, but is not yet equal to the pressure p_2 in the receiver. Hence the initial volume being dealt with is the volume in the compressor cylinder only, and the pressure therefore rises at a more rapid rate, as is shown by the line *be* (Fig. 425). At *e*, which is at the same height (or pressure) as *c*, the discharge valve D opens, and the remainder of the stroke is completed by compressing the total volume of air in the receiver, pipe and cylinder. This part of the stroke is shown by the curve *ef*; the rate of rise of pressure is lower in this stage than in the earlier part of the stroke, owing to the larger volume of air being compressed. Expansion from *f* back to atmospheric pressure takes place along the curve *fg*, and intake of fresh air along *gb* follows as before. Other two successive strokes are shown in Fig. 425 by the curves *bhkmb* and *bnoqb*. The

operations are repeated until the desired pressure is reached in the receiver.

The action of pumping up a bicycle tyre is similar to that described above, with a slight difference owing to the volume of the receiver (the rubber tyre) not being constant, but increasing to some extent as the charging process goes on. The pressure-volume diagram will be altered in this respect only—the curves *bc*, *ef*, *hk*, etc., will not rise so steeply; the other portions of the diagram in Fig. 425 will be unaltered.

Bell-Coleman refrigerating machine.—The earliest commercially successful refrigerating machines operated by taking advantage of the heating and cooling that occurs when a gas is compressed and expanded adiabatically, or approximately so. The action in the Bell-Coleman refrigerating machine may be understood by reference to Fig. 426. A is the chamber which is to be kept at a low temperature. A pump B draws air from this chamber and compresses it; during this operation the temperature of the air rises. The pump delivers the hot air at a pressure of 3·5 to 4 atmospheres into a pipe coil C, which is kept cool by means of cold water circulating round it. The air, after cooling in C, is fed into a motor cylinder D, where it is allowed to expand, doing work in driving a piston, and thus assisting the pump piston in B. During the expansion the air falls in temperature, and at the end of expansion the air is delivered at low temperature into the refrigerator chamber A, where it again takes in some heat from the walls of the chamber, and from the mutton and other substances being chilled. A steam engine, or some other source of power, is used to drive the pump.

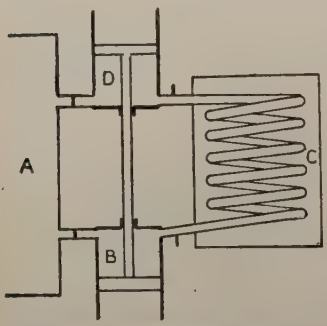

FIG. 426.—Arrangement of the Bell-Coleman refrigerating machine.

EXERCISES ON CHAPTER XXXIII.

1. Define isothermal and adiabatic expansion of a gas. Explain how these operations may be realised approximately.

2. Some gas is contained in a cylinder fitted with a piston, and does work on the piston whilst expanding. If the expansion is isothermal, heat

must be supplied. Explain this, and state what quantity of heat is required in order to maintain constant the temperature.

3. A metal syringe has a nozzle adapted to receive a small piece of tinder. If the piston of the syringe be pushed in very rapidly, the tinder may be ignited. Give a full explanation of this.

4. 880 cubic inches of air at a pressure of 90 lb. wt. per sq. inch absolute are expanded to a volume of 3520 cubic inches. Calculate the final pressure (*a*) if the expansion is isothermal, (*b*) if the expansion follows the law $pv^{1\cdot4}$ = a constant, (*c*) if the law is $pv^{1\cdot2}$ = a constant.

5. In Question 4 (*b*), find the final temperature of the air if the initial temperature is 40° C.

6. Make an outline sketch of the principal parts of a machine for compressing air and describe the action.

7. A bicycle tyre has a capacity of 200 cubic inches when fully inflated, the pressure being then 2·5 atmospheres (absolute). If the tyre initially is quite flat, calculate the volume of air at atmospheric pressure required to inflate it.

8. An air receiver has a capacity of 20 cubic feet, and contains air at one atmosphere absolute pressure. How many cubic feet of air at atmospheric pressure must be pumped into the receiver in order to attain a pressure of 6 atmospheres (absolute). Assume that the temperature does not alter. Sketch approximate pressure-volume diagrams for the first three strokes of the pump.

9. A vessel to be exhausted of air contains 2400 c.c. at a pressure of 76 cm. of mercury absolute. The air-pump is similar to that shown in Fig. 417 and removes 160 c.c. of air during the first upward stroke. What will be the pressure in the vessel at the end of the fifth upward stroke ?

10. The cylinder of an air compressor is 7 inches in diameter, and the piston has a stroke of 10 inches. When the piston is at the end of the stroke, the clearance volume is 5 cubic inches. Air is taken in at a pressure of one atmosphere and is compressed isothermally to 6 atmospheres (both absolute pressures) before being discharged into a receiver. What volume of air, measured at atmospheric pressure, is discharged each stroke ? Sketch a diagram showing the action.

11. Give an outline sketch and explain the action of a refrigerating machine using air.

12. What are adiabatic and isothermal changes ? Explain why the barrel of an ordinary bicycle pump becomes heated when air is pumped into a tyre. Allahabad Univ.

13. Give an account, with a sketch, of some form of gas-pump suitable for the attainment of very low pressures ; and of a gauge which will measure such low pressures. Madras Univ.

14. Explain the action of a mercury air-pump. Show how to find the pressure of the air in the receiver of a simple air-pump at the end of ten strokes of the piston, the volume of the barrel and the receiver being 50 c.c. and 200 c.c. respectively. If the valve at the bottom of the barrel is a metal disc of area 10/6561 sq. inch, and its weight is 1/16 oz., find after how many strokes maximum exhaustion will be reached, assuming the atmospheric pressure to be equal to 14·4 lb. per sq. inch. Presidency College.

15. In a M'Leod pressure gauge (Fig. 422, p. 436), the tube AB has a bore of 1 mm. and is graduated in mm. The total volume between A and C is 100 c.c. In measuring the pressure in a vessel to which this gauge is connected, the following readings were taken: Level of the mercury at K, 56·3 mm.; difference in levels at K and L, 4·6 mm. Find the pressure in the vessel.

16. In Question 10, assume that the compressed air in the clearance space expands in accordance with Boyle's law, and calculate the distance travelled by the piston during the suction stroke before the suction valve opens.

CHAPTER XXXIV

CHANGE OF STATE

Change of state from solid to liquid.—A solid may be conceived as a collection of molecules which preserve their relative mean positions in ordinary circumstances. Each molecule may vibrate in a comparatively small space, but does not leave that space. If heat be imparted continuously to a solid, a temperature is reached at which the molecular motions have increased to such an extent that cohesion is no longer possible. At this temperature a change of state from solid to liquid occurs. That cohesion has broken down is shown by the ease with which the blade of a knife may be passed through water compared with the difficulty experienced in cutting a block of ice. The molecules make extended excursions in a liquid, and currents of molecules are set up easily.

Melting point.—The **melting point** of a substance is the temperature at which change of state from solid to liquid occurs. This temperature is generally the same as that at which solidification of the same substance takes place—a temperature which is called the **freezing** point of the substance. Different substances have different melting points; thus, ice melts at a temperature much lower than paraffin wax.

The melting points of some substances are well defined, and are thus determined easily. Others, such as glass and wrought iron, have an intermediate plastic stage in which the material can be worked into different shapes. Some substances expand when freezing, others contract. Thus the volume of a mass of ice is greater than that of the water from which it was formed (p. 340). Cast iron expands on solidification, a fact which enables sharp, accurate castings to be made; the molten metal is poured into a mould and fills it completely during solidification. Paraffin wax is an example of a substance which contracts while solidifying.

Influence of pressure upon the melting point of a substance.—Water freezes at 0° C. when the pressure is one atmosphere. If the pressure is increased, the expansion which must take place during solidification is prevented partially and the freezing point is lowered; the substance thus remains liquid while the temperature is lowered below 0° C. In general, substances which expand in solidifying have their freezing points lowered by increase in pressure; others which show contraction in freezing have the freezing points raised by increase of pressure. Thus the melting point of ice is lowered by about 0·0072° C. for each increase in pressure of one atmosphere. Paraffin wax melts at 46·3° C. at a pressure of one atmosphere, and at 49·9° C. at 100 atmospheres.

That the freezing point of water is lowered by increase of pressure was proved experimentally by Lord Kelvin, who applied pressure to ice contained in a closed glass vessel by means of a screw tapped into a hole in the cover of the vessel. The vessel contained a thermometer in a case so as to protect it from the effects of the high pressure. The following simple experiment illustrates the same fact.

EXPT. 93.—**The freezing point of water is lowered by pressure.** Let a block of ice rest on two supports; attach a heavy weight to a loop of copper or iron wire and pass the loop round the ice. The pressure of the wire on the ice causes the freezing point to be lowered, and the ice melts under the wire. Ultimately the wire passes completely through the block. During the passage of the wire, the water formed on the lower side of the wire passes round the wire to the upper side, and being relieved of pressure, freezes again. Thus the block of ice at the end of the experiment is still one solid body.

In skating on ice, if the skates are sharp and in good order, the pressure on the sharp edge is sufficient to cause momentary melting of the ice under the edge. Thus a person may be said to be skating on water.

EXPT. 94.—**Determination of the melting point of a substance.** The following method may be used for substances having comparatively low melting points, such as paraffin wax, sulphur, etc. A small quantity of the substance is enclosed in a short piece of thin glass tube of small bore; the tube is then fastened to the stem of a thermometer, near the bulb (Fig. 427). Both are then placed in a test tube fitted with a cork to keep the thermometer steady. The test tube contains some liquid which may be warmed, and has a wire stirrer fitted. The liquid chosen should have a boiling point higher than the melting point of the substance under examination; water may be used for finding the melting point of wax, and oil, or sulphuric acid, in determining the melting point of sulphur.

Heat the test tube gently and stir constantly until the substance is observed to melt. Note the temperature and allow the tube to cool. Note the temperature at which solidification is observed to occur. Repeat the

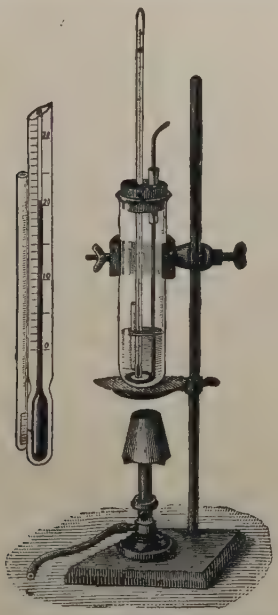

FIG. 427.—Apparatus for determining melting points.

FIG. 428.—Melting point by cooling.

operations several times and take the mean temperature as the melting point of the substance.

EXPT. 95.—**Melting points by cooling experiments.** Referring to Fig. 428, a test tube contains some paraffin wax or naphthalene, and is fitted with a cork and thermometer. The cork has a groove cut up one side, so that it does not fit air-tight. Heat the tube until the substance is melted, and raise the temperature about 10° C. higher than the melting point. Clamp the test tube at some height above the table, and observe the temperature every half-minute during cooling. Continue until the substance has solidified and cooled considerably below the freezing point.

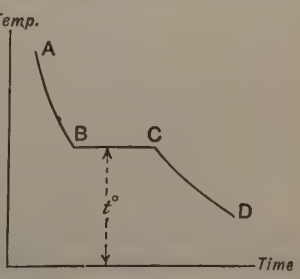

FIG. 429.—Cooling curve, showing the temperature of solidification.

Plot temperature and time, giving a graph resembling Fig. 429. AB shows the fall in

temperature of the liquid; BC indicates steady temperature conditions while solidification is taking place, and CD shows the further fall in temperature of the solid substance. The melting point is t, and is shown by the height of the horizontal line BC.

This method of cooling is much employed in the investigation of metallic alloys, and gives valuable information regarding the temperatures at which various constituents of the alloy change into the solid state.

Latent heat of fusion.—In testing the freezing point of a thermometer (p. 316), it has been observed that the temperature remained steady whilst the ice was melting. Expt. 95 illustrates the same fact with other substances. Since melting and freezing take some time to complete, it is evident that heat is entering the substance during liquefaction and is leaving it during solidification. The **latent heat of fusion** of a substance is the quantity of heat which must be imparted at constant temperature to unit mass of the substance in the solid state in order to effect the change of state from solid to liquid.

EXPT. 96.—**Latent heat of fusion of ice.** Weigh a copper calorimeter; pour in about 300 c.c. of water; weigh again and thus ascertain the mass of the water. Take a piece of ice weighing about 50 grams; wrap it in blotting paper in order to remove moisture. Take the temperature of the water in the calorimeter; and drop the dry ice into the water. Stir gently until the ice has disappeared and note the temperature at this instant. Weigh again in order to find the mass of ice used.

Let m = the mass of the calorimeter, in grams.

s = the specific heat of its material.

m_1 = the mass of water used, in grams.

m_2 = the mass of ice used, in grams.

t_1 = the initial temperature of the water, deg. Cent.

t_2 = the final temperature, deg. Cent.

L = the latent heat of fusion of ice, in calories.

The ice has taken in latent heat while melting, and the resulting water has been raised in temperature from 0° to t_2° C. Assuming that the heat taken up by the ice and resulting water is equal to that given up by the water originally in the calorimeter and by the material of the calorimeter, we have

$$m_2(\mathrm{L}+t_2) = (m_1+ms)(t_1-t_2);$$

$$\therefore \mathrm{L} = \left(\frac{m_1+ms}{m_2}\right)(t_1-t_2) - t_2.$$

The latent heat of fusion of ice is about 80 calories per gram. Compare this value with the result of the experiment.

Expt. 97.—**Latent heat of fusion of paraffin wax.** This experiment is carried out in the same manner as Expt. 96, excepting that melted paraffin wax is substituted for the ice. The calculation differs somewhat, as account must be taken of the heat given up by the liquid wax while cooling to its freezing point. The latent heat given up while solidification is taking place, and the heat given up by the solid wax while cooling to the final temperature of the mixture.

Let m_1 = the mass of water in the calorimeter, in grams.
m_2 = the water equivalent of the calorimeter, in grams.
m_3 = the mass of wax used, in grams.
t_1 = the initial temperature of the liquid wax, deg. Cent.
t_2 = the freezing point of the wax, deg. Cent.
t_3 = the initial temperature of the water, deg. Cent.
t_4 = the final temperature of the mixture, deg. Cent.
s_1 = the specific heat of the wax in the liquid state.
s_2 = the specific heat of the wax in the solid state.
L = the latent heat of the wax, in calories.

The specific heats s_1 and s_2 may be obtained by application of the methods explained in Chapter XXVI. Assuming that the heat given up by the wax is equal to the heat taken up by the water and by the material of the calorimeter, we have:

$$m_3 s_1(t_1 - t_2) + m_3 \mathrm{L} + m_3 s_2(t_2 - t_4) = (m_1 + m_2)(t_4 - t_3)$$

or,
$$\mathrm{L} = \frac{(m_1 + m_2)(t_4 - t_3) - m_3\{s_1(t_1 - t_2) + s_2(t_2 - t_4)\}}{m_3}.$$

Freezing points of solutions.—When a solid is dissolving in a liquid, *e.g.* common salt in water, the solid takes in latent heat of fusion; this heat is derived from the store of heat in the liquid, and hence a cooling effect is produced. If the action is not merely the dissolving of the solid in the liquid, but includes chemical combination of the two substances, heat may be generated by the chemical process, and there may be a rise in temperature. An example of this occurs when a stick of caustic potash is dropped into water, considerable rise in temperature taking place.

The freezing point of a solution is always lower than that of the solvent. Thus, if equal weights of ammonium nitrate and water, both at 0° C., be brought together, the temperature of the resulting liquid will be found to be about −15° C., a temperature which renders the mixture useful in cooling operations. Another useful freezing mixture is produced by mixing equal weights of common salt and snow or pounded ice.

Determination of specific heats by the Bunsen ice calorimeter.—This method is useful when a small quantity of the substance only is available. The apparatus employed (Fig. 430) consists of a test tube A fused into a bulb B. A tube C leads from the bottom of B and has an iron coupling D, which serves as a connection for another fine bore tube S. S has a millimetre scale attached to it. The part of B surrounding A contains pure water which has been boiled, and the lower part of the bulb B and the whole of the tube CD and part of S contain pure mercury. The position of the end of the mercury in S may be adjusted by pushing the tube to a greater or less extent into the collar D. Some of the water in B is frozen by a process of circulating chilled alcohol into and out of the test tube A, or by evaporating ether in the test tube. The whole apparatus is then immersed in fresh pure snow. The shell of ice round A may be from 6 to 10 mm. thick.

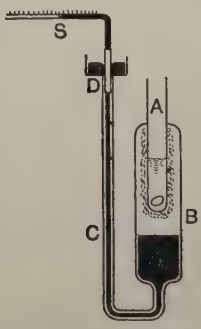

FIG. 430.—Diagram of Bunsen's ice calorimeter.

The apparatus is calibrated by introducing some pure water into A. If the mass of this water is m and its initial temperature t, then it gives out mt units of heat in falling to 0° C. In consequence of this, some of the surrounding ice melts and shrinks in volume, and the mercury in S recedes. Let the recession amount to n scale divisions, then one scale division of movement corresponds to mt/n units of heat. It will be noted that the principle relied upon for the measurement is that of the latent heat of ice, combined with the change of volume which takes place when ice melts.

A fragment of the substance to be tested is now heated in a heater (p. 347) and dropped into some water already in A. A plug of cotton-wool at the bottom of A prevents fracture of the tube. Let m_1 and t_1 be the mass and initial temperature of the substance. Suppose the mercury recedes u_1 scale divisions while the substance is cooling to 0°, and let q be the quantity of heat corresponding to 1 scale division. Then, if s is the specific heat of the substance,

$$m_1 s t_1 = u_1 q,$$

$$s = \frac{u_1 q}{m_1 t_1}.$$

The position of the mercury in S does not remain constant when the instrument is not in use; hence a correction has to be applied. Observe the movement of the mercury during half an hour before the experiment and again for half an hour after finishing the experiment. Let the movements be respectively u_2 scale divisions in t_2

minutes and u_3 scale divisions in t_3 minutes. Then the average movement is given by

$$\text{Average rate of variation} = \tfrac{1}{2}\left(\frac{u_2}{t_2} + \frac{u_3}{t_3}\right).$$

The correction to be applied in the experiment is this quantity multiplied by the duration of the experiment in minutes.

Change of state from liquid to vapour.—In liquids, collisions between the molecules will be frequent, as there is but little space in which a molecule can move. Heat imparted to the liquid increases the speed of the molecules, thus enabling the liquid to store the additional energy. The surface film (p. 298) prevents the escape of most of the molecules, but those molecules which happen to have a speed considerably higher than the mean speed when in the neighbourhood of the surface may break through the surface film and escape. Molecules which have escaped in this manner mingle with the gas over the liquid and behave in the same way as gas molecules. The action is called **evaporation**, and the accumulated escaped molecules are called a **vapour**. If the liquid is contained in an open vessel, evaporation would lead in time to the total disappearance of the liquid.

The process of evaporation may be hastened by raising the temperature of the liquid, thus increasing the mean speed of the molecules and so enabling them to break more easily through the surface film.

Evaporation in a closed vessel.—Suppose there to be some liquid in a closed vessel from which air and all gases, other than vapour formed from the liquid, have been extracted. Molecules will be escaping continually from the liquid ; other molecules in the vapour will occasionally strike the surface of the liquid, and will penetrate the surface film, thus rejoining the liquid ; the latter operation is called **condensation**. Conditions will be attained ultimately in which the number of molecules escaping from the liquid per second is balanced exactly by the number of molecules returning per second. There is then a definite number of molecules per c.c. in the vapour space of the vessel, and the vapour is said to be **saturated**. A **saturated space** is one in which no greater number of molecules can be maintained under the existing conditions.

Saturated conditions are attained very quickly in a closed vessel such as has been assumed above. In fact, the space available for vapour is always saturated. Both liquid and vapour will be at the

same temperature. Raising the temperature of the vessel and its contents causes an increase in the mean speed of both liquid and vapour molecules. The effect of the former is to render easier the escape of more molecules, and of the latter, to increase the pressure on the walls of the vessel due to the bombardment of vapour molecules. At any stated temperature, the saturated vapour space contains a definite number of molecules per unit volume, having a definite mean speed, and hence producing a definite pressure on the walls of the vessel. Hence the pressure of a given saturated vapour at a given temperature is constant.

Saturated vapour.—Reduction of the capacity of the vapour space available in a closed vessel will not lead to any increase in the pressure of the vapour, provided the temperature of the vessel and its contents be maintained constant. Saturated vapour at a given temperature can have one pressure only, and reduction of the capacity of the vapour space will therefore produce condensation of some of the vapour. If such reduction be continued, the whole of the vapour condenses ultimately, the effect being completed when the total capacity of the vessel is sufficient to accommodate only the given quantity of liquid at the given temperature.

Saturation conditions in any space, of whatever capacity, are independent of the volume, but there must be sufficient liquid available to form enough vapour to saturate the space. If the quantity of liquid be insufficient, the whole of it will be evaporated before saturation of the space occurs ; the vapour is then said to be **unsaturated.**

Superheated vapour.—If saturated vapour be conducted away from a closed vessel by means of a pipe, and if heat be then imparted to it without the pressure being allowed to rise, the temperature will increase, accompanied by an increase in the mean speed of the molecules, and the vapour is found to behave more like a perfect gas, *i.e.* it obeys the laws of Boyle and Charles. Such vapour is no longer in a condition in which it may be very easily converted into the liquid state, and is said to be **superheated vapour.** The term is synonymous with the term unsaturated vapour.

EXPT. 98.—**Maximum vapour pressure at the temperature of the room.** Arrange two barometer tubes A and B (Fig. 431) as previously directed (p. 259). Bend the point of a small pipette C, and charge it with the liquid whose vapour pressure is to be examined. Put the point of the pipette

under the open mouth of B, and by blowing introduce a very small quantity of the liquid; this will rise to the surface of the mercury at D, and if the quantity of liquid is small enough, the whole of it will be evaporated. The space above the mercury in B thus becomes filled with unsaturated vapour, and the level of the mercury at D falls a little owing to the pressure exerted by this vapour. Introduce another small quantity of the liquid, when again the whole of it may be evaporated, and the level of the mercury falls further, indicating an increase in the pressure in B. Continue the process until sufficient liquid is introduced to produce a very thin layer of liquid lying on the mercury surface at D. The space in B is then full of saturated vapour at the temperature of the room, which should be noted. The pressure exerted by the saturated vapour is obtained by measuring the difference in levels of the surfaces of the mercury in A and B.

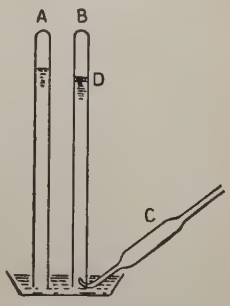

FIG. 431.—Vapour pressure at the temperature of the room.

Verify the fact that no increase in pressure occurs when another small quantity of the liquid is introduced into B. The absence of further fall of the mercury level at D shows that the pressure exerted by the vapour when saturated at the temperature of the room is constant.

The pressure exerted by a vapour when saturated at a given temperature is the maximum pressure which the vapour can exert when at this temperature, and is called the **maximum vapour pressure.**

EXPT. 99.—**The maximum vapour pressure is independent of the volume of the space occupied.** In the apparatus shown in Fig. 431, use a mercury bath deep enough to permit the tube B to be raised, or lowered a few centimetres. Having charged B with sufficient liquid to obtain saturated conditions and measured the difference in mercury levels, lower B in the bath. It will be found that the effect is to diminish the volume of the vapour space; some of the vapour condenses into the liquid state, and on measuring the difference in levels of the mercury in A and B, this will be found to be the same as at first. Now elevate B, thus increasing the vapour space; this should not be carried to excess, or the whole of the liquid will be evaporated and the vapour will be unsaturated. Again measure the difference in mercury level and compare with the initial reading. The result shows that, so long as the vapour is saturated, the pressure at the constant temperature of the room remains unaltered.

EXPT. 100.—**Maximum pressure of aqueous vapour at lower temperatures.** In Fig. 432, ABC is a closed bent tube having a bulb at C. The portion BD and part of the bulb contains mercury; the portion AB is a Torricellian

vacuum (p. 259). The bulb contains some water lying on the surface of the mercury and the remainder of the bulb contains vapour of water. A scale is attached to the straight part of the tube, and the bulb and bent part of the tube are immersed in a beaker of water. The water in the beaker may be heated by means of a bunsen flame, and its temperature is measured by a thermometer E, placed near the bulb. The straight part of the tube and the scale are shielded from the flame and from the beaker.

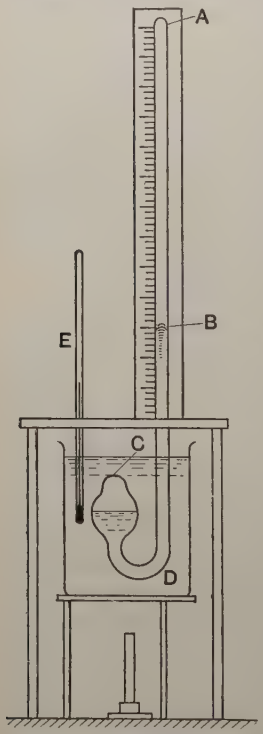

FIG. 432.—Maximum vapour pressure at lower temperatures.

The aqueous vapour in the bulb is saturated so long as there is any water in the bulb, and its pressure is equivalent to the mercury head between the surfaces in B and C. The level in B is read directly on the scale; that in the bulb is taken by means of a metal rod 20 cm. long, arranged to slide up and down along the scale. The lower end of the rod is adjusted to the mercury level in C, and the top end of the rod is read on the scale. The head of mercury is then equal to the scale reading at B minus scale reading at the top end of the rod plus 20 cm.

Maintain steady the temperature of the water in the beaker for a few minutes so as to ensure that the bulb and its contents are at this temperature. Read the temperature and the mercury head as directed above. The result gives approximately the maximum aqueous vapour pressure at the measured temperature.

Since the whole of the mercury column is not at the same temperature, a correction should be applied. Measure the height of the mercury column from B down to the surface of the water in the beaker (the metal rod will assist in taking this measurement); let this be h_a cm. Measure also the difference in levels of the water surface in the beaker and the mercury surface in the bulb; let this be h_w cm. Let the observed temperature of the air in the room be t_a deg. Cent., and that of the water in the beaker t_w. Let β be the coefficient of expansion of mercury (0·000181). Then the corrected head h_0 is given by:

$$h_0 = h_a + h_w - \beta \left\{ h_w t_w + h_a \left(\frac{t_w + t_a}{2} \right) \right\}.$$

Repeat the experiment at intervals of about 10° C. up to the range

of the instrument. Plot a graph showing the relation of the maximum vapour pressure and temperature of aqueous vapour.

EXPT. 101.—**Maximum pressure of aqueous vapour at higher temperatures.** The apparatus employed is shown in Fig. 433. The bent tube at A contains mercury standing in both limbs, and the space in the closed shorter limb contains some water and water vapour only. The tube is immersed in a beaker B containing glycerine, the temperature of which can be raised, with care, considerably above 100° C. The tube A is connected to a reservoir C containing air, and the reservoir is connected to a U gauge D containing mercury, and also to an air pump (not shown in Fig. 433) by means of which air may be forced into the apparatus.

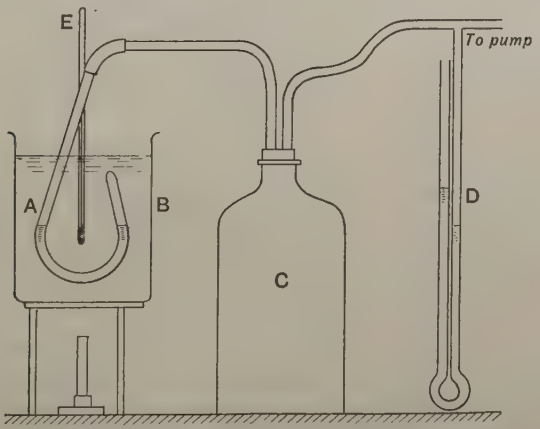

FIG. 433.—Maximum vapour pressure at higher temperatures.

The glycerine bath is brought to the temperature of 100° C., as shown by the thermometer E, and the mercury surfaces in the limbs of A are brought to the same level. During this operation the pump is disconnected and the connection is left open to the atmosphere; hence the pressure on both surfaces in A will be that due to the atmosphere, and will be found by reading the barometer. Connect the pump again and raise the temperature of the bath to say 105° C.; pump in a sufficient quantity of air to restore the mercury to the same levels in A; this level had been disturbed by the increasing aqueous vapour pressure in the shorter limb. When the conditions are steady, read the difference in levels in the gauge D, and obtain the maximum vapour pressure corresponding to the temperature in the bath by adding this difference to the barometer reading.

Repeat the experiment at several different temperatures, and plot a graph showing the relation of maximum vapour pressure and temperature.

Boiling point of a liquid.—Let an open vessel contain some water, and let the pressure of the atmosphere be 76 cm. of mercury. Let heat be imparted continuously to the water, when vapour will be given off constantly. Saturated aqueous vapour under a pressure of 76 cm. of mercury has a temperature of 100° C. Hence, when the water reaches this temperature, it becomes possible for bubbles of saturated water vapour to form just underneath the surface of the water. The water near the bottom of the vessel is at a pressure greater than that of the atmosphere by an amount equal to the head of water in the vessel. Therefore, if the temperature of the water be raised slightly above 100° C., in fact to the saturation temperature corresponding to the pressure near the bottom of the vessel, bubbles of vapour can form at the bottom. These ascend to the surface, enlarging as they travel owing to the diminishing pressure, and disengage themselves from the liquid on reaching the surface. The water is then said to **boil**, and the formation of bubbles in the water is called **ebullition**. The temperature at which a liquid boils under standard atmospheric pressure of 76 cm. of mercury is called the **boiling point** of the liquid, and is **the temperature at which the maximum vapour pressure of the substance is equal to standard atmospheric pressure.**

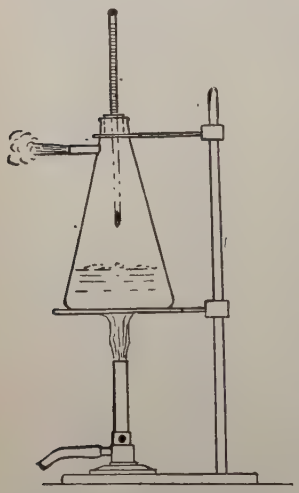

FIG. 434.—Boiling point of a liquid.

EXPT. 102.—**Boiling points of solutions.** Arrange apparatus as shown in Fig. 434. See that the flask is perfectly clean, and introduce some tap water. Heat the water; note the ascending bubbles of air which are liberated when the water becomes hot. When the boiling point is nearly reached, some bubbles of vapour may form near the bottom and collapse before reaching the surface owing to the colder water in the neighbourhood of the surface. When ebullition is taking place freely, note the temperature of the vapour. Push the thermometer down into the water, and verify that the temperature of the boiling water is very nearly equal to that of the vapour.

Boiling with bumping sometimes occurs; a short quiescent period is followed by the formation of a very large bubble which raises the mass

of water; the water falls again and produces the bump. If this occurs, the thermometer will probably show that the temperature of the water has risen a little. Some pieces of broken earthenware dropped into the vessel will remedy bumping by promoting the formation of small bubbles at many parts of the liquid.

Since the bubbles of vapour are under saturated conditions, it follows that the temperature must remain steady throughout the whole period of boiling.

Prepare a number of solutions of common salt in water, say 2, 4, 6, etc. per cent. of salt. Introduce one of these into the vessel (Fig. 434) and have the thermometer in the vapour space. Bring the solution to boiling and note that the thermometer reads the same as if pure water had been in the vessel. Now lower the thermometer into the liquid, when it will be found that the temperature of the liquid is higher than the boiling point of pure water. It follows that the boiling point of a solution should be determined with the bulb of the thermometer immersed in the liquid. Find the boiling points of the remaining solutions. The temperatures

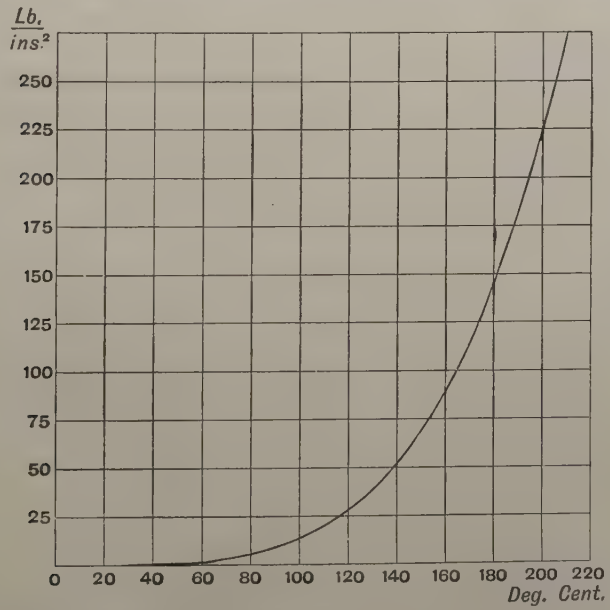

FIG. 435.—Relation of maximum vapour pressure and temperature for aqueous vapour.

should be noted immediately after the solution starts to boil, as the process of evaporation obviously concentrates the solution. Plot a graph showing the relation of boiling points and percentage of salt present.

Influence of pressure on the temperature at which boiling occurs.—Since a liquid does not boil until the temperature reaches the saturation temperature corresponding to the pressure to which the liquid is subjected, it follows that increase in pressure will raise the temperature at which boiling occurs. Thus water under a pressure of one atmosphere boils at 100° C.; if the pressure be raised to 6 atmospheres, the boiling point is raised to about 160° C.; at a pressure of 0·1 atmosphere water boils at a temperature of about 47° C. In Fig. 435 is shown a graph indicating the boiling points of water at various pressures.

Exercises on Chapter XXXIV.

1. What is meant by the melting point of a substance? Give some instances of substances which exhibit different phenomena during the process of melting, or freezing.

2. What are the effects of increased pressure on the melting point of a substance? Give instances.

3. Describe experiments by means of which the melting point of a given substance may be determined.

4. Define the term "latent heat of fusion" of a substance.

5. How would you determine by experiment the latent heat of ice?

6. A piece of ice at 0° C. weighing 83·2 grams is mixed with 364 c.c. of water at 20° C. in a calorimeter having a water equivalent of 42 grams. Calculate the final temperature. (The latent heat of ice is 80 calories per gram.)

7. How much heat must be abstracted from a ton of water at 15° C. in order to convert it into ice at −6° C.? The specific heat of ice is 0·502.

8. Describe how you would carry out an experiment to determine the latent heat of fusion of paraffin wax. Explain how to reduce the results.

9. Certain solids when mixed with water produce a rise in temperature, others a fall. Explain this, and give examples. Give instances of freezing mixtures.

10. Taking the data of Question 7, find the work equivalent to the heat abstracted. If one ton of ice is made per hour, what horse-power is required theoretically?

11. Describe some method for measuring the latent heat of ice, pointing out the precautions to be taken to obtain an accurate result.

Into a copper calorimeter weighing 100 grams and containing 100 grams of water at 16° C.. there are placed 20 grams of ice at −10° C. Will the ice all melt, and, if so, what will be the final temperature of the mixture? (Specific heat of ice = 0·5; specific heat of copper = 0·094.) Sen. Cam. Loc.

12. The melting point of tin is 232° C., its latent heat of fusion is 14 calories, and its specific heat is ·055. How many units of heat are required to melt 100 grams of tin originally at 20° C.?

13. How would you obtain the melting point of a substance from an inspection of its cooling curve ? Hard paraffin wax melts at about 54° C.; draw a rough cooling curve for a quantity of this wax cooling from 80° C. to 30° C.

14. A mixture of crushed ice and water is poured into a vessel containing a thermometer. What will be the effect on the reading of the thermometer :

(*a*) when more water is poured in ;
(*b*) when more ice is put in ;
(*c*) when a handful of salt is stirred in ?

Give reasons in each case for your answer. Adelaide University.

15. Define the terms water equivalent, latent heat of fusion. Explain what becomes of the heat absorbed by a body in the process of melting.

In a copper calorimeter weighing 50 grams there are 200 grams of water at a temperature of 20° C. 20 grams of dry ice are added, and stirred well ; the final temperature is 11° C. Find the latent heat of fusion of ice. The specific heat of copper is 0·095. Tasmania Univ.

16. Give a brief description of the process of evaporation (*a*) in an open vessel, (*b*) in a closed vessel.

17. Distinguish between a saturated and unsaturated vapour. A closed vessel containing initially no substance in the gaseous state is maintained at constant temperature, and a small quantity of liquid at the same temperature is introduced into the vessel. Will the whole of the liquid be evaporated ? Give reasons for your answer.

18. Calculate the quantity of heat required to superheat one pound of aqueous vapour from 100° to 150° C. at the constant absolute pressure of one atmosphere. The specific heat is 0·502. If the initial volume is 26·75 cubic feet, find the final volume, assuming that the laws of perfect gases are followed.

19. Describe how you would find the maximum vapour pressure of alcohol at the temperature of the room. Does the volume of the vapour space provided in the apparatus make any difference in the result ?

20. In an experiment on the maximum pressure of aqueous vapour, using the apparatus shown in Fig. 432 (p. 452), the following readings were obtained : Temperature of the water in the beaker, 80° C. ; difference in levels of the mercury in the bulb and the water in the beaker, 5·2 cms. ; difference in levels of the mercury in the tube and the water in the beaker, 30·6 cms. ; temperature of the surrounding atmosphere, 20° C. Find the vapour pressure. The coefficient of expansion of mercury is 0·00018.

21. Describe an experiment for determining the maximum vapour pressure of aqueous vapour at absolute pressures from about 1 to 2 atmospheres. State the condition which must be satisfied if a liquid is to remain unchanged in presence of its vapour.

22. Define the " boiling point " of a liquid. Some tap water is contained in an open glass vessel ; the initial temperature is about 15° C. and the temperature is gradually raised to the highest possible degree. Describe clearly what may be observed in the vessel. Supposing the water had contained some salt in solution, would this have had any effect on the final temperature ? Explain the phenomena of boiling with bumping.

23. Distinguish carefully between saturated and unsaturated vapours. Into a cylinder exhausted of air and provided with a piston there is introduced just enough water to saturate the space at 20° C. Describe what happens under the following conditions :

(*a*) The volume of the space is increased by pulling out the piston.
(*b*) The volume is diminished by pushing the piston down.
(*c*) The volume remaining as at first, the temperature is increased to 30° C.
(*d*) The temperature falls to 10° C. Calcutta Univ.

24. Taking the density of ice at 0° C. to be 0·917, find what fraction of a block of ice will be immersed when floating in (*a*) fresh water (density = 1), (*b*) sea water (density = 1·03).

25. If the end of the mercury column in a Bunsen ice calorimeter recedes through a volume of 0·2 c.c. when a mass of 3·5 grams of a substance at 98° C. is placed in the test tube, find the specific heat of the substance. Take the density of ice at 0° C. to be 0·917, and the latent heat of water to be 80 calories per gram.

26. Describe Bunsen's ice calorimeter and explain how you would use the instrument to determine the specific heat of a substance. A mass of 0·96 gram of a substance was heated to 100° C. and dropped into the calorimeter. The thread of mercury retreated through a distance of 8·3 mm. in the capillary tube of 1 sq. mm. section. If 1 gram of water expands by 0·0918 c.c. on freezing and evolves 80 calories, calculate the specific heat of the substance. Bombay Univ.

CHAPTER XXXV

PROPERTIES OF VAPOURS—*Continued*

Pressure of a mixture of a vapour and perfect gas.—Dalton first stated the law, and Regnault proved experimentally, that **the pressure which a vapour exerts is very nearly independent of the presence of any other gas or vapour present,** provided there is no chemical action. In a space occupied by a mixture of a vapour and perfect gas which do not react chemically on each other, each exerts the pressure which it would produce if it alone occupied the space, and the total pressure is equal to the sum of these pressures. The law followed is thus the same as for a mixture of two perfect gases (p. 410).

If a closed vessel contains a perfect gas and also some liquid, the whole being at constant temperature, steady conditions will be attained in the space above the liquid when the space is saturated with vapour evaporated from the liquid. The pressure of the vapour is then that corresponding to the given steady temperature, and the total pressure is the sum of this maximum vapour pressure and the pressure exerted by the perfect gas. Increase in the temperature will raise the total pressure, owing partly to the increase in the pressure exerted by the perfect gas, and partly to the increase in the saturation pressure of the vapour.

Experiments on the pressure exerted by a given vapour in a closed vessel must always be arranged in such a way as to exclude from the space any air, or other gas or vapour, since it is now evident that the presence of such would lead to false readings of vapour pressure.

Isothermal line for a mixture of a saturated vapour and perfect gas.—In Fig. 436, AB is the isothermal line in a pressure-volume diagram for a given quantity of a perfect gas at a given temperature, and may be drawn by applying Boyle's law (p. 398). The isothermal line for the same gas when mixed with a saturated vapour

at the same constant temperature may be deduced by adding vertically the constant pressure of the vapour. Thus AC = BD = EF = the constant vapour pressure, and the isothermal line CD is drawn through the points C, F and D. It will be noted that, for points corresponding to equal pressures, BA rises more steeply than DC. If a perfect gas be compressed in a cylinder fitted with a piston, no condensation occurs, and the pressure increases at fairly rapid rates. If a mixture of a perfect gas and a saturated vapour be compressed in the same cylinder, some of the vapour condenses as compression proceeds; the volume of the resulting liquid is negligible, and the pressure increases at less rapid rates than would be the case if the vapour were absent.

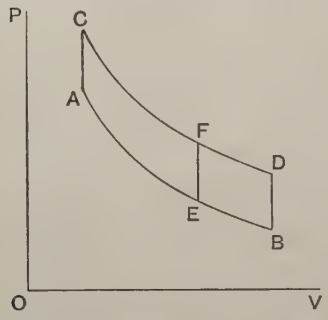

FIG. 436.—Isothermal line for a mixture of saturated vapour and gas.

Collection of gases over water.—When gases are collected over water (Fig. 437), the contents of the graduated collecting vessel consist of a mixture of the gas collected and aqueous vapour saturated at the temperature of collection. The pressure of the gas collected will be equal to that of the atmosphere, as shown by the barometer, if the collecting vessel be adjusted vertically before reading the volume, so that the water surfaces inside and outside the vessel are at the same level.

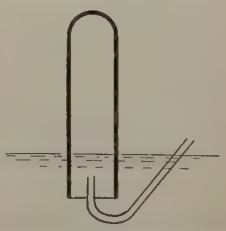
FIG. 437.—Collection of a gas over water.

Let v_1 = the measured volume of the mixture, in c.c.

v_2 = the volume of dry gas collected, at standard temperature and pressure, in c.c.

h_a = the barometer reading, in cm. of mercury.

$t°$ C. = the temperature of the water.

h_v = the pressure of saturated aqueous vapour at temperature $t°$ (from the Table, p. 533), in cm. of mercury.

The sum of the pressures of the gas and the aqueous vapour is equal to h_a; hence

Pressure of the measured volume of dry gas at $t° = h_a - h_v$.

Applying the law $p_1v_1/T_1 = p_2v_2/T_2$, we have

$$\frac{(h_a - h_v)v_1}{t+273} = \frac{76v_2}{273},$$

or

$$v_2 = \frac{273(h_a - h_v)v_1}{76(t+273)}.$$

Vapour density.—Strictly speaking, the density of a substance is the mass of unit volume (p. 4). In dealing with substances in the gaseous state, the density may be measured as the mass in grams per litre (1000 c.c.). **Vapour density** is generally understood to mean the ratio of the mass of a given volume of the substance in the gaseous state, under stated conditions of pressure and temperature, to the mass of an equal volume of dry air at the same pressure and temperature. Sometimes dry hydrogen, or oxygen, is taken instead of air in making the comparison. The densities of these standard gases at 0° C. and 76 cm. of mercury are as follows:

Dry air,	0·001293 gram per c.c.
Dry hydrogen,	0·00008987 gram per c.c.
Dry oxygen,	0·001429 gram per c.c.

Expt. 103.—Vapour density of an unsaturated vapour by the method of Dumas. A glass bulb having a finely drawn neck is employed (Fig. 438). At first the bulb contains air only at the temperature and pressure of the room, and is weighed. This weight represents the weight of the material of the bulb (supposed to be obtained by weighing *in vacuo*) diminished by the buoyancy of the air displaced by the material of the flask.

Fig. 438.—Dumas's bulb.

Let w_1 = the observed weight in air, in grams.

w_b = the weight of the material of the bulb which would be obtained *in vacuo*, in grams.

w_a = the buoyancy, in grams, of the air displaced by the material of the bulb.

$t°$ C. = the temperature of the room.

h_a = the barometer reading, cms. of mercury.

Then, $$w_1 + w_a = w_b,$$

or $$w_1 = w_b - w_a. \quad \ldots\ldots(1)$$

About 2 c.c. of the substance to be examined (alcohol, say) is now introduced into the bulb, which is then placed in a bath of water and held down by means of a metal frame. The water is boiled until the liquid in the bulb has disappeared entirely. The bulb is now full of alcohol vapour only, at the temperature of 100° C. and under a pressure h_a cm. of

mercury. This temperature is considerably above the saturation temperature of alcohol vapour at ordinary atmospheric pressure, and the vapour is therefore unsaturated. The neck of the bulb is now sealed hermetically, and the bulb is allowed to cool to the temperature of the room. It is then weighed again; the result represents the weight of the material of the bulb *in vacuo*, together with the weight of the vapour which filled the bulb at 100° C., diminished by the buoyancy of the air displaced by the outside volume of the whole bulb.

Let w_2 = the observed weight of bulb and vapour, in grams.

w_A = the buoyancy in grams of the air displaced by the closed bulb.

w_v = the weight in grams of the contained vapour, or the liquid condensed from the vapour.

Then, $$w_2 + w_A = w_b + w_v,$$

or $$w_2 = w_b + w_v - w_A. \quad \text{.....(2)}$$

From (1) and (2), $$w_2 - w_1 = w_v - (w_A - w_a). \quad \text{.....(3)}$$

Now $(w_A - w_a)$ is the weight of air at $t°$ C. and h_a cm. of mercury which would occupy the interior volume of the bulb. The interior volume of the bulb is now determined by immersing the closed flask in water and breaking off the neck. When full of water, the bulb is weighed again (the piece of glass broken off must be included).

Let w_3 = the observed weight, in grams.

Then, Weight of contained water $= (w_3 - w_1)$ grams.

$\therefore$ Volume of interior of bulb $= (w_3 - w_1)$ c.c. .....(4)

To obtain the weight of the air contained by the bulb at temperature t and pressure h_a, let d' be the density of air under these conditions, and d_0' its density at normal temperature and pressure (p. 461). Then, from equation (5) (p. 412),

$$d' = \frac{273 h_a d_0'}{76(t+273)};$$

$$\therefore \text{ Weight of the contained air} = (w_3 - w_1)\frac{273 h_a d_0'}{76(t+273)}. \quad \text{.....(5)}$$

Substituting this for $(w_A - w_a)$ in (3), we have

$$w_2 - w_1 = w_v - \frac{273 h_a d_0'(w_3 - w_1)}{76(t+273)},$$

or $$w_v = w_2 - w_1 + \frac{273 h_a d_0'(w_3 - w_1)}{76(t+273)} \text{ grams.} \quad \text{.....(6)}$$

The volume occupied by this mass of vapour being $(w_3 - w_1)$ c.c. at 100° C. and h_a cm. of mercury, we have for the vapour density of alcohol under these conditions

$$d_{100} = \frac{w_2 - w_1}{w_3 - w_1} + \frac{273 h_a d_0'}{76(t+273)} \text{ grams per c.c.} \quad \text{.....(7)}$$

The density of hydrogen at normal temperature and pressure is 0·00008987 gram per c.c., and at 100° C. and h_a cm. of mercury is

$$d_H = \frac{273h_a}{76(100+273)} \times 0{\cdot}00008987$$

$$= 0{\cdot}0000008654\ h_a \text{ grams per c.c.} \quad \ldots\ldots(8)$$

Hence the vapour density of alcohol at 100° C. and h_a cm. of mercury, with hydrogen as standard, is

$$D_{100} = \frac{d_{100}}{0{\cdot}0000008654h_a}. \quad \ldots\ldots(9)$$

Assuming that the alcohol vapour follows the laws of a perfect gas, the density at normal temperature and pressure is given by

$$d_0 = \frac{373 \times 76}{273h_a} \cdot d_{100}, \text{ (p. 412)} \quad \ldots\ldots(10)$$

and the vapour density under these conditions with hydrogen as standard is

$$D_0 = \frac{d_0}{0{\cdot}00008987}. \quad \ldots\ldots(11)$$

Expt. 104.—**Vapour density of an unsaturated vapour by Victor Meyer's method.** The apparatus employed is shown in Fig. 439. A is a glass tube having a bulb at its lower end. The upper end is connected by a rubber tube having a clip B to a short piece of glass tube which is open to the atmosphere. A branch tube C, having an enlargement, connects A to a graduated gas jar D, inverted over a trough of water. A is placed inside another glass or copper vessel E, which contains some water; the water is boiled and the steam escapes through F. The bulb of A can thus be maintained at 100° C.

The liquid to be tested (say alcohol) is contained in a small phial shown enlarged at G; the phial has a ground glass stopper which can be forced out easily when the pressure in the interior of the phial exceeds that of the atmosphere by a small amount. The phial is inserted at the top end of A after the water in E has been boiling quietly for a few minutes, and the clip is instantly closed. The phial drops down A, and its fall is arrested by a cushion of asbestos fibre in the bulb, which prevents fracture. The stopper of the phial is blown out almost immediately by the pressure of the vapour, and the alcohol evaporates rapidly. The vapour formed drives air out of A through the branch C into the jar D; the volume of air thus measured gives, after corrections have been applied, the volume of the vapour.

Let w_1 = the weight of the phial and stopper when full of air, in grams.
w_2 = the same when containing alcohol, in grams.
V = the volume of air collected in c.c., at h_a and t.
h_a = the barometer reading, in cm. of mercury.
t° C. = the temperature of the room.

The weight of the vapour formed is equal to the weight of alcohol in the phial, hence

Mass of the vapour $= w_2 - w_1$ grams. .....(1)

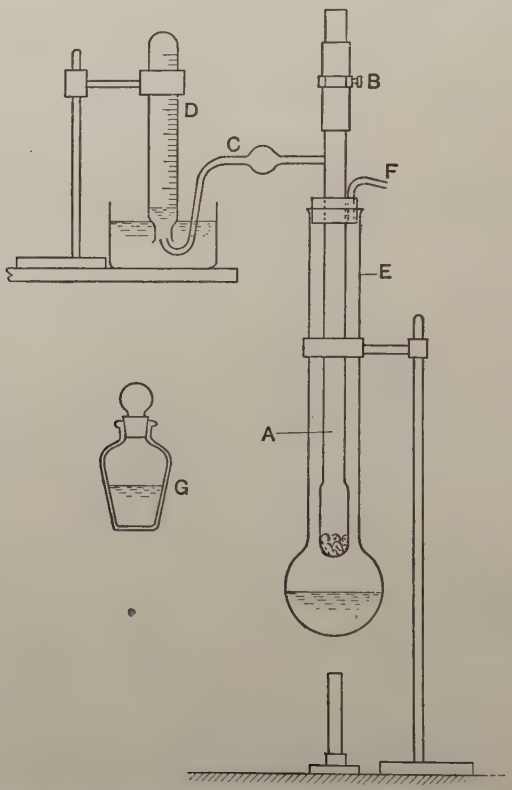

FIG. 439.—Victor Meyer's apparatus for vapour density determinations.

The contents of the graduated jar consist of a mixture of air and saturated water vapour (p. 460). Obtain from the Table (p. 533) the pressure h_v of saturated water vapour at temperature t. The pressure of dry air at temperature t which would occupy the volume V if the aqueous vapour were absent is $(h_a - h_v)$ cm. of mercury. Applying Boyle's law to obtain the volume V′ of this air at pressure h_a and temperature t, we have

$$(h_a - h_v)\mathrm{V} = h_a \mathrm{V}';$$

$$\therefore \mathrm{V}' = \left(\frac{h_a - h_v}{h_a}\right)\mathrm{V} \text{ c.c.} \quad \text{.....(2)}$$

From (1) and (2), the density of the alcohol vapour at 100° C. and h_a cm. of mercury is

$$d_{100} = \frac{(w_2 - w_1)h_a}{(h_a - h_v)V} \text{ grams per c.c.} \quad \ldots\ldots(3)$$

The vapour formed in the tube A is at a temperature considerably above the saturation temperature, and is therefore unsaturated vapour. Assuming that it follows the laws of perfect gases, the density at 0° C. and 76 cm. of mercury may be obtained from equation (5), (p. 412),

$$d_0 = \frac{76(t+273)}{273h_a} d_{100} \text{ grams per c.c.} \quad \ldots\ldots(4)$$

And the density relative to hydrogen, both being at normal temperature and pressure, is

$$D_0 = \frac{d_0}{0{\cdot}00008987}. \quad \ldots\ldots(5)$$

Specific volume of saturated vapours.—The density of saturated vapours cannot be determined easily by experiment; generally

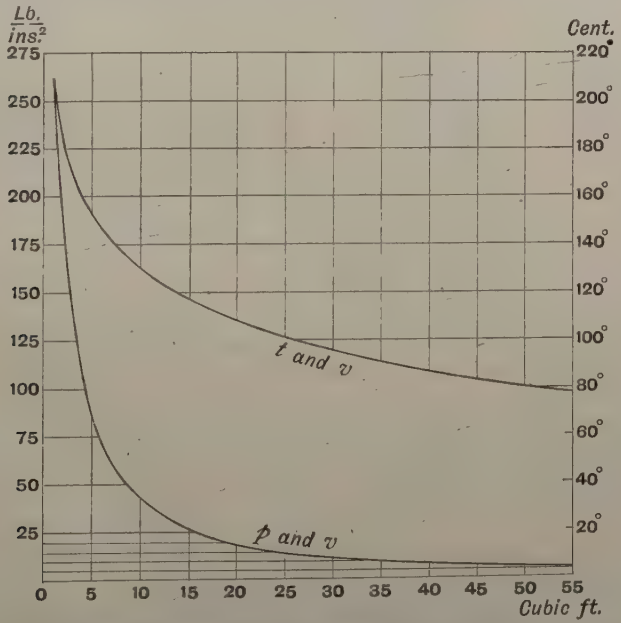

FIG. 440.—Relation of specific volume, pressure and temperature of saturated aqueous vapour.

indirect methods are employed, by means of which the density may be calculated from other experimental data.

The volume occupied by unit mass of a substance in the gaseous state, under given conditions of pressure and temperature, is called the **specific volume.** The specific volume of a saturated vapour is defined when the temperature has been stated, since the saturated vapour of a given substance at a stated temperature can exist at one pressure only. If d is the density of a saturated vapour in grams per cubic centimetre, the specific volume is $1/d$ cubic centimetres per gram.

The graph given in Fig. 440 shows the specific volume of saturated aqueous vapour at various pressures and temperatures.

Latent heat of vaporisation.—If a mass of liquid changes state from liquid to gaseous at constant pressure, there is no change in

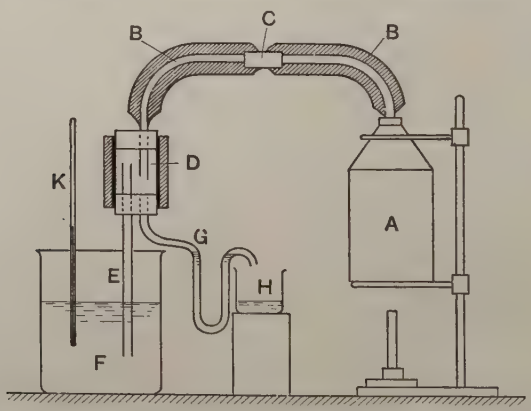

FIG. 441.—Apparatus for determining the latent heat of vaporisation of water.

temperature. The **latent heat of vaporisation** of a substance is the heat which must be imparted at constant temperature to unit mass of the substance in the liquid state in order to effect the change of state from liquid to gaseous.

EXPT. 105.—**Latent heat of vaporisation of water, boiling under a pressure of one atmosphere.** In Fig. 441, a small copper boiler A contains some water which can be heated by means of a bunsen flame. The resulting steam flows out of the boiler through a tube B, consisting of two parts connected at C by a short piece of rubber tube. B is "lagged," *i.e.* heat insulated, by winding strips of flannel round the tube; this helps to prevent condensation of the steam flowing through the tube. The tube B leads into a separator D, consisting of a short piece of wide bore glass tube having

rubber stoppers at both top and bottom. The tube B reaches nearly to the bottom stopper. Another tube E leads from D into a measured quantity of water contained in a calorimeter F. A tube G, bent as shown, is pushed partly into the lower stopper of D, and drains any water from D into a beaker H. There will always be some water in G which will serve to seal the apparatus and will prevent steam being blown through G. The arrangement helps to prevent water from entering the calorimeter, and renders the entering steam as dry as possible. The temperature of the water in the calorimeter is observed by a thermometer K.

Weigh the empty calorimeter; pour in some water and weigh again; by differences find the mass of the water. While this is being done, the water in the boiler is being heated. Let steam discharge freely from the mouth of the tube E into the atmosphere for three or four minutes. Take the temperature of the water in the calorimeter; put the tube E into the calorimeter (the rubber joint at C assists this operation) so that its mouth is below the surface of the water. A crackling noise will be heard, caused by the collapse of steam bubbles undergoing rapid condensation. Continue until the water in the calorimeter has risen in temperature about 20 Centigrade degrees. Remove the tube E; read the temperature of the water; weigh the calorimeter and its contents, and by differences find the mass of the steam which has been condensed.

Let m = the mass of the empty calorimeter, in grams.

s = the specific heat of its material.

m_1 = the mass of the water used, in grams.

m_2 = the mass of the steam condensed, in grams.

t_1° C. = the initial temperature of the water.

t_2° C. = the final temperature.

L = the latent heat of vaporisation of saturated aqueous vapour at 100° C.

The steam has given up latent heat in condensing into water at 100° C., and the resulting water has given up additional heat in cooling from 100° C. to t_2. Assuming that the total heat thus given up is equal to the heat taken up by the mass m_1 of water and the material of the calorimeter, we have

$$m_2\{\mathrm{L} + (100 - t_2)\} = (m_1 + ms)(t_2 - t_1).$$

$$\therefore \ \mathrm{L} = \left(\frac{m_1 + ms}{m_2}\right)(t_2 - t_1) - (100 - t_2).$$

This result is in calories per gram mass of steam.

Careful experiments show that one gram of steam at a pressure of one atmosphere and at 100° C. has a latent heat of 539 calories. Compare the result of your experiment with this number.

The graph in Fig. 442 shows the latent heat of saturated aqueous vapour at other temperatures, ranging from 0° C. to 210° C. It will be noted that the latent heat decreases as the temperature is increased.

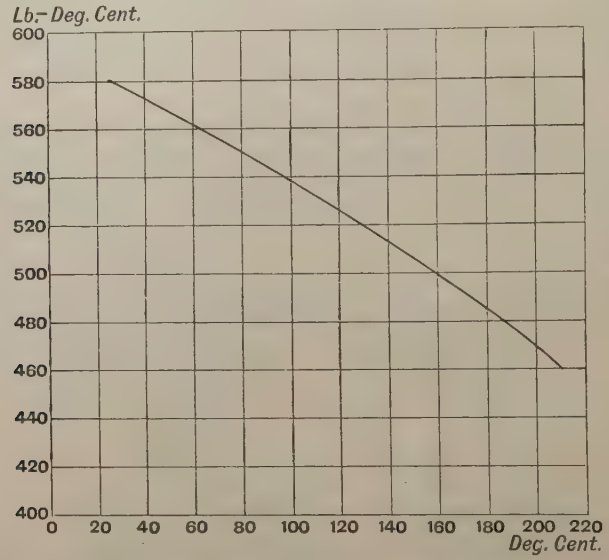

FIG. 442.—Relation of latent heat and temperature of saturated aqueous vapour.

EXPT. 106.—**Freezing water by evaporation of ether.** Stand a beaker containing some ether in a small pool of water spilled on the bench. Use a glass tube and foot-bellows to blow air through the ether. Rapid evaporation of the ether will take place and the heat will become latent so quickly that the water will freeze.

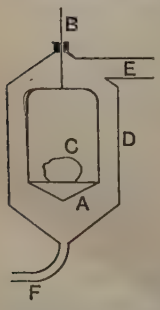

FIG. 443.—Diagram of Joly's steam calorimeter.

Joly's steam calorimeter.—In this apparatus (Fig. 443) a pan A is suspended by means of a fine wire B from one arm of a delicate balance. C is a substance whose specific heat is to be determined. The arrangement is enclosed in a chamber D, which can be supplied with steam from a boiler through a pipe E; the pipe F drains off the water of condensation. A plug of plaster of Paris is fitted at G in order to prevent water of condensation from interfering with the free vertical movements of the suspending

wire B; a small coil of platinum wire heated electrically prevents condensation of steam on the wire beyond the plug.

Expt. 107.—**Specific heat of a substance by Joly's method.** With air only in the chamber, place the substance in the pan, and determine its mass by weighing. Read the temperature as shown by a thermometer in the chamber. Admit steam suddenly so as to fill the chamber quickly with saturated steam, then reduce the steam supply so as to avoid steam currents which would interfere with the following weighing. In a few minutes, the condensation which is taking place inside the chamber ceases, and the substance arrives at the temperature of the steam. Weigh again, thus determining the mass of steam which has been condensed on the substance and pan.

Let m_1 = the mass of the substance, in grams.

m_2 = the mass of the steam condensed, in grams.

t_1° C. = the initial temperature of the chamber.

t_2° C. = the temperature of the steam, obtained by reading the thermometer in the chamber, or by consulting the Table (p. 533).

L = the latent heat of the steam, in calories.

s = the specific heat of the substance.

c = the capacity for heat of the pan (p. 345).

Then, $$m_1 s(t_2 - t_1) + c(t_2 - t_1) = m_2 \mathrm{L}.$$

$$m_1 s(t_2 - t_1) = m_2 \mathrm{L} - c(t_2 - t_1).$$

$$s = \frac{m_2 \mathrm{L}}{m_1(t_2 - t_1)} - \frac{c}{m_1}.$$

Make another experiment with the pan empty; a similar calculation will then give the value of the capacity for heat of the pan. Insert this value in the above equation and calculate the value of s. In accurate work, corrections are applied for the buoyancies of the air and steam in the weighing operations.

The **differential Joly calorimeter** is used for determining the specific heat of gases at constant volume. The apparatus (Fig 444) is arranged so as to eliminate corrections as far as possible. Two equal copper globes are suspended from the opposite arms of the balance, and hang in the steam chamber. One globe is filled with the gas under test and the other is exhausted. Small pans are fitted to the globes in order to catch the water condensed on their surfaces. Greater condensation takes place on the globe containing gas, and the excess gives the data required in calculating the quantity of heat necessary in order to raise the temperature of the enclosed gas.

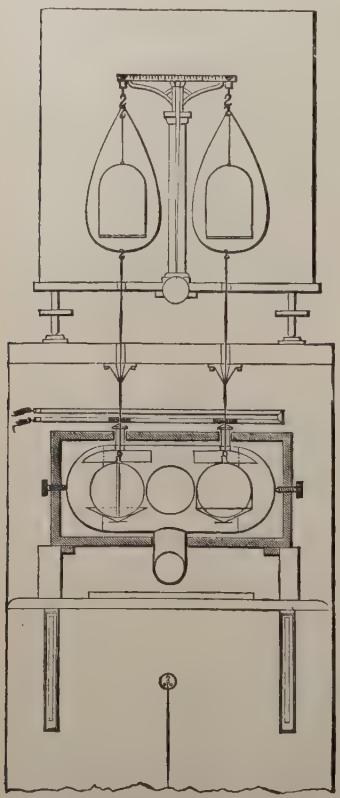

FIG. 444.—Differential Joly calorimeter.

Let m_1 = the mass of the gas filling the sphere.

m_2 = the difference between the masses of the steam condensed on the globes.

t_1 = the initial temperature.

t_2 = the temperature of the steam.

p_1 = the initial pressure of the gas.

p_2 = the final pressure of the gas.

L = the latent heat of the steam.

s = the specific heat of the gas at constant volume for the temperature range t_1 to t_2, and the pressure range p_1 to p_2.

Then $$m_1 s(t_2 - t_1) = m_2 \mathrm{L},$$

$$s = \frac{m_2 \mathrm{L}}{m_1(t_2 - t_1)}.$$

EXERCISES ON CHAPTER XXXV.

1. State Dalton's law for the pressure of a mixed gas and vapour. A closed vessel contains air saturated with aqueous vapour; the temperature is 70° C. and the absolute pressure inside the vessel is 78·2 cm. of mercury. Find the pressure exerted by the air in the vessel.

2. Draw the isothermal line for 4 cubic feet of dry air at 15 lb. wt. per square inch absolute pressure and 94° C. when compressed to a volume of one cubic foot. If the air is saturated with aqueous vapour, draw the isothermal line for the mixture.

3. Some hydrogen is collected over water; the measured volume is 245 c.c.; the temperature of the water is 16° C.; the barometer reads 75·43 cm. of mercury. Find the volume of dry hydrogen at 0° C. and 76 cm. of mercury.

4. In a test on the vapour density of alcohol, using the method of Dumas (p. 461), the following observations were made: Weight of bulb, 9·77 grams; weight of the bulb and vapour filling it at 100° C., 9·889 grams; weight of the bulb when full of water, 141·65 grams; temperature of the room, 15° C.; barometric height, 75·1 cm. Find the vapour density of alcohol at 100° C. and 75·1 cm. pressure, using hydrogen as the standard.

5. Describe an experiment for determining the vapour density of a given substance.

6. Using Victor Meyer's method (p. 463), the following readings were taken in an experiment on the vapour density of alcohol: Weight of the empty phial, 1·415 grams; weight of the phial containing alcohol, 1·738 grams; volume of air collected, 171·2 c.c.; barometric height, 76·29 cm.; temperature of air, 15·2° C. Find the vapour density of alcohol at 0° C. and 76 cm. Use hydrogen as the standard.

7. Take the required data from the Table (p. 534), and plot graphs showing the relation of specific volume and (*a*) pressure, (*b*) temperature for saturated aqueous vapour.

8. Define latent heat of vaporisation. Describe an experiment for determining the latent heat of vaporisation of water at ordinary boiling point.

9. Find the quantity of heat which must be supplied to one pound of water at 20° C. in order to convert it into dry saturated steam at a pressure of 146 lb. wt. per square inch absolute. Take the quantities required from the Table on p. 534.

10. A tank contains 20 gallons of water at 30° C. which is heated by discharging dry saturated steam at atmospheric pressure into it. If the final temperature is 80° C., find the weight of the steam used. Consult the Table on p. 534 for any quantities required.

11. What is meant by the statement, "The maximum pressure of aqueous vapour at 15° C. is 13 mm."?

Some nitrogen is collected in a tube over water at 15° C. and is found to occupy 50 c.c., the gaseous pressure in the tube being 743 mm. Calculate the volume which the dry nitrogen would have at 0° C. and 760 mm. pressure. Sen. Cam. Loc.

12. Explain the meaning of latent heat. How would you determine the latent heat of fusion of ice? Explain the cooling effect of a fan.
Calcutta Univ.

13. Explain how you would determine the latent heat of steam. Would you expect it to depend in any way on the temperature at which water is boiled? 7 grams of ice float in water in a calorimeter of thermal capacity 5 calories. If, when 4·5 grams of steam (at 100° C.) are passed into the calorimeter, the final temperature becomes 50° C., how much water was there in the calorimeter? (Latent heat of steam at 100° C. = 540.)
Calcutta Univ.

14. Define "vapour density." Describe how to find the vapour density of a liquid, using the method of Dumas. Prove the formula.

15. Describe Victor Meyer's method of finding the vapour density of a liquid. Prove the formula. Bombay Univ.

16. Draw a curve indicating generally the change in maximum vapour pressure of water between 0° and 100° C.

Given that, at any temperature, the maximum vapour pressure is slightly diminished upon dissolving a salt in water, show that the boiling point of the solution at any pressure is higher than that of pure water. L.U.

17. Describe the properties of saturated and non-saturated vapours.

A barometer tube dipping into a mercury reservoir contains a mixture of air and saturated vapour above a column of mercury which is 70 centimetres above the level in the reservoir, the atmospheric pressure being 76 centimetres. What is the height of the mercury column when the tube is depressed so as to reduce the volume occupied by the air to one half its original value, the pressure of the saturated vapour being 1·5 centimetres ?
L.U.

18. Give the definitions of the expressions, specific heat, latent heat of fusion, latent heat of vaporisation, thermal conductivity.

Fifty grams of steam at 100° C. are passed into a mixture of 100 grams of ice and 200 grams of water at 0° C. Find the rise of temperature, the latent heat of vaporisation of water at 100° C. being 537 and the latent heat of fusion of ice 80. Sydney Univ.

19. Describe how the specific heat of a body may be found by the Joly steam calorimeter. Given the following observations, find the specific heat of the sample of calcite : Preliminary experiment ; mass of the dry pan in the steam calorimeter, 28·395 grams ; mass of pan and condensed steam, 28·565 grams. Calcite experiment ; mass of the dry pan and dry calcite, 41·46 grams ; mass of pan, calcite and condensed steam, 42·016 grams. Initial temperatures in both experiments, 24° C.

20. Describe an experiment for determining the specific heat of a gas at constant volume.

CHAPTER XXXVI

ATMOSPHERIC CONDITIONS. HYGROMETRY

Evaporation from free surfaces of water.—Evaporation takes place constantly at all temperatures from open surfaces of water. Molecules escape from the surface of the liquid and may be carried away by currents of air, so that there is no chance of return to the liquid. The continual loss of liquid from open surfaces of water is compensated by the return of rain water contained in rivers and streams. Increase in temperature of the water will increase the quantity evaporated in a given time. Increase in the speed of the wind also causes increased rates of evaporation by further diminishing the opportunities of return to the liquid. A familiar illustration of this fact occurs in the drying of the wet interior of a tube by blowing air into it by means of bellows.

The general result of evaporation from open sheets of water is that the atmosphere always contains aqueous vapour.

Different liquids contained in open vessels under like conditions possess different rates of evaporation. Liquids which evaporate more easily are said to be **volatile**. Ether, alcohol and petrol are examples of volatile liquids.

Mist, cloud, dew.—Our perceptions regarding the dryness of the atmosphere depend on its **hygrometric state**, a quantity which may be expressed as the ratio of the mass of water vapour present per cubic centimetre of air to the mass of water vapour which would be required to saturate one cubic centimetre of air if the temperature remained unaltered. If the air be not saturated with water vapour at a given temperature, it may become so if the temperature falls; less vapour is required to produce saturation at low temperatures. Hence the presence of a cold body in an atmosphere which has not quite reached saturation point may cool the air in its immediate vicinity sufficiently to produce saturation or over-saturation. Some

of the vapour will then condense on the surface of the cold body, producing the phenomenon of **dew**.

If a large body of the atmosphere be cooled slowly, saturation point will be reached throughout the mass simultaneously. The atmosphere is charged more or less with particles of dust, and condensation takes place on each dust particle, giving rise to a **mist**. The fogs of large towns are caused by condensation of this kind on soot particles floating in the atmosphere.

Clouds are caused by similar condensation of water vapour in the upper strata of the atmosphere; heated air containing water vapour, but above the saturation point, becomes colder as it ascends and expands, and the water vapour condenses to a mist or cloud on the saturation point being reached.

Evaporation of snow and ice.—Supposing that a solid exposed to the atmosphere is undergoing the process of melting, and that the maximum vapour pressure of the substance at the temperature of the atmosphere is equal to, or greater than, the pressure of the atmosphere, then it is impossible that the substance can remain in the liquid state, and the phenomenon is presented of the substance changing direct from the solid to the gaseous state. The process is called **sublimation**; iodine among other substances possesses this property. Snow and ice evaporate slowly; in Arctic regions this is the only method of evaporation possible. There is an appreciable vapour pressure of water substance below 0° C., and consequently ice can pass directly from the state of solid to that of vapour without any intermediate liquid state.

Hoar-frost is deposited instead of dew if the temperature of the slowly-cooling atmosphere reaches the freezing point of water before saturation occurs. The process of formation of hoar-frost and of snow does not consist in the freezing of dew, but the direct change of state from aqueous vapour into the solid form.

Ventilation.—The ordinary atmosphere contains carbonic acid gas to the amount of 3 or 4 parts in 10,000 parts of air. Indoors the proportion is usually higher, especially if many persons be present in a room. An adult person at rest produces about 0·6 cubic feet of carbonic acid gas per hour, and the conditions in a room become objectionable unless these exhalations are diluted largely by a plentiful supply of air admitted to the room. Should the proportions exceed 10 parts of carbonic acid gas in 10,000 of air, there will be discomfort. In ventilation problems a limit of 6 to 7 parts of

carbonic acid gas in 10,000 of air is aimed at. To dilute sufficiently the gas expelled by an adult requires from 1800 to 3600 cubic feet of air per hour, depending upon the particular conditions existing in the building.

In ordinary houses, open doors, windows, and the chimney of open fireplaces generally suffice. None of these are very effective; and it should be noted that fireplaces produce the peculiar condition that a greater number of persons in the room will require more energetic ventilation, produced by urging the fire, despite the fact that the people by their presence are assisting to raise the temperature of the room.

In mechanical methods of ventilation, a fan is employed to propel air into the room, or to withdraw air from it. Fresh air is led into the room by means of properly arranged ducts. The incoming air may be warmed by passing it over pipes through which steam or hot water circulates, and may be brought to a proper hygrometric state by passing it over water, or by spraying water into it.

Dew point.—The temperature at which dew begins to form when the atmosphere undergoes cooling is called the **dew point.** Experiments on the determination of the dew point are based generally on the principle of cooling artificially a portion of the atmosphere until it is observed that dew is commencing to form. The cooling is carried out at constant pressure—that shown by the barometer—and Charles's law may be assumed to be followed by the vapour molecules until the temperature of saturation is reached.

The perception of dampness in the atmosphere is a consequence of a near approach to saturation conditions, under which evaporation goes on very slowly. The actual quantity of aqueous vapour present is only one of the factors to consider; warm air may contain much more aqueous vapour than colder air, and yet may convey the impression of dryness because the atmosphere is not nearly saturated.

Relative humidity may be defined as the ratio of the pressure of the aqueous vapour actually present at the existing temperature to the pressure of the vapour which would be present if the vapour were saturated at the same temperature.

Let t_1° C. = the observed temperature of the atmosphere.

t_2° C. = the dew point.

p_1 = the pressure of aqueous vapour when saturated at t_1 (see Table, p. 533).

p_2 = the pressure of aqueous vapour when saturated at t_2 (see Table).

Assuming that the barometric pressure has remained constant in the process of determining the dew point by cooling a portion of the atmosphere, p_2 is the pressure of the aqueous vapour actually present in the atmosphere. Hence

$$\text{Relative humidity} = \frac{p_2}{p_1}.$$

Inspection of the following short Table shows that the masses per cubic metre at two given temperatures have nearly the same ratio as the pressures corresponding to the same temperatures. Hence it is often sufficiently accurate to take

$$\text{Relative humidity} = \frac{\text{mass of water vapour per unit volume of air at observed temp.}}{\text{mass of water vapour per unit volume of saturated air at same temp.}}.$$

PROPERTIES OF SATURATED AQUEOUS VAPOUR.

Temp. ; deg. Cent.	0	5	10	15	20
Pressure of sat. vapour ; mm. of mercury	4·58	6·54	9·20	12·78	17·51
Mass of sat. vapour ; grams per cub. metre	4·84	6·76	9·33	12·71	17·12
Temp. ; deg. Cent.	**25**	**30**	**35**	**40**	
Pressure of sat. vapour ; mm. of mercury	23·69	31·71	42·02	55·13	
Mass of sat. vapour ; grams per cub. metre	22·80	30·04	39·18	50·7	

Hygrometry is the determination of the state of the atmosphere as regards the hygrometric state, dew point, etc.

EXPT. 108.—**Determination of the dew point by Regnault's hygrometer.** A form of Regnault's apparatus is illustrated in Fig. 445. A brightly polished silver vessel A contains some ether B and has a tube C dipping into the ether. Air may be pumped into the ether through C by means of a perfume spray bulb D ; this air accelerates evaporation from the ether and the resultant mixture of air and vapour escapes through a tube E. The evaporation of the ether cools the vessel and also the atmosphere in

the immediate neighbourhood, and will ultimately cause saturated conditions to be attained by the air near the vessel. A film of dew will then be deposited on the bright surface of the vessel. The temperature at which dew appears may be read by means of the thermometer F. Another similar but empty silver vessel G is mounted on the same stand and aids, by comparison of the surfaces, the detection of the first appearance of dew.

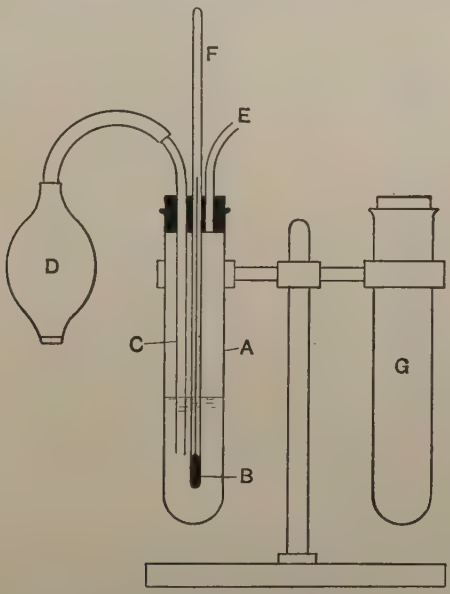

FIG. 445.—Regnault's hygrometer.

The temperature of the atmosphere in the room is read by another thermometer. A large sheet of glass should be interposed between the observer and the instrument in order to prevent his breath temporarily increasing the amount of moisture present.

Lower the temperature slowly and observe the reading of the thermometer F when dew appears. Allow the temperature to rise and note the reading of F when the dew disappears. Repeat several times and take the mean temperature as the dew point. Note the temperature of the atmosphere in the room in the neighbourhood of the apparatus.

Calculate the relative humidity of the atmosphere at the time of the experiment (p. 475).

EXPT. 109.—**Determination of the dew point by Daniell's hygrometer.** This instrument (Fig. 446) consists of two glass bulbs A and D connected by

a tube C, from which all air has been exhausted. A contains some ether, the remainder of the interior contains ether vapour only. A thermometer B is situated inside A, its bulb dipping into the ether. The internal surface of A is blackened, or gilt, in order to enable the formation of dew on the external surface to be observed easily. The thermometer E enables the temperature of the atmosphere to be observed. The bulb D is covered with muslin, and is saturated with some ether. Rapid evaporation of this ether takes place, and consequently the bulb D is cooled. The ether vapour inside D is thus cooled and condensed, and more vapour is evaporated from the ether in A to take its place. Thus the bulb A is cooled slowly, and presently dew appears on the surface. Take similar readings to those observed with the Regnault hygrometer and estimate the dew point and the relative humidity. The sheet of glass should be employed as before between the observer and the instrument.

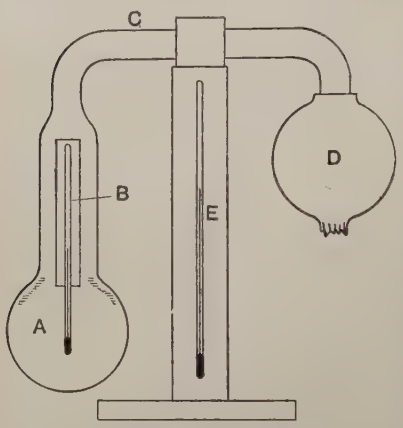

FIG. 446.—Daniell's hygrometer.

Wet and dry bulb method.—The pressure of the aqueous vapour present in the atmosphere, and hence the dew point, may be found by the **wet and dry bulb thermometer method.** Two thermometers are arranged on a stand ; one is an ordinary thermometer and shows the temperature of the atmosphere. The bulb of the other thermometer has a piece of clean lamp wick wrapped round it, which is led to a small basin of water ; this keeps the bulb moistened. If there be much aqueous vapour present in the atmosphere, there will be but little evaporation from the moisture near the bulb of the thermometer, and therefore only a small cooling effect. The difference in reading of the thermometers will thus depend on the hygroscopic state of the atmosphere. Both thermometers are read when steady conditions have been attained, and the aqueous vapour pressure is found by use of specially constructed tables. The method does not give very accurate results.

EXPT. 110.—**Chemical hygrometer.** The apparatus required is shown in Fig. 447. A and B are drying tubes charged with phosphorus pentoxide ; C is also charged with the same substance and serves to seal the drying tubes from the vessel D. D is a large stone jar furnished with a tap at the

bottom. By filling D with water and then making the connections as shown, a volume of air equal to the capacity of D will be drawn through the apparatus, entering at E. D is called an aspirator. Practically all moisture in the entering air is absorbed in A; any escaping A will be absorbed in B. C absorbs any moisture which might be travelling backwards from the vessel D when the flow of water from the tap is stopped. The temperature of the air entering the apparatus is observed by the thermometer G.

Pour water into D until it is quite full. Disconnect A and B at H and weigh both together in order to determine the mass m_1 grams. Connect up again and run the water entirely out of D through the tap; read the

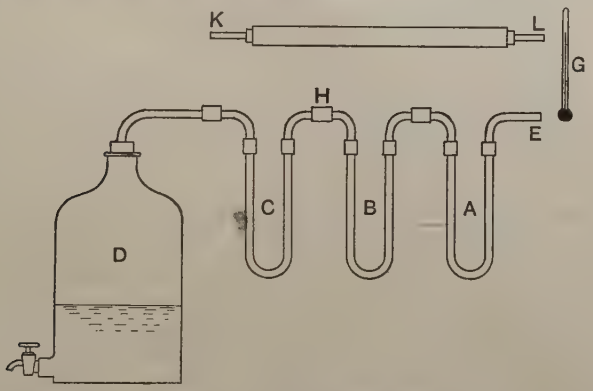

FIG. 447.—Chemical hygrometer.

thermometer G while the water is escaping. Disconnect at H and again weigh A and B together; let the mass be m_2 grams. Then the mass of moisture absorbed is $(m_2 - m_1)$ grams.

Fill D again and make the connections. KL is a wide bore glass tube (Fig. 446) having a rubber stopper at each end fitted with short pieces of tube of smaller bore. The tube is charged with broken pumice and water; any air passed through this tube will become saturated with water vapour. Connect the tube K to the apparatus at E and repeat the operations described above. Let m_3 be the final mass of A and B together, then $(m_3 - m_2)$ will be the weight of saturated aqueous vapour which can be held by air at the existing temperature.

Since the volume of air passing through the apparatus is practically the same in both experiments, the relative humidity of the atmosphere γ (p. 476), obtained by dividing the actual mass of vapour per cubic centimetre of air by the mass of vapour required to saturate one cubic centimetre of air, is given by

$$\gamma = \frac{m_2 - m_1}{m_3 - m_2}.$$

Exercises on Chapter XXXVI.

1. State the cause of the presence of aqueous vapour in the atmosphere. Describe the formation of dew, mist, fog and clouds.

2. There are six adults in a room measuring 16 feet by 12 feet by 10 feet, and each requires 2500 cubic feet of air per hour. Find the number of times per hour that the air in the room should be changed. Supposing each person exhales 0·6 cubic foot of carbon dioxide gas per hour and that the air in the room contains at first 4 parts of this gas per 10,000, in what time will the proportion reach 10 parts per 10,000 if ventilation be absent entirely?

3. Define the dew point. State the conditions upon which it depends. What is meant by "relative humidity"?

4. Describe Regnault's hygrometer. In an experiment with this instrument, the mean of three tests gave the temperature of dew formation 9·6° C., and of the disappearance of dew 10·5° C. The temperature of the atmosphere was 18° C. What is the dew point? Find also the relative humidity.

5. Describe the Daniell hygrometer. In an experiment with this instrument, the dew point was found to be 11·13° C. and the temperature of the atmosphere was 19° C. Find the relative humidity.

6. Describe the wet and dry bulb method of determining the hygrometric state of the atmosphere.

7. Describe the method of carrying out an experiment with a chemical hygrometer. The following readings were obtained with an instrument of this type: Weight of U tubes at first, 85·48 grams; weight of U tubes together with water vapour from ordinary air, 85·61 grams; weight of U tubes together with water vapour from saturated air, 85·88 grams; temperature of air, 18° C. Find the hygrometric state and the dew point.

8. Describe an experiment to determine the dew point. What do you understand by a table of saturation pressures of aqueous vapour? Explain how relative humidity of the air can be determined by the dew point and such a table. Panjab Univ.

9. Calculate the dew point when the air is $\frac{2}{3}$ saturated with water vapour, the temperature being 15° C., given that for pressures of 7, 9, 11, 13 mm. of mercury, the corresponding boiling-points of water are 6°, 10°, 13°, and 15° C. respectively. Sen. Cam. Loc.

10. What is meant by "sublimation"? Explain how hoar-frost is formed.

11. Calculate the mass of 7·6 litres of moist air at 27° C. given that the dew point is 15° C., and the barometric height 762·75 mm. Calculate also the humidity of the air. Vapour pressure of water at 27° C. and 15° C. = 25·5 mm. and 12·75 mm. respectively. Bombay Univ.

12. Describe the systems of ventilation used (*a*) in your physical lecture room, (*b*) in your physical laboratory. Make reference to the state of the atmosphere in these rooms both in cold weather and in warm weather. In your opinion are the means employed satisfactory? If not, give reasons.

CHAPTER XXXVII

EXPANSION AND COMPRESSION OF VAPOURS. REFRIGERATORS

Formation of vapour at constant pressure.—In Fig. 448 (*a*) is shown a vertical cylinder fitted with a piston having an area of one square unit. Underneath the piston is unit mass of a given substance in the liquid form; if the volume of this quantity of liquid is v, then the height occupied by the liquid will be equal to v. The piston is loaded so as to produce a constant pressure p on the liquid, and the temperature is taken to be that of the saturated vapour of the substance corresponding to the pressure p. This temperature is kept constant in what follows.

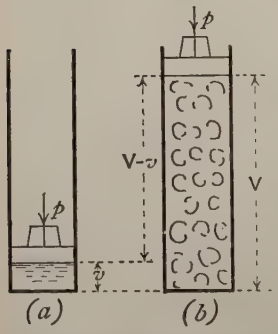

FIG. 448.—Formation of vapour at constant pressure.

Let heat be imparted to the substance; the effect will be the formation of vapour at constant pressure, and the piston will rise in order to accommodate the increased volume. Stop the supply of heat at the instant when the whole of the liquid has been converted into vapour. The cylinder will then be full of saturated vapour (Fig. 448 (*b*)). Let the volume of the vapour (which will still have unit mass) be V, then the length of cylinder now occupied will be equal to V, and the height through which the piston has been displaced will be $(V - v)$ (Fig. 448 (*b*)). The quantity of heat taken in is equal to the latent heat of vaporisation of the substance under the given conditions of pressure and temperature. Let this be L units of heat. Some of the heat has been used in doing external work, and the remainder has gone to increase the internal energy. Since the external work done is $p(V - v)$, the increase in internal energy is given by

$$\text{Increase in internal energy} = L - \frac{p(V - v)}{J} \text{ thermal units, ...(1)}$$

J being the mechanical equivalent of heat.

Supposing we had started with the liquid in the cylinder at 0° C., we should have had to supply h units of heat in order to raise the temperature to that at which vaporisation begins; adding this to the latent heat L, we obtain a quantity which may be called the **heat of formation** of the vapour. Thus

$$\text{Heat of formation} = \mathsf{H} = h + \mathsf{L} \text{ thermal units.} \quad \ldots\ldots\ldots\ldots(2)$$

The heat of formation of aqueous vapour was, until recently, called the **total heat** of steam. The term total heat is now taken to include the relatively small quantity of work which must be done in order to force the liquid into the cylinder against the resistance offered by the pressure p. This work, expressed in thermal units, is pv/J. Hence, for unit mass of a saturated vapour,

$$\text{Total heat} = \mathsf{I} = h + \mathsf{L} + pv/\mathsf{J} \text{ thermal units.} \quad \ldots\ldots\ldots\ldots(3)$$

During the process of warming the liquid, the heat h has been stored in the form of internal energy, neglecting the very small amount of external work which has been done owing to the expansion of the liquid. Hence the increase in internal energy, reckoned from the initial temperature of 0° C., is

$$\text{Increase in internal energy} = h + \mathsf{L} - \frac{p(\mathsf{V} - v)}{\mathsf{J}} \text{ thermal units.} \quad \ldots(4)$$

There are no means available for estimating the total internal energy of a substance under given conditions. It is therefore convenient to choose the arbitrary **zero state** in which the substance is in the liquid form at 0° C. and under a pressure equal to that of the saturated vapour at 0° C. A substance in the zero state is described as having no internal energy. Internal energies for temperatures below 0° C. are reckoned negative.

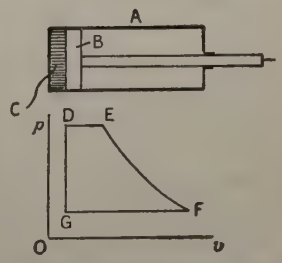

FIG. 449.—Expansion of a vapour in a cylinder.

Expansion and compression of a vapour.—In Fig. 449 is shown a cylinder A fitted with a piston B and containing unit mass of a substance in the liquid state, under pressure produced by the force P. In the pressure-volume diagram D is the corresponding point. Let the liquid be at the temperature t_1 of vaporisation corresponding to the pressure, and let the piston move outwards, at the same time permitting heat at temperature t_1 to pass into the cylinder. The liquid thus takes up latent heat of vaporisation. If P be maintained uniform, the operation is represented by the constant

pressure line DE, and at E vaporisation is complete. It is evident that this process is isothermal.

The cylinder now contains saturated vapour, and any further outward movement of the piston will produce expansion of this vapour, accompanied by falling pressures as shown by the curve EF. There are various ways in which the expansion may be performed. If heat is supplied continuously in quantity sufficient to maintain constant the temperature during expansion, the vapour at F would be superheated ; its temperature would still be t_1, and this temperature is higher than the saturation temperature corresponding to the lower pressure at F. Such expansion is isothermal.

In practice an attempt is generally made not to superheat the vapour during expansion, but merely to supply heat sufficient to maintain the vapour in the dry saturated state.

If no heat at all be supplied during expansion, *i.e.* if the expansion is adiabatic, then the vapour, which is dry at E (Fig. 449), may be wet at F. This is the case with water vapour ; work has been done against the resistance of the piston, and the heat necessary to do this work has been abstracted from the vapour.

Supposing the vapour to be dry and saturated at F, its temperature being t_2, condensation may be effected by pushing the piston inwards, at the same time permitting heat to flow freely out of the cylinder. The vapour will give out its latent heat at constant temperature t_2, and the pressure will be constant as shown by FG (Fig. 449). This operation is isothermal, and at G we have again unit mass of liquid, but at a lower temperature t_2. The original conditions may be restored by applying pressure and imparting heat to the water, thus bringing us back again to the point D.

A complete series of operations of this kind, starting with given conditions and bringing the substance back again to the same conditions, is called a **cycle of operations.**

Critical temperature.—Water expands when heated, and the volume of saturated aqueous vapour which can be produced from a given mass of water diminishes as the temperature and pressure increase. It would therefore appear that a **critical temperature** can be attained at which the volume of the water and the volume of the saturated vapour formed from the water are equal. At this temperature, which is about 365° C., the water and the aqueous vapour cannot be distinguished from one another. The reader will remember that

the latent heat of aqueous vapour diminishes as the temperature of the saturated vapour increases. At the critical temperature the latent heat becomes zero. The pressure of aqueous vapour at the critical temperature is about 194·6 atmospheres (2860 lb. weight per square inch).

A substance in the gaseous state and at a temperature higher than the critical temperature cannot be liquefied by mere pressure. Before liquefaction can commence the temperature must be brought below the critical temperature. The critical temperature may be defined as the highest temperature at which the substance in the gaseous state may be liquefied by application of pressure. The **critical pressure** is the pressure of the saturated vapour of the substance at the critical temperature. The term **gas** is usually applied when the substance is at temperatures higher than the critical temperature, and **vapour** at temperatures below the critical temperature. Hence we may say that a vapour can be liquefied by pressure alone, but a gas cannot.

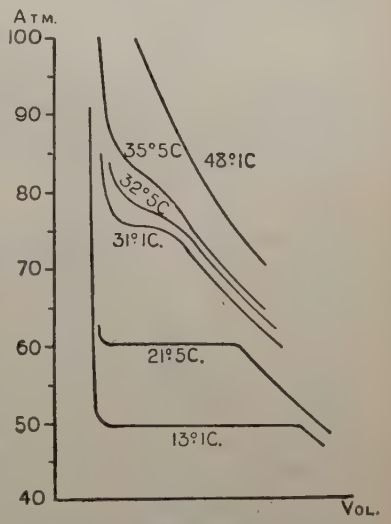

FIG. 450.—Isothermal lines for CO_2.

Andrews found that carbon dioxide could be liquefied by applying pressure, provided that the temperature did not exceed 31·1° C.; at this temperature a pressure of 73 atmospheres (1074 lb. weight per square inch) is required. Some isothermal lines for CO_2 are shown in Fig. 450. If any part of an isothermal line becomes horizontal, it may be inferred that latent heat is being imparted to, or removed from the substance, which accordingly is undergoing evaporation or liquefaction. It will be noted that liquefaction may be obtained at 13·1° C. with a pressure of about 50 atmospheres. No liquefaction occurs if the temperature exceeds 31·1° C., which is thus the critical temperature of CO_2. Some critical temperatures are given in the following Table.

CRITICAL TEMPERATURES AND PRESSURES.*

Substance.	Critical temp., C.	Critical pressure, atmospheres.
Hydrogen - -	−234·5	20
Oxygen - -	−118	50
Air - - -	−140	39
Water - - -	365	194·6
Carbon dioxide -	31·1	73
Ammonia - -	130	115
Sulphur dioxide -	155·4	78·9

Liquefaction of gases.—In liquefying a gas sufficient pressure may be obtained by means of a pump, and the temperature may be lowered below the critical point by taking advantage of the cooling

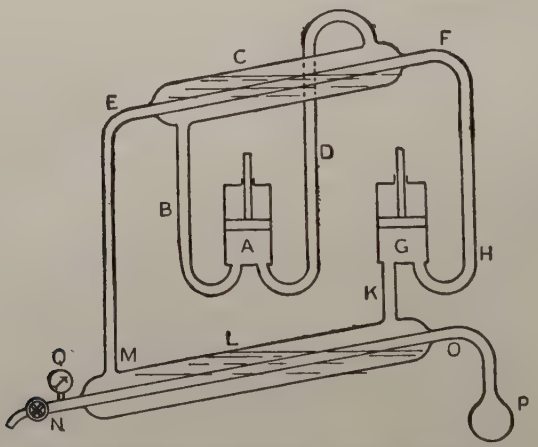

FIG. 451.—Pictet's apparatus for liquefying oxygen.

effect produced when a liquid evaporates. Pictet's method of liquefying oxygen may be understood by reference to Fig. 451. A pump A draws SO_2 vapour (which can be liquefied very easily) from a tube C, forming a jacket round another tube EF, compresses the vapour and delivers it back again through B into C as liquid SO_2. Evaporation of this liquid takes place in C, and by the taking up of latent heat, chills the tube EF and its contents.

*For fuller data, see *Physical and Chemical Constants*, by Kaye and Laby; Longmans.

Another pump G draws CO_2 vapour through K from a tube L, which forms a jacket round the tube NO ; the vapour is compressed by the pump G, and is delivered through H into FE ; here the chilling which occurs causes the CO_2 vapour to condense into the liquid state, and it is returned as liquid through the tube EM into L. Evaporation of the CO_2 takes place in L during the suction stroke of the pump G, and the taking up of latent heat chills the contents of the pipe ON. The complete arrangement constitutes a two-stage cooling device for chilling the tube ON. P is a strong steel vessel connected to ON and containing potassium chlorate, which gives off oxygen when heated. The tube ON is closed by a valve at N, and has a pressure gauge fitted at Q. On heating P, oxygen is given off, and, since the tube ON is closed, a high pressure of gas is produced. This pressure, combined with the chilling of the tube ON, is sufficient to liquefy the oxygen.

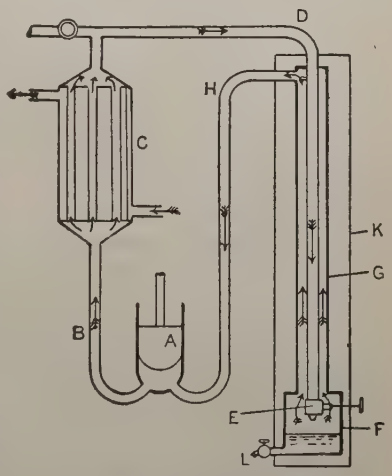

FIG. 452.—Linde's apparatus for liquefying air.

Using an improved form of Pictet's apparatus, Dewar liquefied both oxygen and air in considerable quantities. The resulting liquids can be stored in the vacuum vessel devised by Dewar (p. 386).

Linde's apparatus for liquefying air.—Dr. Joule and Lord Kelvin showed by experiment that, when certain gases are expanded from a higher to a lower pressure by passage through a porous plug, a small fall in temperature occurs. In the case of air, the fall in temperature is about 0·25 degree Centigrade for each atmosphere difference in pressure on the two sides of the plug. Advantage is taken of this fact in Dr. Linde's apparatus for liquefying air. Air which has been cooled slightly by expansion through a small orifice is used to cool more air approaching the orifice. The same air is expanded again and again through the orifice, and undergoes successive cooling during each passage. The arrangement is shown in outline in Fig. 452. A pump A compresses the air and delivers it through a pipe B into a cooler C, where its temperature, which has risen during compression, is lowered. The air passes from the cooler into a pipe DE ; there is a throttling valve at E, by means of which the air experiences a drop in pressure and expands into a box F.

The air, thus cooled somewhat, passes upwards through a pipe G, which encloses the pipe DE and abstracts some heat from the air passing through DE on its way to the throttling valve. The air is then withdrawn from the pipe G through H, and is again compressed by the pump. The action is continuous, and if it is maintained long enough, will result in liquid air collecting in the box F. A valve L enables the liquid air to be withdrawn. The pipes G and DE are enclosed in a case K, the space being packed with heat-insulating substance, and constitute an interchanger. Usually in practice a two-stage air compressor is used, in which one pump delivers air partially compressed into another pump which completes the compression. The air is cooled during its passage from pump to pump, and the second pump delivers the air into a cooler surrounded by a mixture of ice and salt. The interchanger may consist of three tubes arranged concentrically and coiled into a spiral. With this modified apparatus, liquid air may be obtained in a few minutes ; the production is accelerated if carbonic acid snow is used for preliminary cooling.

Refrigerating machines using a vapour.—In modern refrigerating machines vapour is used as the working fluid, and alternately is condensed into the liquid form and evaporated again into vapour. In Fig. 453 is shown an outline diagram of such a machine. The vessel A contains a pipe coil which is surrounded by the substance to be chilled—usually calcium chloride brine. The working fluid is allowed to evaporate inside this coil, and in doing so obtains its latent heat of vaporisation at the expense of the heat in the brine, which accordingly is chilled further.

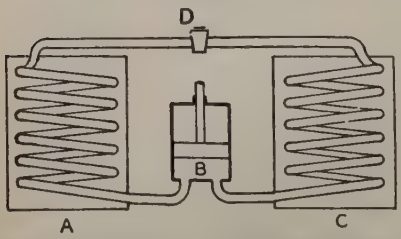

FIG. 453.—Diagram of a refrigerating machine using a vapour.

A pump B draws the resulting vapour from the coil in A and compresses it, the temperature rising as the pressure increases. The pump delivers the compressed vapour into another pipe coil contained in a vessel C, which is kept cool by means of circulating water. The pressure being high, and the temperature being lowered in this pipe coil, the vapour condenses again, giving up its latent heat to the circulating water. The resulting liquid is fed through a regulating valve D into the pipe coil in A, where, the pressure being much lower than in C, evaporation takes place again.

The chilled brine is taken from the vessel A through pipes, and is circulated through the room which has to be maintained at low temperature. The brine becomes heated somewhat during its passage by abstraction of heat from the walls and substances in the room, and is returned again to A to be chilled further. Pumps are used in order to maintain the circulation of the brine.

Substances used in refrigerating machines.—The substances generally used in vapour refrigerating machines are anhydrous ammonia,

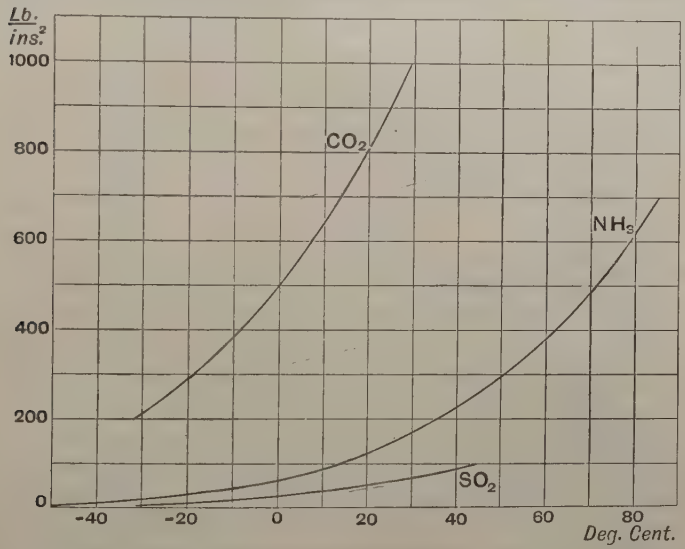

FIG. 454.—Relation of pressure and temperature for refrigerating substances.

NH_3, carbon dioxide, CO_2, and sulphur dioxide, SO_2. It is obvious that water cannot be employed since it would freeze at the low working temperatures. The substances mentioned possess physical properties which render them suitable as refrigerating agents. In Fig. 454 the relations of the pressure and temperature of the saturated vapours of the three substances are shown. It will be noticed that the pressure of sulphur dioxide saturated vapour becomes very low at temperatures below the freezing point of water. Thus at $-25{\cdot}8°$ C. the pressure is 7 lb. weight per square inch absolute, and the volume occupied by one pound of the vapour is 10·3 cubic feet (Fig. 455). It follows that at refrigerating temperatures there is a possibility of air leaking into the apparatus and thereby interfering

with the working. Further, the dimensions of the machine must be large, since the volume of the vapour is large. Sulphur dioxide machines are convenient for dairy installations in which the working temperatures are not required to be very low.

The saturated vapour of ammonia has a pressure of 16 lb. weight per square inch absolute at $-31{\cdot}2°$ C., hence there is no risk of air leaking into an ammonia machine. The volume at the above pressure is 16·56 cubic feet per pound of saturated vapour. Carbon

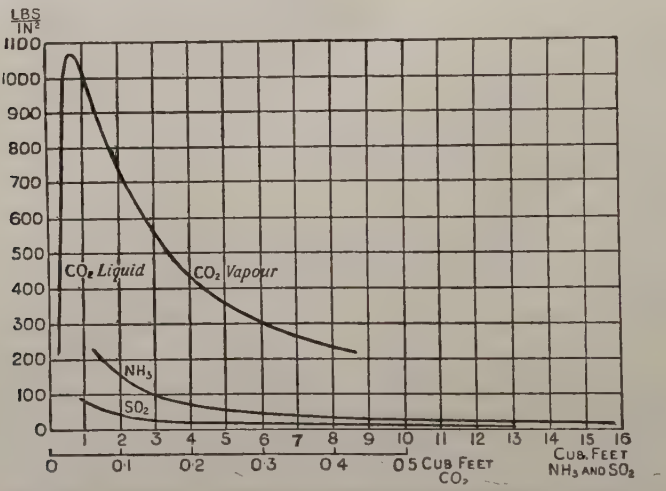

FIG. 455.—Relation of pressure and specific volume for refrigerating substances.

dioxide saturated vapour has a pressure of 225 lb. weight per square inch absolute at $-28{\cdot}3°$ C., and the volume is 0·409 cubic feet per pound. The high working pressures in carbon dioxide machines offer no vital objection in practice, and the small volume of vapour to be dealt with enables the machine to be very compact. Carbon dioxide machines are much used on board vessels carrying mutton, etc.; ammonia machines are employed extensively in land installations.

The latent heats of vaporisation of the three refrigerating substances are given in graph form in Fig. 456. The three graphs passing through the point whose co-ordinates are 0, 0, show the heat which must be added to, or abstracted from the liquid at 0° C. in order to bring the liquid to other temperatures.

Coefficient of performance in refrigerating machines.—The coefficient of performance in a refrigerating machine is the ratio of the refrigerating effect to the work done in the compressor. The refrigerating effect is the heat taken up by unit mass of working substance in the pipe coil in A (Fig. 453), and may be stated as the difference between the total heat I_1 of the working substance on leaving A and

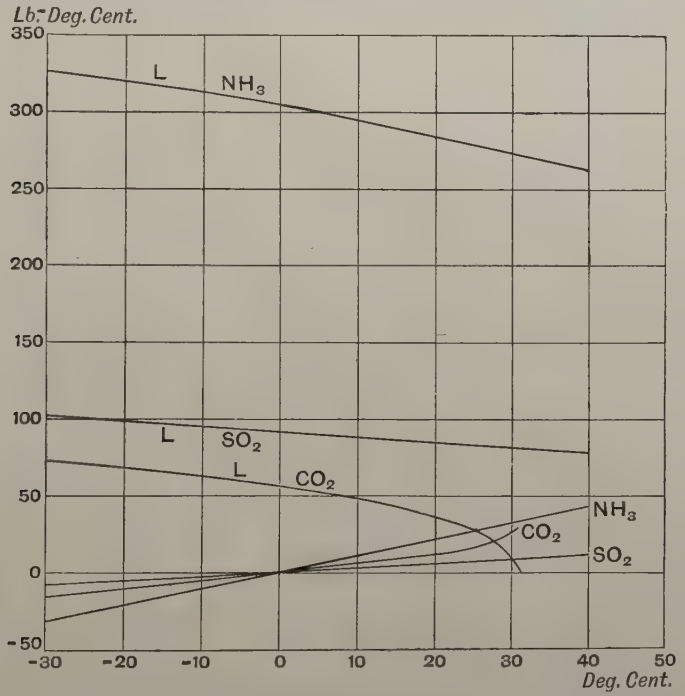

FIG. 456.—Relation of latent heat, heat of the liquid, and temperature for refrigerating substances.

the total heat I_2 on entering A. The work done in the compressor may be measured by the heat equivalent to this work, and may be stated as the difference between the total heat I_3 of the working substance on leaving the compressor and the total heat I_1 on leaving A (Fig. 453). Thus

$$\text{Coefficient of performance} = \frac{I_1 - I_2}{I_3 - I_1}.$$

This fraction is always greater than unity, and may have values of 3 to 7 in practice, depending upon the precise working conditions.

Exercises on Chapter XXXVII.

1. One gram of water at 160° C. is subjected to a pressure of 6323 grams weight per square cm., this being the pressure of saturated aqueous vapour at the given temperature. The water is converted into saturated vapour at 160° C. and takes in 499 calories of latent heat. If the volume of the vapour is 306·5 cubic cm. find the external work done during evaporation; express the result in calories. Find the increase which has taken place in the internal energy.

2. Define the following terms: Heat of formation of a vapour; total heat of a vapour; zero state; internal energy of a vapour.

3. Explain the changes which take place when a vapour, initially saturated and containing no liquid, is expanded (*a*) isothermally, (*b*) adiabatically. Give reasons as far as you can.

4. Some saturated aqueous vapour at a given temperature is contained in a cylinder fitted with a piston. If the piston is pushed inwards and compresses the vapour isothermally, what will happen ultimately? Give reasons for your answer.

5. What is meant by a "cycle of operations"? Give an example.

6. What is meant by the critical temperature and critical pressure of a given substance? State what you know of the properties of a substance at its critical temperature.

7. Describe a method of liquefying oxygen; give an outline sketch of the apparatus.

8. Give a sketch and describe the operation of the Linde apparatus for liquefying air. State clearly the principle on which the action depends.

9. Describe the mode of action of a refrigerating machine in which the vapour of a given substance is used. Give an outline sketch.

10. What are the principal substances used in practical refrigerating machines? Refer to their principal properties and to any advantages possessed by each substance.

11. Explain the meaning of the term "coefficient of performance" as applied to a refrigerating machine.

12. Define an isothermal curve. Draw the isothermals of a substance (*e.g.* carbonic dioxide) partly in the liquid and partly in the gaseous form, for temperatures (*a*) a little below, (*b*) a little above the critical temperature. Under what conditions can the substance be made to pass from the gaseous to the liquid form without discontinuity. Allahabad Univ.

13. One pound mass of saturated steam is contained in a cylinder fitted with a piston, and is permitted to expand from a pressure of 7 kilogram wt. per square cm. to 3 kilogram wt. per square cm. Sufficient heat is supplied to prevent any condensation taking place during expansion. Take what quantities are required from the table (p. 534), and plot a pressure volume graph.

14. Steam is expanded isothermally in a cylinder fitted with a piston, and does 12,500 foot-lb. of work upon the piston. Calculate the quantity of heat which must be supplied during the expansion.

15. Dry air at 0° C. and 76 cm. of mercury has a density of 0·001293 grams per c.c. Saturated aqueous vapour at 200° C. and 15·89 kilograms wt. per sq. cm. pressure has a volume of 0·1288 cubic metres per kilogram. Find the density of the vapour under these conditions, and compare the value with the density of dry air under the same conditions. Take the density of mercury as 13·6 grams per c.c.

CHAPTER XXXVIII

HEAT ENGINES

Heat engines.—Any contrivance having for its object the transformation of heat energy into mechanical work is called a **heat engine.** Heat engines operate by taking in a supply of heat at a comparatively high temperature, converting some of it into work, and discharging the remaining heat at a comparatively low temperature. In order to convey the heat into and away from the engine a working substance must be employed. In steam engines the working substance is a vapour; oil and gas engines use a combustible mixture of gases; hot air engines use permanent gases.

Efficiency of a heat engine.—The efficiency of a heat engine is the ratio of the mechanical work done and the heat energy supplied, both stated in thermal units. If a heat engine takes in Q_1 and discharges Q_2 units of heat, then $(Q_1 - Q_2)$ units of heat have disappeared in the engine and have had the chance of being converted into mechanical work. Hence the maximum efficiency which this heat engine can have is $(Q_1 - Q_2)/Q_1$.

In any heat engine the working substance goes through certain changes of pressure, volume, temperature, etc., and either returns to the initial conditions, or may be imagined to do so. The complete series of operations from the start to the return to the initial conditions constitutes the **cycle** of operations.

Carnot's cycle.—In Carnot's ideal cycle of operations all the heat taken in by the engine enters at constant temperature T_1, and all the heat discharged is given out at constant temperature T_2. The engine is supposed to be such as to enable isothermal and adiabatic operations to be realised, and is therefore an ideal engine (p. 429).

In Fig. 457, A is an imaginary Carnot engine. Heat is supplied from a hot body B, which is maintained at temperature T_1; a cold

body C is maintained at temperature T_2 and receives the heat discharged from the engine. Let the initial temperature of the working substance in the engine be T_2.

The **first operation** is to compress the working substance adiabatically until its temperature is T_1.

The **second operation** is to allow the working substance to expand isothermally, heat being taken in from B at constant temperature T_1. This operation is stopped when Q_1 units of heat have been taken in.

The **third operation** is to allow the working substance to expand adiabatically until the temperature has fallen to T_2.

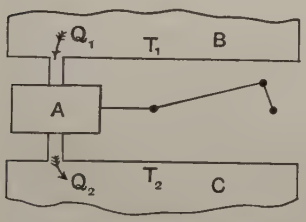

FIG. 457.—Action in a Carnot engine.

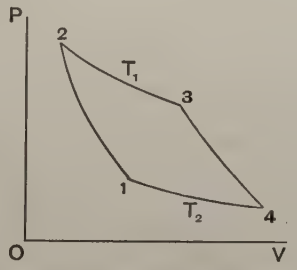

FIG. 458.—Pressure-volume diagram illustrating Carnot's cycle.

The **fourth operation** is isothermal compression of the working substance at constant temperature T_2; during this operation Q_2 units of heat are discharged to the cold body C, and the operation is stopped when the initial conditions of pressure, volume, etc., are attained. This completes the cycle.

Reference to the pressure-volume diagram (Fig. 458) may assist in visualising the cycle. The stages are as follows:

1 2. Adiabatic compression from T_2 to T_1.

2 3. Isothermal expansion at T_1; heat taken in $= Q_1$.

3 4. Adiabatic expansion from T_1 to T_2.

4 1. Isothermal compression at T_2; heat discharged $= Q_2$.

External work is done by the working substance during the operations 2 3 and 3 4, and work is done upon it during the operations 4 1 and 1 2. The heat which disappears in the engine is $(Q_1 - Q_2)$, and this quantity is the equivalent of the net external work done.

Carnot's cycle is reversible.—It is important to notice that the operations in Carnot's cycle can be performed in inverse order. Thus, starting the cycle at the point 2 (Fig. 458), we have :

2 1. Adiabatic expansion from T_1 to T_2.

1 4. Isothermal expansion at T_2; heat taken in from the cold body $= Q_2$.

4 3. Adiabatic compression from T_2 to T_1.

3 2. Isothermal compression at T_1; heat discharged to the hot body $= Q_1$.

This reverse order is rendered possible by the working substance taking in and giving out heat at constant temperatures only, and being brought to the temperature of either the hot or the cold body before heat flow starts.

If a Carnot engine be reversed, the cold body loses a quantity of heat equal to that which it gained when the cycle was performed in the forward direction. The hot body receives a quantity of heat equal to that which it supplied formerly. $(Q_1 - Q_2)$ represents the heat equivalent of the external work which must be done upon the working substance. When working in the reverse direction the engine may be described as a **heat pump.**

Efficiency of a Carnot engine.—No heat engine working between given temperatures T_1 and T_2 can have an efficiency higher than that of any reversible engine working between the same temperatures. In Fig. 459, A_1 is a heat engine which cannot be reversed, and A_2 is a reversible heat engine. A_1 runs in the forward direction, taking in Q_1' heat units from the hot body B and discharging Q_2' heat units into the cold body C. A_2 runs in the reverse direction, and takes in Q_2 heat units from C and discharges Q_1 heat units into B. Assuming for the moment that A_1 has an efficiency higher than that of A_2, let A_1 drive A_2, so that the whole arrangement constitutes a self-acting machine.

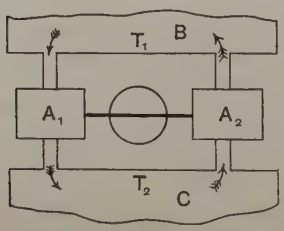

FIG. 459.—Efficiency of a Carnot engine.

Since the efficiency of A_1 has been supposed to be greater than that of A_2, we have :

$$\frac{Q_1' - Q_2'}{Q_1'} > \frac{Q_1 - Q_2}{Q_1}. \quad \text{..... (1)}$$

(*a*) Suppose that $Q_1' - Q_2' = Q_1 - Q_2$, ...(2)

then $Q_1' < Q_1$. .....(3)

Also from (2) and (3) $Q_2' < Q_2$ .....(4)

From (3), the hot body B has gained $(Q_1 - Q_1')$ heat units, and from (4), the cold body C has lost $(Q_2 - Q_2')$ heat units. Hence the general effect of the arrangement is that the cold body is losing heat and the hot body is gaining heat during each cycle of the engines.

Now it is contrary to all experience to find a self-acting machine able continuously to transfer heat from a cold body to another body at a higher temperature. This statement constitutes the **second law of thermodynamics.**

(*b*) Suppose in (1) that Q_1' and Q_1 are equal, then for (1) to be true, we must have $Q_1' - Q_2' > Q_1 - Q_2$, ...(5)

and therefore, $Q_2' < Q_2$. .....(6)

Hence, from (6), the cold body C has lost $(Q_2 - Q_2')$ heat units, and by supposition the hot body B neither gains nor loses heat. Thus the work of driving A_2 had been accomplished by abstracting heat from the cold body. Another statement of the second law of thermodynamics is that it is impossible to derive useful work continuously by abstracting heat from the coldest of the surrounding bodies.

Admitting the truth of this law, we may assert that it is impossible that any heat engine can have an efficiency higher than that of a reversible heat engine working between the same temperatures. Hence all reversible heat engines working between the same temperatures have the same efficiency.

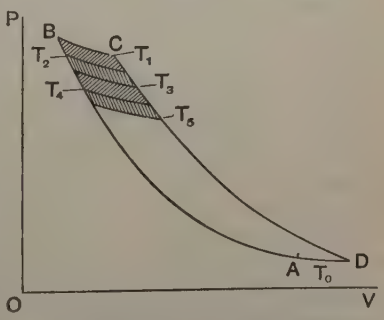

FIG. 460.—Pressure-volume diagram for a series of Carnot's engines.

Kelvin's absolute scale of temperature.—In Fig. 460, let ABCD be the pressure-volume diagram for a Carnot's heat engine working between absolute temperatures T_1 and T_0. Let there be another series of Carnot's engines, arranged so that the first engine takes in Q_1 heat units from the hot body and discharges Q_2 heat units into the second engine,

which thus receives its supply of heat. The second engine thus takes in Q_2 heat units from the first engine, and is arranged to discharge Q_3 heat units into the third engine, and so on.

The external work done by the first engine is $(Q_1 - Q_2)$, and by the second engine $(Q_2 - Q_3)$, etc. Let the initial and final temperatures for the first engine be T_1 and T_2, and for the second engine T_2 and T_3, etc. In Lord Kelvin's absolute scale of temperature, if all the engines in the series perform equal quantities of work during each cycle, the temperature ranges through which they work are equal; *i.e.*

If $(Q_1 - Q_2) = (Q_2 - Q_3) = (Q_3 - Q_4) = \text{etc.}$

Then $(T_1 - T_2) = (T_2 - T_3) = (T_3 - T_4) = \text{etc.}$

This absolute scale of temperature is independent of the properties of any particular substance.

Absolute zero of temperature.—In Fig. 460, the small shaded areas represent the work done by the engines of the series, and are equal to one another. If there be 100 engines between the boiling and freezing points of water, then each engine has a range of one degree Centigrade. Continuing downwards by equal steps of temperature, the last engine will take in heat at 1° absolute, will perform external work, and will have no heat left to discharge after performing the external work. **Absolute zero** in the Kelvin scale may be defined as the lower limit of temperature in a Carnot engine in which the internal energy of the working substance has been used entirely in the performance of external work. Kelvin's absolute zero is found by experiment to coincide with the zero of the gas thermometer scale (p. 402); temperatures measured from it are called **absolute temperatures.**

Efficiency in terms of temperature.—In the above series of engines the quantity of heat Q_1 has been divided equally among all the engines of the series, *i.e.* the heat drop is the same in each engine, and the temperature drops are also equal, hence the quantity of heat is proportional to the Kelvin absolute temperature. The efficiency of the first engine of the series is $(Q_1 - Q_2)/Q_1$, and this may be written $(T_1 - T_2)/T_1$, or, for any Carnot engine working through a temperature range from T_1 to T_2,

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1}.$$

If the engine takes in Q_1 heat units, the maximum work which can be derived from the engine is given by :

$$\text{Maximum work obtainable} = Q_1\left(\frac{T_1 - T_2}{T_1}\right).$$

The conclusions derived from the Carnot cycle are valid, being in accord with experience, but the cycle cannot be realised in practice ; hence other cycles have been devised which afford fairer comparison with the performances of actual engines.

Hot air engines.—The action in a hot air engine is of interest, and may be understood by reference to Fig. 461. The cylinder A is fitted with a piston B which is connected by a rod C to a crank D. D is fixed to a revolving shaft E, and another crank F is fixed to the shaft and makes an angle of 90° with the crank D. Another cylinder G has a displacer H which is driven up and down by the crank F, to which it is connected by the rods K and L. The lower end of G is heated by means of a bunsen M, or a furnace may be used ; the upper end is kept cool by means of cold water circulating through pipes N. The cylinders A and G are in permanent communication by the passage O.

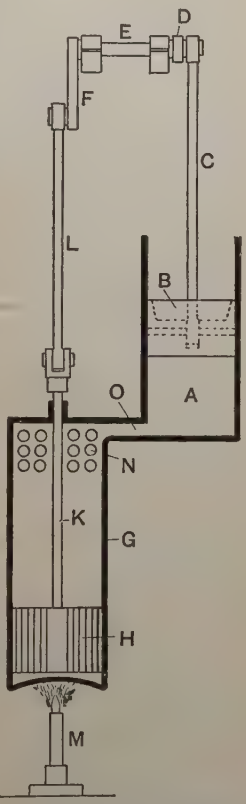

FIG. 461.—Diagram of a hot air engine.

A is the motor cylinder, and work is done on the piston B by the pressure of the air enclosed in the cylinders. This pressure is increased during the upward stroke and diminished during the downward stroke of B in the following manner. The displacer H is so constructed that air can pass through it. In the position shown in Fig. 461, H is at the bottom of the stroke and has therefore caused the air to flow to the top end of G, where it is in contact with the cold tubes. The consequent cooling has the effect of lowering the pressure. B has still about half of the downward stroke to be completed. While this stroke is being completed, H begins to rise, and air flows to the lower end of G, where it is in contact with hot walls. The rise in temperature which occurs causes the pressure to increase, and the upward stroke of B is thus accomplished under

the action of the augmented pressure. The pressure varies only slightly above and below that of the atmosphere, and the same air is used over and over again. The engine is suitable for very small powers only.

Particular interest is attached to the action of the displacer H, which is a form of the **regenerator** invented by Stirling. The heated air, in passing upwards through H, loses a considerable portion of its heat to the material of which the displacer is constructed. This heat is taken in again by the air during its downward passage through H. The temperature of the displacer, when matters have settled down, is graded from bottom to top, being approximately that of the hot cylinder bottom at the lower end and that of the cold pipes at the upper end. The air thus loses heat gradually in passing upwards, and is in contact with matter at nearly its own temperature throughout the passage. During the downward passage, the initially cold air is in contact with cool material at the upper end of the displacer, and gradually becomes warmer as it passes downwards; in this passage, once again, the air is in contact with material at nearly the same temperature as itself throughout the passage. Owing to the regenerator, there is considerable reduction in the wastage of heat, and the action of the regenerator in alternately heating and cooling the air is very nearly reversible. It will be noted that the displacer H does not produce any change in volume, but merely transfers air from one place to another.

Exercises on Chapter XXXVIII.

1. Describe what is meant by a heat engine. During each cycle, a certain heat engine takes in 24·36 units of heat and discharges 19·42. Find the efficiency.

2. Describe the Carnot cycle; make reference to a pressure-volume diagram.

3. State exactly what is meant by the term "reversible" as applied to a heat engine. Describe the reversed Carnot cycle.

4. State the second law of thermodynamics. Making use of your statement of the law, show that no heat engine can have an efficiency higher than the efficiency of a reversible heat engine working between the same temperatures.

5. Explain the Kelvin scale of absolute temperature. Define absolute zero on this scale.

6. Find the efficiency of a perfect (Carnot) heat engine which takes in 50,000 lb.-deg.-Cent. units of heat per hour at a temperature of 180° C. and rejects heat at 80° C. How much work could be done in foot-lb. per hour by this engine?

7. Describe the action of a hot-air engine. Explain the function of the regenerator.

8. Name three properties of materials which vary in consequence of variation of temperature, and explain how some one of these variations may be employed to define a scale of temperature.

On what principle is it possible to define a scale of temperature independently of any particular property of matter ? Adelaide University.

9. Assuming the properties of an engine working in a simple reversible cycle, explain the construction of the scale of absolute temperature. Allahabad Univ.

CHAPTER XXXIX

STEAM ENGINES AND BOILERS

Steam engine cycle.—The cycle in a steam-power plant may be followed by reference to Fig. 462, in which is given a diagram of an engine and boiler, together with the necessary auxiliaries. Starting with water in a tank A—called the hot-well—the water is pumped by a feed pump B through a feed water heater C. This heater may consist of rows of pipes placed in the flue leading from the boiler; its object is to heat the comparatively cold feed water to a temperature nearer that of the water in the boiler. The water leaves the feed heater through the pipe D and is discharged into the long cylindrical drum E which forms part of the boiler. The boiler shown is of the Babcock & Wilcox type. A number of inclined pipes F and H are connected to the drum E, and rows of inclined tubes G are connected to F and H. K is the furnace; the hot gases leaving the burning fuel pass upwards among the inclined tubes G, and are directed by baffle plates so that the gases pass downwards among the tubes, then upwards again. The gases then pass round the tubes of the feed water heater C, and thence to the chimney L. The water in the boiler stands at the level of the axis of the drum E. Owing to the inclination of the tubes G and the hot gases passing round the tubes at the higher ends, a circulation of water is produced, downwards through the tubes F, through the tubes G, and then upwards through the tubes H into the drum E again. Boilers of this kind are called **water-tube boilers.**

Steam collects in the upper part of the drum E and leaves through a stop valve N. The steam passes through a pipe P into another row of tubes M, round which the hot furnace gases circulate. The steam is thus heated to a temperature higher than that of the saturated steam in the drum E, and becomes superheated steam (p. 450). The steam leaves the superheater M through the pipe Q, and is led

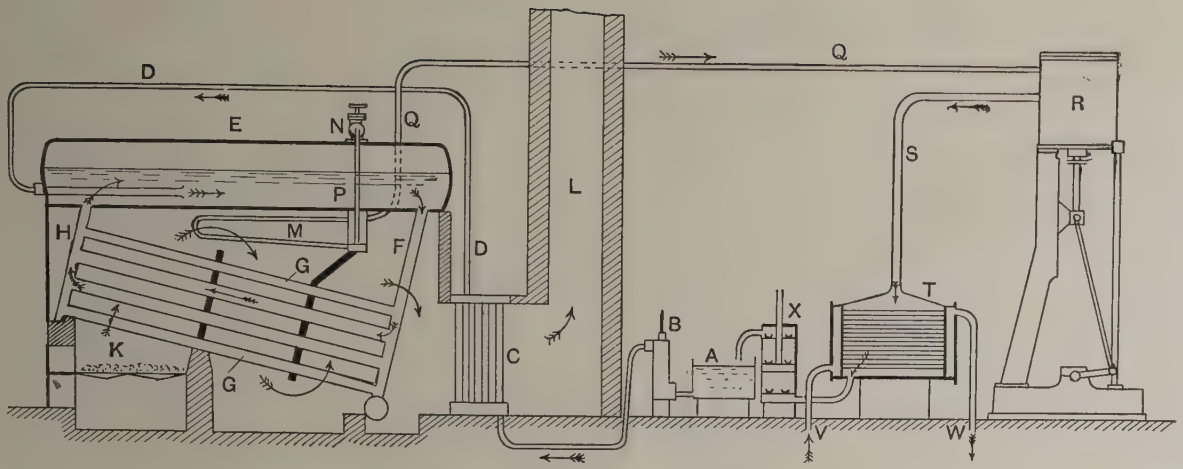

FIG. 462.—Diagram of a steam power plant.

A, hot well.
B, feed pump.
C, feed water heater.
D, feed pipe.
E, water and steam drum.
F, G, H, pipes for circulating water in the boiler.
K, furnace.
L, chimney.
M, superheater.
N, steam stop valve.
P, steam pipe to superheater.
Q, steam pipe to engine.
R, steam engine.
S, exhaust pipe.
T, surface condenser.
V, circulating water inlet to condenser.
W, circulating water outlet from condenser.
X, air pump.

to the engine R, where it does work and is discharged at much lower pressure and temperature through the pipe S into the condenser T. The condenser shown consists of a vessel having a large number of brass tubes through which cold water flows ; this circulating water enters at V and leaves at W. The steam from the engine passes round outside the tubes, and is condensed by the action of the cold surfaces of the tubes ; condensers of this type are called **surface condensers.** The steam space in the condenser is maintained at a partial vacuum by means of an air pump X, which withdraws air and also the water condensed from the steam. The air pump discharges the water into the hot well A.

It will thus be seen that the heat liberated from the burning fuel is utilised (*a*) in heating the feed water, (*b*) in completing the heating of the feed water and evaporating it in the boiler, (*c*) in superheating the resulting steam. Heat is thus taken in by the working substance at all temperatures lying between the temperature of the water in the hot well and the temperature of the steam in the pipe Q. (Compare this with Carnot's cycle, in which all the heat is taken in at the higher temperature only.)

The heat discharged by the engine is extracted by the circulating water in the condenser, which rises in temperature during its passage through the tubes.

EXAMPLE.—In a certain steam plant, the engine gives one horse-power for an hour for each $1\frac{1}{2}$ pounds of coal fed into the boiler furnace. The coal has a heating value of 8000 lb.-deg.-Cent. units per pound. What percentage of the heat energy of the coal is converted into mechanical work ?

Energy derived from 1 horse-power $= 33000$ foot-lb. per minute.

$$= \frac{33000 \times 60}{J}$$

$$= \frac{33000 \times 60}{1400}$$

$= 1414$ lb.-deg.-Cent. per hour.

Energy supplied per hour to produce this result

$= 8000 \times 1\frac{1}{2}$.

$= 12000$ lb.-deg.-Cent.

$$\therefore \text{ Required percentage} = \frac{1414}{12000} \times 100$$

$$= \underline{11{\cdot}78}.$$

The remaining 88·22 per cent. of the heat contained in the coal has been wasted in various ways.

Action in the steam engine.—The action in a steam engine may be understood by reference to the outline diagram in Fig. 463. The cylinder A has a piston B which is urged to and fro in the cylinder by the action of the steam pressure. The piston is connected by a piston rod C to a crosshead D, which is guided by part of the frame H so as to travel in a straight line. The crosshead is connected by a connecting rod E to a crank F which is fixed to a revolving crank shaft G. The reciprocating motion of the piston is thus converted into motion of rotation of the crank shaft.

Steam is brought from the boiler into a steam chest K through a pipe L, and enters the cylinder through ports M and N. A third port P permits the discharge of the steam after it has done work on the piston. These ports are opened and closed at the proper

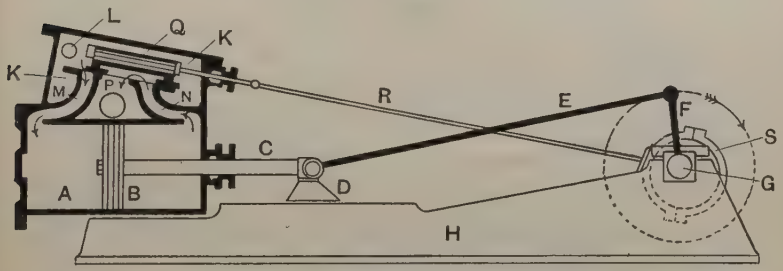

FIG. 463.—Outline diagram of a steam engine.

instants by a valve Q, which is reciprocated by means of an eccentric S, or small crank, fixed to the crank shaft; S is connected to the valve by the eccentric rod R and a valve rod.

As shown in Fig. 463, the valve has been drawn towards the right and has uncovered the steam port M; steam is thus flowing into the left-hand end of the cylinder, and is urging the piston towards the right. At the same time, the cavity in the valve has uncovered the steam port N to the exhaust port P, and the steam in the right-hand end of the cylinder is being discharged. During the backward stroke of the piston, the valve has been pushed to the left by the action of the eccentric, and uncovers the steam port N to steam, and the steam port M to exhaust.

Steam is prevented from leaking from one side of the piston to the other side by means of spring rings fitted into grooves on the rim of the piston. These rings press outwards against the walls of the cylinder. The piston rod and valve rod pass through stuffing boxes in the end of the cylinder and valve chest, and are thus rendered steam tight.

Steadiness of rotation of the crank shaft G is secured by means of a heavy flywheel which is fixed to the shaft. The engine is

maintained at approximately constant speed by means of a governor A (Fig. 464). Two heavy balls FF are mounted at the ends of arms GG, which are attached at their upper ends by pins to a spindle H. The spindle is rotated by a belt from the crank shaft running on the pulley K, the motion being transmitted to H by means of bevel wheels L. Other arms MM connect the balls to a sleeve N which can slide on the spindle H. A heavy weight P is carried by the sleeve. A bent lever Q has one arm connected to a collar embracing the sleeve, and the other arm is connected by a rod S to the throttle valve lever E.

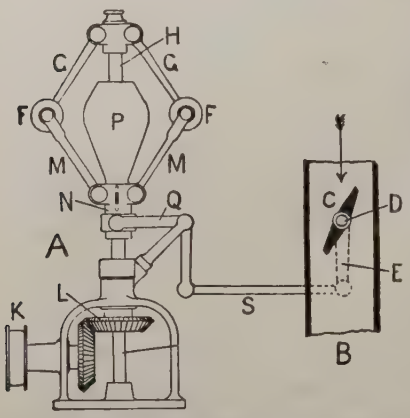

FIG. 464.—Diagram showing how the governor controls the steam supply.

When the engine is running, centrifugal force acts on the balls, causing them to move outwards until a steady position is reached which depends upon the speed of rotation. Any increase in speed will cause further outward movement of the balls, producing an upward movement of the sleeve N. This movement is transmitted to the throttle valve C, and causes it to close partially the steam-pipe B. Thus the supply of steam to the engine is reduced, and the speed falls again. Reduction in speed below the normal causes the balls to move inwards, and the throttle valve is consequently opened to a greater extent, thus increasing the supply of steam to the engine.

The student is referred to the section on Dynamics, Chaps. XV. and XVI., for the theory of the action of flywheels and governors.

Thermal efficiency of a steam engine.—In practice the heat supplied to a steam engine is taken as the difference between the total heat of the steam entering the cylinder and the heat of the same weight of water at the temperature of the exhaust steam. Both these quantities are reckoned from the arbitrary zero state of water at 0° C. The thermal efficiency of the engine is calculated by taking the ratio of the heat equivalent of the work done on the engine piston and the heat supplied.

EXAMPLE 1.—In testing a small engine, the steam was supplied at an absolute pressure of 95·5 lb. wt. per sq. in., temperature 162·3° C., total

heat 661·5 lb.-deg.-Cent. per pound of steam. The exhaust steam had a temperature of 102·4° C. The heat of water at this temperature is 103 lb.-deg.-Cent. per pound. The engine used 36·25 pounds of steam per horse-power per hour. Find the thermal efficiency.

$$\text{Heat supplied per pound of steam} = 661{\cdot}5 - 103$$
$$= 558{\cdot}5 \text{ lb.-deg.-Cent.}$$
$$\text{Heat supplied per horse-power per hour} = 558{\cdot}5 \times 36{\cdot}25$$
$$= 20{,}246 \text{ lb.-deg.-Cent.}$$
$$\text{Work obtained per horse-power per hour} = 33000 \times 60 \text{ ft.-lb.}$$
$$= \frac{33000 \times 60}{1400}$$
$$= 1414 \text{ lb.-deg.-Cent.}$$
$$\text{Thermal efficiency} = \frac{1414}{20246} \times 100$$
$$= \underline{6{\cdot}98} \text{ per cent.}$$

EXAMPLE 2.—Find the efficiency of a Carnot engine working between the same limits of temperature as the above engine.

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1}$$
$$= \frac{(162{\cdot}3 + 273) - (102{\cdot}4 + 273)}{162{\cdot}3 + 273}$$
$$= \frac{59{\cdot}9}{435{\cdot}3} = 0{\cdot}137$$
$$= \underline{13{\cdot}7} \text{ per cent.}$$

In practice the **Rankine cycle** is taken as the standard of comparison in preference to the Carnot cycle. In the Rankine cycle, the ideal engine is taken as receiving steam at the same temperature and pressure as the actual engine, up to the point of cut off; expanding this steam adiabatically until the pressure falls to that of the exhaust of the actual engine, and discharging at this pressure. In the above examples, the efficiency of the ideal Rankine engine is about 12·9 per cent.

Some sources of waste in steam engines.—The principal source of waste in the steam engine is the action of the cylinder walls on the steam. During the exhaust stroke, the walls have been cooled considerably, with the result that a considerable portion of the heat in the steam admitted during the next stroke is abstracted in heating the walls. If the incoming steam is saturated, some of it will condense, and there will be a mixture of steam and water in the cylinder before expansion starts. During expansion, the temperature of the

steam is falling, and a point is reached beyond which the walls are hotter than the steam. Heat then flows from the walls into the mixture, and evaporates some of the water. When the exhaust valve opens, the pressure and temperature of the steam fall considerably and the evaporation becomes very energetic. The result is that a large amount of heat is carried away in the exhaust steam and is wasted.

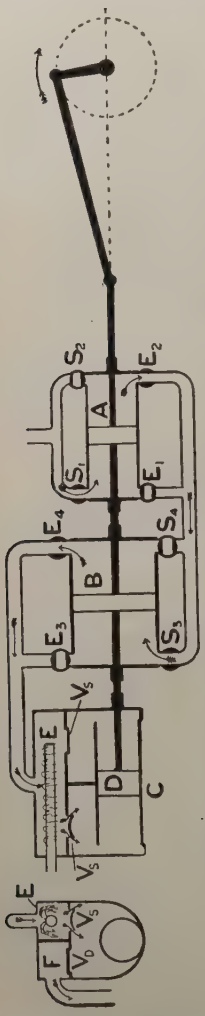

FIG. 465.—Diagram of a tandem compound condensing engine with jet condenser.

The action of the walls is lessened by using superheated steam, which may be cooled considerably before condensation begins. The action is smaller in large cylinders, since the area of the wall surface is less proportionally to the volume of steam in large than in small cylinders. Higher speed of revolution diminishes the action of the walls by giving less time for the action to take place. The evils of cylinder condensation were first discovered by James Watt. Prior to his inventions, the steam was actually condensed in the cylinder by means of jets of water. Watt fitted a separate condenser, and endeavoured to keep the cylinder of the engine as hot as the entering steam by surrounding the cylinder with a jacket containing steam taken from the boiler, and by clothing the outside of the cylinder with substances which were bad conductors of heat.

Compound engines.—The efficiency of the engine is increased by increasing the upper limit of temperature, *i.e.* by using steam at higher pressures. There is a limit to the extent which this can be done economically in a single cylinder, since the range in temperature becomes very great, and the wall action becomes correspondingly great. The plan adopted is to expand the steam partially in one cylinder and to continue the expansion in a second cylinder. A third and fourth cylinders are

employed in the case of great initial pressures. Such engines are called **compound**, or **multiple-expansion engines.**

A compound engine is shown in outline in Fig. 465. A is the high-pressure cylinder, in which the early stage of expansion is performed. The steam is shown entering by the valve S_1, and the other side of the piston is exhausting through the valve E_2. The steam so exhausted is led through the valve S_3 and acts on the piston of the low-pressure cylinder B. The contents of B on the other side of the piston are exhausting through the valve E_4 into a condenser E, where jets of cold water condense the steam; condensers of this kind are called **jet-condensers.** The resulting water and air are withdrawn by the air pump C. The pistons of A and B and also the piston D of the air pump are all fixed to the same piston rod, and the axes of all three cylinders coincide. The arrangement is described as a **tandem engine.** More usually, each of the cylinders A and B operate on separate cranks, and the air pump is either worked by a separate engine or motor, or by a connection to the low-pressure piston rod.

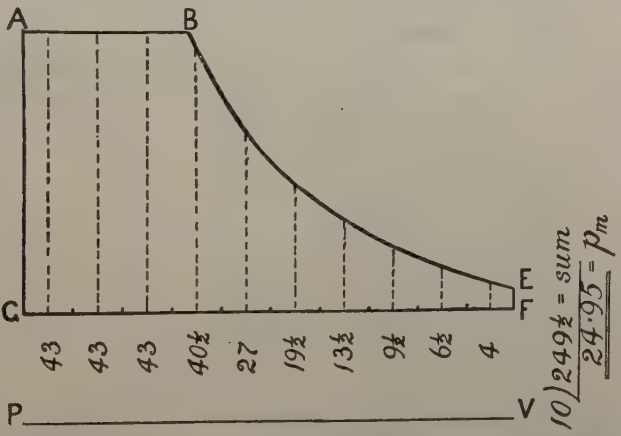

FIG. 466.—Diagram showing a method of estimating the mean pressure.

Work done on the piston of a steam engine.—In Fig. 466 a diagram of work done has been drawn for a steam engine. PV is the perfect vacuum line; PA represents the absolute initial pressure, which is maintained constant during the admission of steam to the cylinder. The steam is cut off at B, and expansion follows as shown by the curve BE. The constant back pressure acting on the other side of the piston is shown by the height of the line FG above PV. The total work done on each square inch of piston area is represented by

the area PABEV ; part of this work is done against the back pressure, and is represented by the area PGFV. The net work done is therefore represented by the area ABEFG.

The mean effective pressure acting on the piston is obtained by dividing GF into ten equal parts and measuring the height (using a scale of pressures) at the centre of each part. The sum of these heights, divided by ten, gives the mean pressure p_m. The work done and the horse-power developed on the piston are calculated as follows:

Let p_m = the mean effective pressure, in lb. wt. per sq. in.
A = the area of the piston, in sq. inches.
L = the length of the stroke of the piston, in feet.
N = the number of working strokes per min.

Then Resultant mean force = p_mA lb. wt.
Work done per stroke = p_mAL foot-lb.
Work done per minute = p_mALN foot-lb.

$$\text{Horse-power} = \frac{p_m \text{ALN}}{33000}.$$

Horse-power calculated in this way is called the **indicated-horse-power** from the circumstance that an instrument called an indicator is used to obtain the actual diagram of work done in the cylinder.

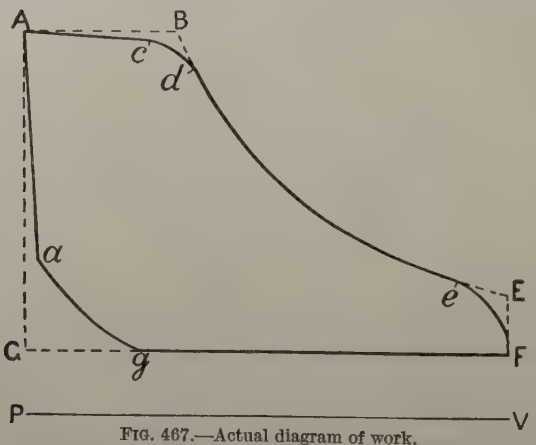

FIG. 467.—Actual diagram of work.

Actual diagrams of work.—The actual diagram of work differs somewhat from that shown in Fig. 466 ; the differences will be seen by inspection of Fig. 467, which shows in dotted lines the diagram formerly considered. From A to *c* the pressure generally falls

somewhat during admission ; the corner at *cd* is rounded owing to the slowly closing valve throttling the steam. The exhaust valve opens before the end of the stroke as shown by the curve at *e*F. At *g* the

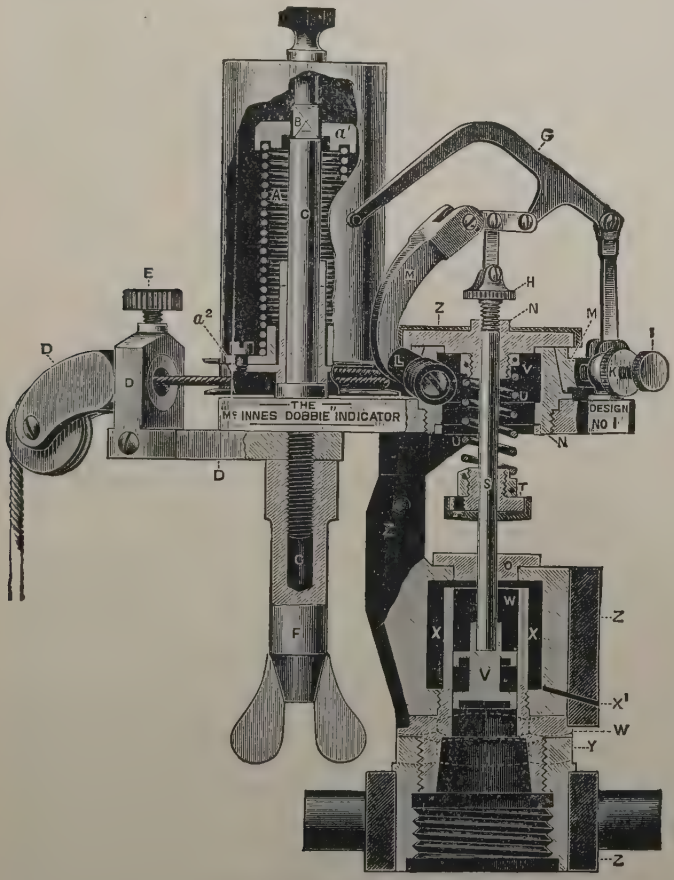

FIG. 468.—McInnes-Dobbie indicator.

exhaust valve closes and entraps some of the steam which is compressed by the returning piston and serves as a cushion for bringing the piston to rest quietly. The valve begins to admit steam at *a*.

The indicator.—An example of an indicator is given in Fig. 468. A small cylinder X is fitted with a piston and is in communication

with the engine cylinder. The pressure of the steam urges the indicator piston upwards against the resistance of a spring U. The height through which the piston rises is thus proportional to the pressure. The movement of the piston is small, and is magnified by means of a parallel-motion mechanism G, to which the end of the piston rod is attached at H. A piece of paper is wrapped round a drum which reciprocates through about nine-tenths of a revolution ; the drum is pulled in one direction by a cord attached to the engine

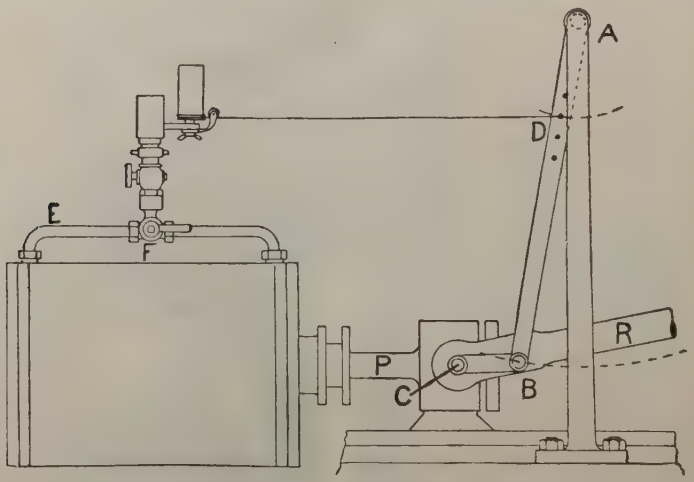

FIG. 469.—Arrangement for driving the indicator drum.

crosshead and is brought back again by means of a spring A. A pencil attached to the left-hand end of G draws the diagram on the paper. Vertical heights on the paper represent pressures, and horizontal distances represent the movements of the piston.

As a rule the crosshead of the engine has too great a travel to permit of direct connection to the indicator drum. An arrangement such as is shown in Fig. 469 is often used. The crosshead C drives a lever AB to which it is connected by a short link BC. The indicator cord is attached at D, and thus the drum obtains the motion of the piston on a reduced scale. The pipe arrangement and cock at E and F enable the indicator to be connected to either end of the cylinder ; a diagram from each end is generally taken on the same paper. Such a diagram is shown in Fig. 470. The fraction $\frac{1}{80}$ means that the strength of the spring used in the indicator was such that the

scale of pressures on the diagram is 80 lb. wt. per square inch for each inch of vertical height. From the diagrams, the mean pressure for one side of the piston is 64 and for the other side 60 ; the average for both sides is thus 62 lb. wt. per square inch. It is obviously impossible for the indicator to draw the perfect vacuum line on the

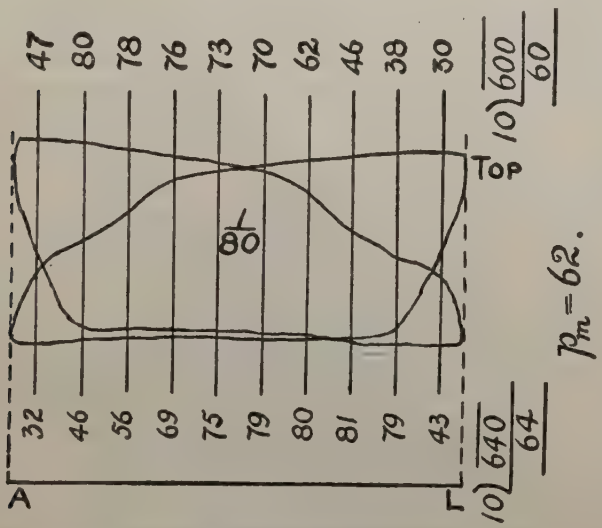

FIG. 470.—Example of an indicator card.

paper. The line AL is drawn by putting the indicator cylinder into communication with the atmosphere by means of the cock ; AL thus represents the pressure of the atmosphere. If the perfect vacuum line is required, it may be drawn on the diagram at a distance below AL equal (to scale) to the pressure of the atmosphere, obtained by reading the barometer at the time the diagram was taken.

Brake horse-power.—The indicated horse-power of an engine is a measure of the power developed on the piston of the engine, and does not represent the power which the engine is capable of giving out in the useful form of driving machinery. To obtain the latter, the energy given out by the engine may be absorbed by the resistance offered by a brake. A type of brake suitable for testing engines of comparatively small power is shown in Fig. 471. A double rope is passed round the flywheel and is kept in position by means of four wooden blocks which embrace the rim of the wheel. A load W is

attached to one end and a spring balance to the other end of the rope. As arranged in Fig. 471, the wheel rotates anticlockwise. The load W resists the rotation of the wheel, and the pull P of the spring balance is assisting the rotation; both these forces are communicated to the wheel by the frictional forces between the ropes and the rim of the wheel.

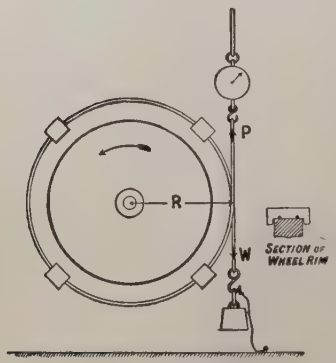

FIG. 471.—A common form of rope break.

Let R = the radius in feet, measured to the centre of the rope.

P = the pull of the spring balance, in lb. weight.

W = the load, in lb. weight.

N = the revolutions per minute.

Then, Net force retarding the wheel $= W - P$.

Work done per revolution $= (W - P)2\pi R$.

Work done per minute $= (W - P)2\pi RN$ foot-lb.

$$\text{Horse-power developed} = \frac{(W - P)2\pi RN}{33000}.$$

This horse-power, having been obtained by means of a brake, is called the **brake horse-power**, and is the useful horse-power which the engine can give out. The work done against the frictional resistance of the brake is converted into heat, and the wheel rim becomes warm in running a test. If the load is not too large, the heat is dissipated into the atmosphere with sufficient rapidity to prevent the wheel from becoming overheated. Brake wheels are sometimes made of hollow rim section, and cooling water is run into the rim and scooped out again by means of a sharp-edged pipe.

Mechanical efficiency of an engine.—The difference between the indicated and brake horse-powers represents the horse-power required to overcome frictional resistances of the mechanism of the engine. Writing I.H.P. and B.H.P. respectively for these powers, we have:

Power wasted in the engine mechanism = I.H.P. − B.H.P.

Energy given to the piston per minute = I.H.P. × 33000.

Energy given out by the engine per minute = B.H.P. × 33000.

The ratio of these quantities of energy gives the efficiency of the engine mechanism, and is called the **mechanical efficiency** of the engine.

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.} \times 33000}{\text{I.H.P.} \times 33000}$$
$$= \frac{\text{B.H.P.}}{\text{I.H.P.}}.$$

Steam turbines.—It is only during the last twenty-five years that the steam turbine has been developed. The action consists in blowing steam against blades fixed to a wheel, and thus causing the

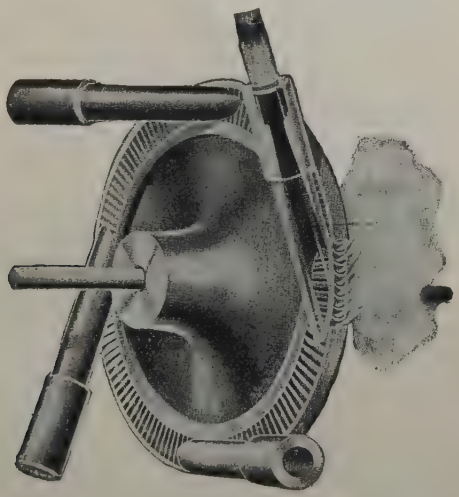

FIG. 472.—Action of the de Laval steam turbine.

wheel to revolve. Part of a de Laval steam turbine is shown in Fig. 472. In this type of turbine, the steam is brought to a nozzle and allowed to expand in passing through it. The expansion is approximately adiabatic, and the work which would have been done by the steam had it been expanded behind a piston is converted into kinetic energy in the steam. The result is that the steam leaves the nozzle with a very high speed. In Fig. 472 there are four nozzles, each of which blows a jet of steam against the blades which are fixed round the circumference of the wheel.

If steam is expanded in a properly designed nozzle from a pressure of 150 to 1 lb. wt. per square inch absolute, it will leave the nozzle with a velocity of nearly 4000 feet per second (assuming no losses). Theoretically the speed of the rim of the wheel should be one half of

this, viz. 2000 feet per second. Such speeds are not practical, and a compromise is effected by running the wheel at rim speeds of from 500 to 1400 feet per second. The 5 horse-power de Laval turbine wheel runs at 30,000 revolutions per minute, and the 300 horse-power wheel runs at 10,600 revolutions per minute. These speeds of revolution are too high to be used direct, and gearing is introduced into the turbine in order to reduce the speeds to meet practical requirements.

Reduction in the speed of turbine wheels may be obtained by the expansion of the steam in stages, each stage involving a small drop in pressure only. In each stage the steam passes through the blades of a revolving wheel, and all the wheels are fixed to the same shaft. The velocity acquired by the steam in each stage is comparatively small, and such turbines may be run at speeds which can be used for direct driving of electrical machinery. The principle is illustrated in diagram form in Fig. 473. A number of rotating bladed wheels are arranged to cut at right angles a nozzle through which steam is expanding. There are fixed guide blades in the nozzle between each pair of wheels so as to give the steam the correct direction of flow in entering the wheel blades.

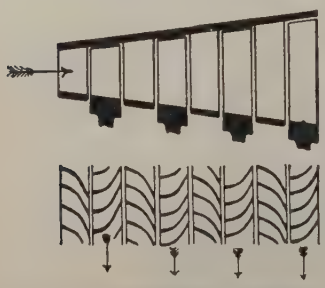

FIG. 473.—Diagrammatic representation of a reaction steam turbine.

The Parsons steam turbine is of this type (Fig. 474). In the example shown, the top cover of the machine has been folded back so as to enable the wheels to be inspected. The half rings of guide blades may be seen inside the cover, the other half rings are fixed in the lower part of the casing. The steam enters the turbine at the end where the wheels have the smallest diameters and shortest blades, and leaves at the other end. The diameter is increased and the blades made longer in order to provide accommodation for the passage of the increasing volume of the steam as it expands.

Lancashire boiler.—The Lancashire boiler is much used in modern practice. An example is shown in Fig. 475. The boiler consists of a large cylinder about 30 feet in length and about 8 feet in diameter. Two furnace tubes, each about 40 inches in diameter, pass from end to end of the boiler. There is a furnace about 7 feet long at the front end of each of these tubes. Brickwork flues are arranged round the boiler. The hot gases from the furnaces pass along the furnace tubes and emerge at the back end. Here they pass downwards into the bottom flue E, and then along the bottom of the boiler to the front end. At this end, the gases divide, and flow into two side flues

FF, and pass along these to the back end of the boiler, where they reunite in the flue G leading to the chimney.

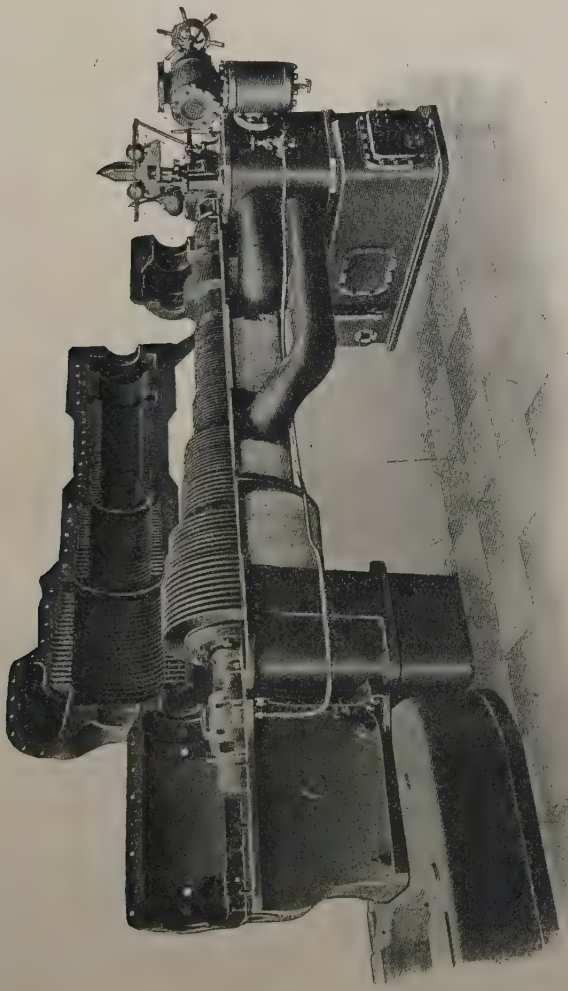

FIG. 474.—A Parsons steam turbine, with the top cover opened.

A boiler of this type can burn about 800 pounds of coal per hour, and can evaporate about 7000 pounds of water per hour. The efficiency of a good boiler is about 75 per cent., *i.e.* about 75 out of

every hundred heat units liberated from the burning fuel pass out of the boiler with the steam generated.

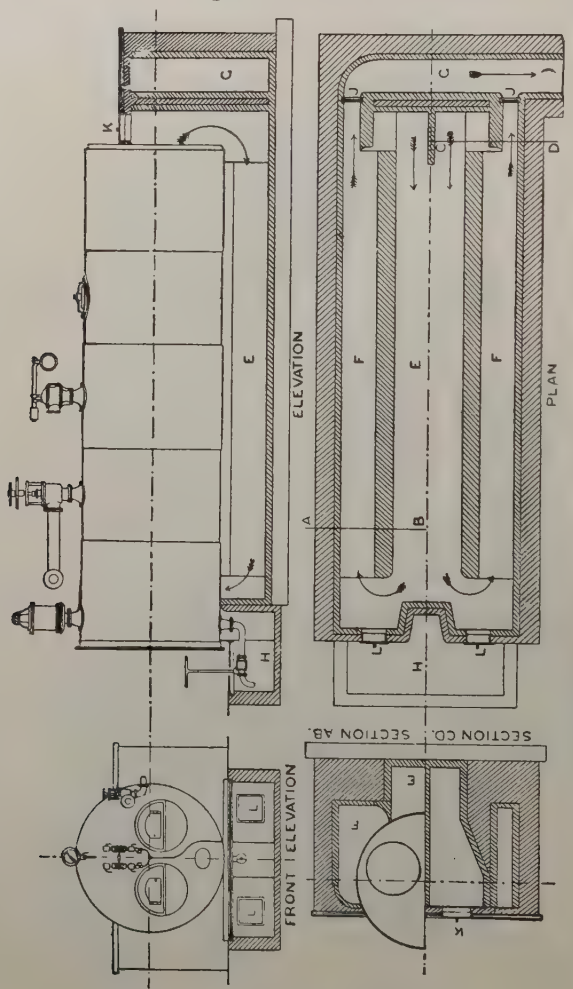

FIG. 475.—Lancashire boiler, showing the brickwork seating and flues.

EXERCISES ON CHAPTER XXXIX.

1. Trace the various stages undergone by the working substance in a steam power plant, from the pumping of the feed water into the boiler to the discharge of the condensed steam into the hot well.

2. Describe the working of a steam boiler. Make an outline sketch showing the passage of the hot gases.

3. In a certain steam boiler, 400 pounds of coal are burned per hour; the heating value of the coal is 7600 lb.-deg.-Cent. units per pound. The boiler is fed with 3500 pounds of water per hour at a temperature of 30° C., and this water is evaporated into steam at 146 lb. per square inch absolute pressure. What percentage of the heat supplied in the coal leaves the boiler in the steam? (See the Table, p. 534, for any quantities required.)

4. The boiler given in Question 3 supplies all the steam generated to a steam engine which develops 220 horse-power. Find the percentage of the heat supplied in the coal which is turned into useful work by the engine.

5. Describe the action of the steam in the cylinder of a steam engine, making reference to an outline sketch.

6. Explain the functions of (*a*) the flywheel, (*b*) the governor in a steam engine.

7. How is the thermal efficiency of a steam engine calculated in practice? A steam engine is supplied with saturated steam at 165° C. and the exhaust steam has a temperature of 70° C. The engine used 15 pounds of steam per horse-power per hour. Find the thermal efficiency. What would be the thermal efficiency of a Carnot engine working between the same temperatures? (Take the quantities required from the Table, p. 534.)

8. Describe the principal sources of waste in a steam engine. What means are taken to reduce these?

9. Make an outline sketch and briefly explain the action in a compound steam engine.

10. Sketch an approximate indicator diagram for a steam engine in which steam is supplied at 75 lb. wt. per square inch absolute and is cut off at one-third stroke. The pressure of the exhaust steam is 17 lb. wt. per square inch absolute. How would you obtain the mean pressure from the diagram?

11. Sketch in outline the principal parts of a steam engine indicator. Give a brief description of the method of using the indicator.

12. A steam engine has a cylinder 20 inches diameter. The piston has a stroke of 3 feet. The engine runs at 180 revolutions per minute, and the mean pressure is 46 lb. wt. per square inch. Find the indicated-horse-power.

13. In testing an engine by means of a brake, the load on the brake was 70 lb. weight and the pull of the spring balance was 22 lb. weight. The brake wheel was 5 feet in diameter, measured to the centre of the rope, and had a speed of 210 revolutions per minute. Find the brake-horse-power. If the mechanical efficiency of the engine was 84 per cent., find the indicated-horse-power and the horse-power wasted in engine friction.

14. Describe the action in a steam turbine of the de Laval type.

15. Describe the principle of the action in a Parsons steam turbine, explaining clearly the method by which a speed of revolution is obtained lower than that of a turbine of the de Laval type.

16. Explain the meaning of the terms *temperature slope* (or *gradient*), *coefficient of thermal conductivity.*

Calculate these quantities in the case of a boiler whose plates are 1·8 cm. thick, if the side exposed to the furnace has an area of 7·6 square metres and is kept at a temperature of 105° C., steam being generated at the rate of 380 kilograms per hour at atmospheric pressure. Latent heat of steam, 537 calories per gram. How would you account for the comparatively low temperature of the exposed surface ? **L.U.**

CHAPTER XL

INTERNAL COMBUSTION ENGINES

Cycles used in internal combustion engines.—Any engine in which the combustion of the fuel is carried out in the engine cylinder is called an **internal combustion engine.** The fuel employed is gaseous, and may be lighting gas, or power gas made specially for use in the engine, or oil vapour.

There are two cycles in general use. In the **four-stroke**, or Beau-de-Rochas cycle, there are four strokes of the piston in the complete cycle.

1. **Charging stroke.** The piston moves outwards and draws into the cylinder an explosive mixture of gaseous fuel and air.

2. **Compression stroke.** The piston moves inwards and compresses the mixture.

3. **Explosion and expansion.** The mixture is exploded and the piston is pushed outwards by the expansive action of the gases. This is the only working stroke in the cycle.

4. **Exhaust.** The piston moves inwards and discharges the exhaust products of combustion. This cycle occupies two revolutions of the engine shaft.

In the **two-stroke** cycle, the whole action is completed in one revolution as follows:

1. **Explosion and expansion.** The previously compressed charge is exploded and expanded behind the piston, which is moving outwards. Towards the end of the stroke, the piston uncovers openings in the walls of the cylinder, and the gases exhaust outwards through these openings. Immediately afterwards other openings are uncovered and the fresh charge is forced into the cylinder.

2. **Compression.** The piston now moves inwards and compresses the fresh charge.

Diagram of a four-stroke cycle.—A typical indicator diagram is shown in Fig. 476. AL is a datum line showing atmospheric pressure, 1 is the charging stroke, in which the pressure falls a little below that of the atmosphere. 2 is the compression stroke, at the end of which

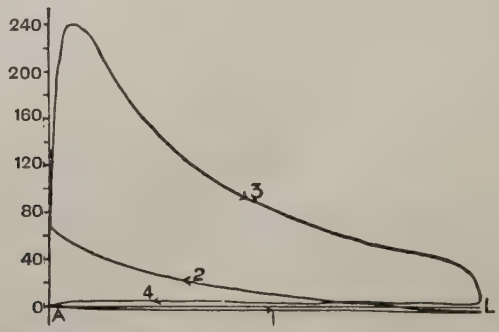

FIG. 476.—Indicator diagram showing the cycle in a gas engine.

the pressure may be from 50 to 200 lb. wt. per square inch above that of the atmosphere, depending on the type of engine. 3 is the explosion and expansion stroke. 4 is the exhaust stroke, during which the pressure rises slightly above that of the atmosphere.

Arrangement of a small gas engine.—Referring to Fig. 477, A is a cylinder fitted with a piston B which is connected to the crank C by

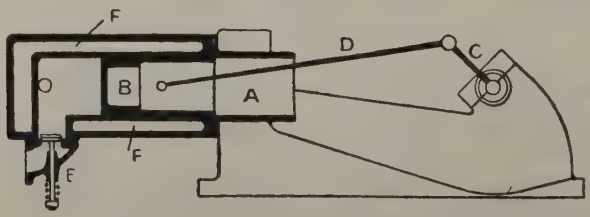

FIG. 477.—Outline diagram of a gas engine.

the connecting rod D. The cylinder is open at the end facing the crank, and there is no piston rod. The cycle takes place on the left-hand side of the piston only, and the right-hand side of the piston is constantly exposed to the pressure of the atmosphere. A jacket F surrounds the cylinder, and water circulates in this jacket for the purpose of preventing the temperature of the cylinder walls from rising too high. Valves are provided for admitting gas and air to the cylinder and for exhausting the waste products of

combustion. The exhaust valve is shown at E in Fig. 477; the other valves are of the same type—mushroom—and are kept closed by means of springs; the valves are opened at the proper instants by means of levers, operated by cams fixed on a side shaft driven by the crank shaft. In Fig 478, F is a cross section of the cylinder, A is the exhaust valve, B is the lever for opening the exhaust valve, and

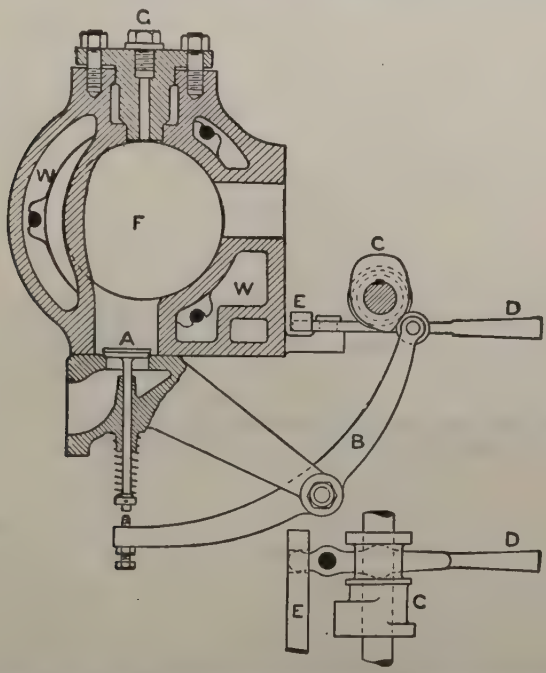

FIG. 478.—Exhaust valve and starting gear of a Crossley gas engine.

C is the cam on the side shaft. The side shaft runs at half the speed of revolution of the crank shaft, since each valve is required to open once only in two revolutions of the crank shaft.

The charge in a gas engine consists roughly of one part by volume of gas to about eight parts of air. Ignition of the explosive mixture may be effected by means of a tube heated externally by a bunsen. The interior of the tube is put into communication with the engine cylinder near the end of the compression stroke, and the charge becomes ignited by contact with the hot walls of the tube. Electric ignition is often used, the charge being exploded by means of an electric spark which is passed between platinum points in the cylinder.

Steady speed is maintained by a governor, which may operate either by cutting off entirely the supply of gaseous fuel during one or more cycles, air only being drawn into the cylinder, or by reducing the supply of gas, thus giving weaker explosions.

In modern gas engines, about 35 per cent. of the heat available in the gaseous fuel is converted into work done on the piston ; about 25 per cent. passes into the jacket water, and the remaining 40 per cent. is carried off by the exhaust gases. This efficiency of 35 per cent. is roughly three times the efficiency of a good boiler and steam engine plant. The efficiency of internal combustion engines depends to a large extent upon the pressure reached at the end of compression, and becomes greater as this pressure is increased.

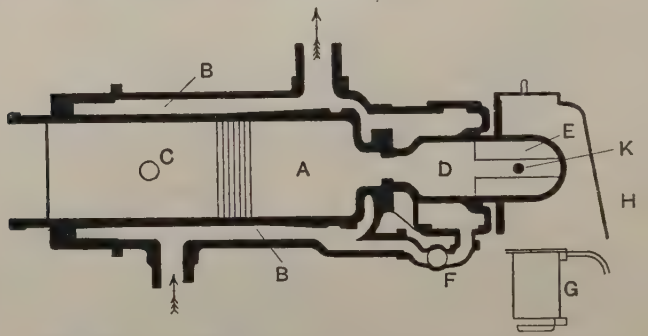

FIG. 479.—Section of the cylinder of a Hornsby-Akroyd oil engine.

Oil engines.—A large number of internal combustion engines using oil fuel operate by converting the oil into vapour and mixing the vapour with air so as to form an explosive charge. The mixture is then used in the cylinder in the same way as the charge in a gas engine. Heavy oils are vaporised by spraying and heating the spray ; lighter oils can be vaporised by spraying into air which is only slightly warmed. The heat required for vaporising may be obtained by using some of the heat developed during explosion, or from the hot exhaust gases, or from an external lamp.

Hornsby-Akroyd oil engine.—This engine is a good example of the type using heavy oil. A section of the cylinder A is given in Fig. 479 ; B is the water jacket and C is the piston. The cylinder is in constant communication with a bulb D at the rear end. The portion of the bulb near D is water jacketed, being supplied with water from the

main jacket through a pipe and regulating cock at F. The portion E has no water jacket, and therefore becomes hot when the engine is working. This portion forms the vaporiser, and oil is injected into it at the proper time through a small opening at K. The engine will not start unless the vaporiser is hot, and a lamp G is provided in order that E may be heated before attempting to start the engine; the flame from the lamp is directed on to the bulb E by means of a hood H.

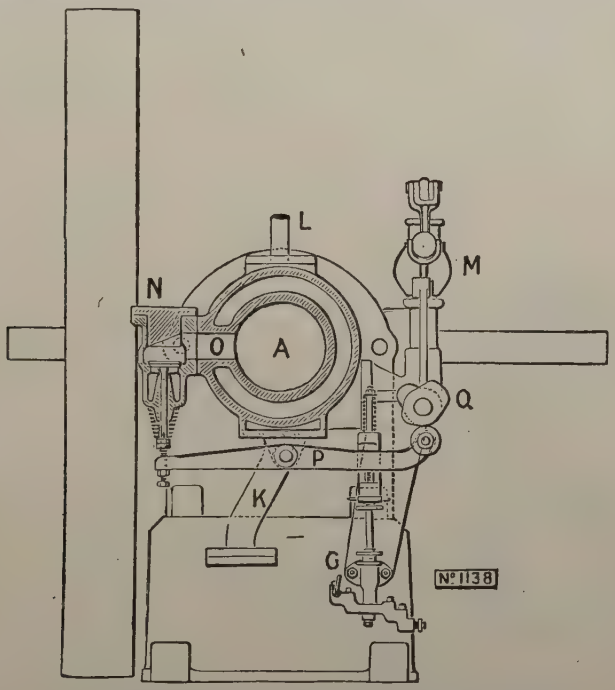

FIG. 480.—Sectional end elevation of the Hornsby-Akroyd oil engine.

A cross section of the cylinder is given in Fig. 480. In this view is shown a pump G which draws oil fuel from a tank and pumps it into the hot bulb. This pump is operated by the lever P, which also opens the valve N, through which and the port O air is taken into the cylinder. When the lever P is pushed down by the cam on the side shaft Q, which happens at the commencement of the suction stroke of the piston, a supply of oil is sprayed into the hot bulb, and air at the same time enters the cylinder as the piston moves forward. The oil becomes vaporised by the action of the hot walls of the bulb and mixes with the air in the cylinder.

During the next inward stroke of the piston, the charge is compressed, and the degree of compression is so arranged that the charge is exploded by the hot walls of the bulb E (Fig. 479) at the instant that the compression stroke is completed. The succeeding outward stroke of the piston is the working stroke, and is accomplished by the expansive action of the gases generated in the explosion. The following inward stroke of the piston is the exhaust stroke, and the exhaust gases are discharged through the port O (Fig. 480) and a valve which is placed near the air valve N, the valve being operated by a cam on the side shaft. In Fig. 480, M is a governor which adjusts the supply of oil fuel entering the cylinder to meet the demand for power, and thus maintains steady speed. It will be noticed that the complete cycle occupies four strokes.

In this type of engine, as in the gas engine described on p. 522, the heat is added to the working substance whilst the piston is practically stationary; the volume of the working substance is thus nearly constant during the operation of taking in heat. Oil engines of this kind use about 0·8 pound of oil per brake horse-power per hour.

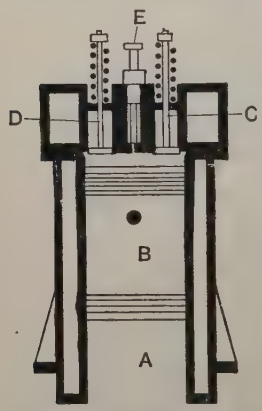

FIG. 481.—Section of the cylinder of a four-stroke Diesel engine.

Diesel oil engine.—The Diesel oil engine uses heavy oil, but operates on a different plan. Both the four-stroke and the two-stroke cycles are used in this engine. In Fig. 481 is shown the section of the cylinder A used for the four-stroke cycle. B is the piston, and is cooled by circulating water, as also is the cylinder. There are three valves at the top of the cylinder, an air valve C, an exhaust valve D, both opening downwards, and a fuel admission valve E opening upwards. All these are operated by cams from a side shaft. The cycle is as follows:

First stroke, downwards, charging with air only. The air valve C is open during this stroke, and air alone is drawn into the cylinder.

Second stroke, upwards, compression. During this stroke, the air is compressed to about 500 lb. wt. per square inch, and the temperature rises to about 600° C.

Third stroke, downwards, admission of fuel and expansion. During a small part of this stroke, the fuel valve is opened and oil fuel is forced into the cylinder by the action of air which has been compressed by a pump to a pressure of about 800 lb. wt. per square inch.

The compressed air in the cylinder, being at a temperature of 600° C., is sufficiently hot to cause the oil fuel to ignite and burn during admission. The pressure rises but slightly, if at all, during this stage, since the piston is moving downwards, and the rise in temperature is compensated by the increase in volume. After a small fraction of the stroke has been completed, the oil fuel is cut off by the closing of the valve E, and the remainder of the stroke is completed under the action of the expanding gases.

Fourth stroke, upwards, exhaust. During this stroke the valve D is opened, and the waste products of combustion escape into the atmosphere.

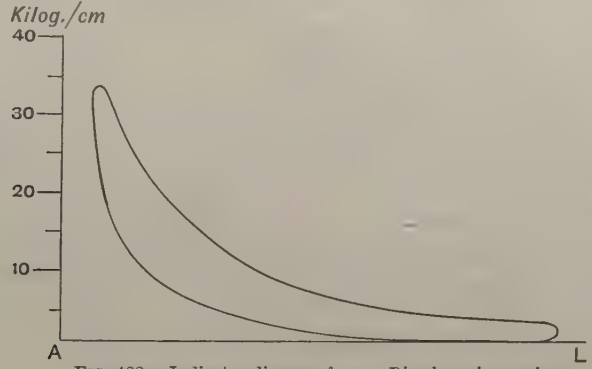

FIG. 482.—Indicator diagram from a Diesel marine engine.

The fuel consumption of the Diesel oil engine is very low—about 0·45 pound of fuel oil per brake-horse-power per hour. The efficiency, reckoned on the work done on the piston, is about 40 per cent., or reckoned on the useful work which the engine is capable of giving out, about 30 per cent. This engine is developing rapidly for marine purposes.

An indicator diagram is given in Fig. 482. From the attached scale of pressures, it will be seen that the pressure at the end of the compression is about 33 kilograms per square centimetre. The shape of this diagram should be compared with that for the gas engine, given in Fig. 476.

The **Semi-Diesel oil engine** is a development from the Akroyd and Diesel oil engines. The advantages of the latter are great as regards fuel economy, but the pressures required are very high. Lower pressures can be employed by having a hot bulb, as in the Akroyd engine, and the consumption of oil fuel is still kept low.

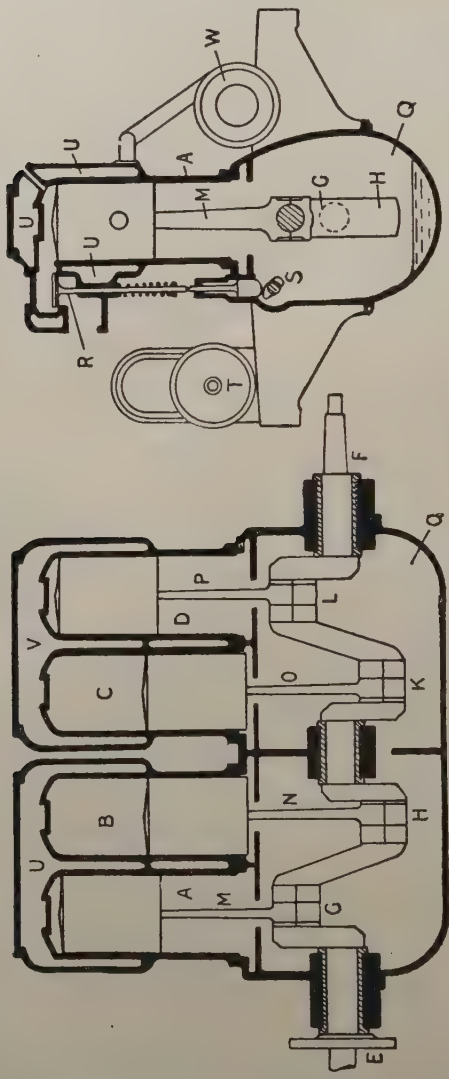

FIG. 483.—Outline sections of a four-cylinder petrol engine.

Petrol engines.—Under this heading is included engines using fuel oil which does not require to be heated considerably in order to vaporise. A set of petrol engines for a motor car is shown in outline

in Fig. 483. There are four cylinders A, B, C and D ; the pistons are connected to the crank shaft EF, which is furnished with four cranks G, H, K, L, arranged as shown. M, N, O and P are the connecting rods. The cranks work in a chamber Q, the lower part of which forms a tank for holding lubricating oil ; the lower end of each connecting rod has a scoop which dips into the oil at the lowest part of the stroke and splashes it upwards. The inlet and exhaust valves, one of which is shown at R, are arranged side by side, and are operated by cams on a side shaft S, which is driven at half the speed of the crank shaft. U and V are water jackets surrounding the cylinder ; water is pumped through these by a centrifugal pump W, driven from the crank shaft. The water after leaving the jackets passes through a radiator placed at the front of the car, where air may circulate freely round the radiator pipes and thus cool the water. Ignition of the charge is accomplished electrically, the current being supplied by a magneto machine T driven by the engine.

Action of the carburettor.—Air charged with petrol vapour is said to be carburetted. The principle of a carburettor may be understood by reference to Fig. 484. A is a passage leading to the inlet valve on the engine cylinder. During the suction stroke of the engine, air is drawn through this passage, the supply coming from the atmosphere through the orifice B. C is a pipe of fine bore kept supplied with petrol from a small tank D. The air rushing past the mouth of C causes a jet of petrol to issue and mingle with the air. The mouth of B is placed fairly close to one of the engine exhaust pipes, and the entering air is thus warmed slightly, sufficiently to vaporise the issuing jet of petrol. The tank D is supplied with petrol from the main tank on the car through a pipe connected to E. The level in D is kept just a little below the mouth of C by means of a float F. When enough petrol has entered D, the float rises and pushes the levers H, H ; these in turn push downwards a needle valve G and close the entrance E.

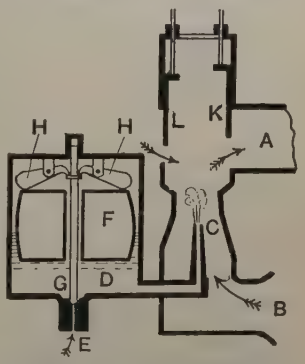

FIG. 484.—Diagram of a carburettor.

Part only of the air supply enters at B, and the remainder is sucked through an opening which can be adjusted by means of a sliding valve L. Another adjustable valve K, called the throttle valve,

serves to regulate the quantity of mixture entering the cylinder. The quantity of petrol which issues from the jet depends upon the speed of the air current past the pipe C, and does not vary proportionally with the speed. Too much petrol is taken up at high speeds of flow of the air. Hence the necessity for having the valve at L, which, when fully opened, reduces the speed of the air entering at B. The valve at L is often made so as to be automatic, opening against the resistance of a light spring, thus producing an **automatic carburettor.** If the speed of the engine is high, the automatic inlet valve at L opens more fully, and *vice versa.* Ordinary working proportions in the working mixture are about 2 of petrol vapour to 98 of air by volume.

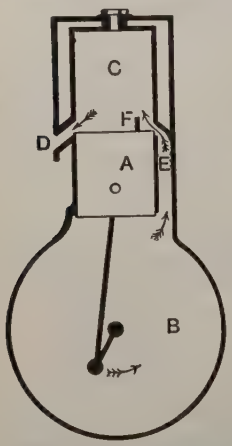

FIG. 485.—Diagram of a two-stroke petrol engine.

Engine speeds up to 2000 revolutions per minute at full power are common. The efficiency of transmission of power from the engines to the road driving wheels in a motor car is about 70 per cent. on the average. The principal resistance at high speeds of the car is due to the passage of the vehicle through the air, which may amount to 50 per cent. of the total resistance. At low speeds the air resistance may be as low as 10 per cent. of the total resistance.

The two-stroke cycle.—A two-stroke cycle petrol engine is shown in outline in Fig. 485. During the upward stroke of the piston A, the charge is drawn into the crank chamber B and is compressed somewhat during the downward stroke. Near the end of the downward working stroke, the piston uncovers an exhaust port D, and the gases commence to rush out of the cylinder. Immediately afterwards the inlet port at E is uncovered, and the slightly compressed charge enters the cylinder through the passage leading from the crank chamber. There is a baffle F on the piston which serves to direct upwards the entering charge and partially to prevent mixture with the waste gases leaving the cylinder through D. The charge in C is compressed during the upward stroke of the piston and is ignited at the top of the stroke. Expansion follows during the downward stroke. The whole cycle is thus accomplished in two strokes of the piston. In Diesel engines working on the two-stroke cycle, a separate pump driven by the engine is used to

compress slightly and deliver the charge to the cylinder. It is evident that, since an explosion occurs every revolution, more power can be obtained from a two-stroke engine than from an engine of the same size and speed working on the four-stroke cycle.

Horse-power of an internal combustion engine.—The indicated-horse-power of a gas, or oil, engine may be obtained from the indicator diagram in the same way as that employed for steam engines (p. 510), the only difference being that the number of explosions per minute is used instead of the number of strokes.

Let p_m = the mean pressure obtained from the indicator diagram, in lb. wt. per sq. in.

A = the area of the piston, in sq. in.

L = the length of the stroke, in feet.

E = the number of explosions per minute.

Then $$\text{I.H.P.} = \frac{p_m \text{ALE}}{33000}.$$

The indicator diagrams obtained from internal combustion engines working under given conditions often show considerable variation, and thus there is difficulty in obtaining the indicated-horse-power accurately. For this reason it is customary to rate internal combustion engines by stating the brake-horse-power, which can be determined accurately by experiment in the manner described on p. 513.

Exercises on Chapter XL.

1. Describe with reference to an indicator diagram each of the strokes in a four-stroke cycle internal combustion engine.

2. Describe how the four-stroke cycle is carried out in the cylinder of a gas engine. What is the object of the water-jacket? Show by a sketch how one of the valves is operated.

3. The cylinder of a gas engine is 6·69 inches in diameter and the piston has a stroke of 1·187 feet; the engine runs at 188 revolutions per minute. In a test, the mean pressure from the indicator diagram was 69·3 lb. wt. per square inch and there were 76 explosions per minute. Find the indicated horse-power.

4. In the gas engine test given in Question 3, find the brake-horse-power from the following particulars: Diameter of the brake wheel, 4·719 feet; load, 103 lb.; pull of the spring balance, 39 lb.; revolutions per minute, 188.

5. In the gas engine test given in Questions 3 and 4, the consumption of gas was 22·1 cubic feet per indicated-horse-power per hour. If the heating value of the gas is 300 lb.-deg.-Cent. per cubic foot, what percentage of the heat supplied in the gas is converted into work done on the piston?

6. Describe briefly how the oil fuel is prepared for entering the cylinder in engines using (*a*) heavy oil, (*b*) light oil.

7. Describe briefly the action in an Akroyd oil engine.

8. Give a brief description of the Diesel oil engine, explaining the various points in the cycle.

9. Give a short general explanation of the mode of working in a petrol engine.

10. Give a short account of the two-stroke cycle. Illustrate your answer by an outline sketch.

11. An oil engine gave 6·2 brake-horse-power for an oil consumption of 5·64 pounds of oil per hour. If the heating value of the oil is 10,500 lb.-deg.-Cent. units, what percentage of the heat supplied in the oil is converted into useful work ?

12. The piston in a Diesel oil engine is 52 cm. in diameter and has a stroke of 90 cm. The mean pressure from the indicator diagram was 7·2 kilograms per square centimetre, and the speed was 112 revolutions per minute. Find the indicated-horse-power. The engine works on the four-stroke cycle and there is a working stroke every two revolutions.

13. In Question 12, the complete sets of engines have twelve similar cylinders, and the engines drive a ship on a voyage lasting $14\frac{1}{2}$ days. If the oil fuel consumption is 0·42 pound per indicated-horse-power per hour, find how many tons of oil are burned on the voyage.

14. Give some instances of the conversion of mechanical energy into heat, and also some instances of the conversion of heat into mechanical energy.

15. What are the essentials of a heat-engine, and what is the theory of its action ?

Illustrate your answer by reference to some one type of heat-engine actually used. Adelaide Univ.

Tables of Properties of Aqueous Vapour.*

Boiling points of water at pressures near standard atmospheric pressure.

The pressures are given in mm. of mercury at 0° C. at the sea level in latitude 45°.

Pressure mm.	Temp. degree C.	Pressure mm.	Temp. degree C.	Pressure mm.	Temp. degree C.	Pressure mm.	Temp. degree C.
733	98·99	745	99·44	757	99·89	769	100·33
734	99·03	746	·48	758	·93	770	·36
735	·07	747	·52	759	·96	771	·40
736	·11	748	·56	760	100·00	772	·44
737	·14	749	·59	761	·04	773	·47
738	·18	750	·63	762	·07	774	·51
739	·22	751	·67	763	·11	775	·55
740	·26	752	·70	764	·15	776	·58
741	·29	753	·74	765	·18	777	·62
742	·33	754	·78	766	·22	778	·65
743	·37	755	·82	767	·26	779	·69
744	·41	756	·85	768	·29	780	·73

Pressure of Saturated Water Vapour, from 0° to 100° C., in mm. of mercury.

Temp. C.	Pressure mm.	Temp. C.	Pressure mm.	Temp. C.	Pressure mm.	Temp. C.	Pressure mm.
0	4·58	26	25·13	51	96·99	76	301·3
1	4·92	27	26·65	52	101·9	77	314·1
2	5·29	28	28·25	53	107·0	78	327·2
3	5·68	29	29·94	54	112·3	79	340·9
4	6·10	30	31·71	55	117·8	80	355·1
5	6·54	31	33·57	56	123·6	81	369·7
6	7·01	32	35·53	57	129·6	82	384·9
7	7·51	33	37·59	58	135·9	83	400·5
8	8·04	34	39·75	59	142·4	84	416·7
9	8·61	35	42·02	60	149·2	85	433·4
10	9·20	36	44·40	61	156·3	86	450·8
11	9·84	37	46·90	62	163·6	87	468·6
12	10 51	38	49·51	63	171·2	88	487·1
13	11·23	39	52·26	64	179·1	89	506·1
14	11·98	40	55·13	65	187·4	90	525·8
15	12·78	41	58·14	66	195·9	91	546·1
16	13·62	42	61·30	67	204·8	92	567·1
17	14·52	43	64·59	68	214·0	93	588·7
18	15·46	44	68·05	69	223·6	94	611·0
19	16·46	45	71·65	70	233·5	95	634·0
20	17·51	46	75·43	71	243·8	96	657·7
21	18·62	47	79·37	72	254·5	97	682·1
22	19·79	48	83·50	73	265·6	98	707·3
23	21·02	49	87·80	74	277·1	99	733·3
24	22·32	50	92·30	75	289·0	100	760·0
25	23·69						

* For fuller tables of aqueous vapour pressures, see *Physical and Chemical Constants*, by Kaye and Laby (Longmans).

Properties of Saturated Steam. (Centigrade Units.)*

Temp. deg. Cent.	Pressure.		Volume.		Heat Units per Unit Mass.		
	Kilograms wt. per sq. cm.	Lb. wt. per sq. inch.	Cubic metres per kilogram.	Cubic ft. per pound.	Of water.	Latent heat.	Total heat of vapour.
0	0·0063	0·089	204·97	3283	0	594·7	594·7
5	0·0089	0·127	146·93	2353·6	5·0	592·1	597·1
10	0·0125	0·178	106·62	1707·9	10·0	589·4	599·4
15	0·0173	0·246	78·23	1253·2	15·0	586·8	601·8
20	0·0236	0·336	58·15	931·48	20·0	584·1	604·1
25	0·0320	0·455	43·667	699·48	25·0	581·5	606·5
30	0·0429	0·610	33·132	530·72	30·0	578·8	608·8
35	0·0569	0·809	25·393	406·76	35·0	576·1	611·1
40	0·0747	1·06	19·650	314·77	40·1	573·4	613·5
45	0·0971	1·38	15·346	245·82	45·1	570·7	615·8
50	0·125	1·78	12·091	193·68	50·1	567·9	618·0
55	0·160	2·28	9·607	153·89	55·1	565·2	620·3
60	0·202	2·88	7·695	123·26	60·1	562·4	622·6
65	0·254	3·61	6·211	99·49	65·2	559·6	624·8
70	0·317	4·51	5·050	80·89	70·2	556·8	627·0
75	0·392	5·58	4·1353	66·24	75·3	553·9	629·2
80	0·482	6·86	3·4085	54·60	80·3	551·0	631·5
85	0·589	8·38	2·8272	45·29	85·3	548·1	633·5
90	0·714	10·16	2·3592	37·79	90·4	545·2	635·6
95	0·862	12·26	1·9797	31·712	95·5	542·2	637·6
100	1·033	14·70	1·6702	26·754	100·5	539·1	639·7
105	1·232	17·52	1·4166	22·692	105·6	536·1	641·7
110	1·462	20·79	1·2073	19·339	110·7	532·9	643·6
115	1·726	24·55	1·0338	16·560	115·8	529·8	645·5
120	2·027	28·83	0·8894	14·247	120·9	526·6	647·4
125	2·371	33·72	0·7681	12·304	126·0	523·3	649·2
130	2·760	39·26	0·6664	10·675	131·1	520·0	651·0
135	3·200	45·51	0·5800	9·291	136·2	516·6	652·8
140	3·695	52·56	0·5071	8·123	141·3	513·2	654·5
145	4·248	60·42	0·4450	7·128	146·4	509·7	656·1
150	4·868	69·24	0·3917	6·274	151·6	506·2	657·8
155	5·557	79·04	0·3460	5·542	156·7	502·6	659·3
160	6·323	89·93	0·3065	4·910	161·9	498·9	660·8
165	7·170	101·98	0·2724	4·363	167·1	495·2	662·3
170	8·104	115·27	0·2429	3·891	172·2	491·4	663·7
175	9·131	129·87	0·2171	3·478	177·4	487·6	665·0
180	10·258	145·90	0·1945	3·116	182·6	483·7	666·3
185	11·491	163·44	0·1748	2·800	187·9	479·8	667·6
190	12·835	182·56	0·1575	2·523	193·1	475·7	668·8
195	14·300	203·4	0·1423	2·279	198·3	471·7	670·0
200	15·89	226·0	0·1288	2·063	203·6	467·5	671·1
205	17·61	250·5	0·1170	1·874	208·9	463·3	672·2
210	19·49	277·2	0·1063	1·703	214·1	459·1	673·2
215	21·57	306 8	0·0965	1·546	219·4	454·7	674·1

* Taken by permission from *The Steam Engine and Other Heat Engines*, by Sir Alfred Ewing. (Camb. Univ. Press.)

PART III

LIGHT

CHAPTER XLI

PROPAGATION OF LIGHT

Radiation.—In Chap. XXIX it was seen that all surfaces emit radiation; its absorption, reflection and transmission by various substances were considered, and its detection and measurement were touched upon. Such instruments as the blackened bulb, or the thermopile (p. 381) measure the total radiant energy absorbed. Consider the temperature of a body to be gradually raised; as the temperature of the emitting surface rises, the total radiation increases, but its character also changes. Although at this stage the change in character in the radiation cannot be discussed in any detail, still the eye tells us that there is some change.

Consider an iron ball in a dark room; it cannot be seen; but, if its temperature be raised, it becomes luminous, appearing a dull red. At a higher temperature it becomes bright red, and finally white at the highest temperatures, when it is said to be "white hot."

The hot iron is a source of light. Other sources of light are the candle, the electric incandescent lamp, the arc lamp, etc., in all of which cases the light is emitted on account of the high temperature of some substance. The character of the light depends upon the temperature of the source, but we are not here concerned with this; attention will be devoted to the laws of propagation of this radiation which produces vision, and is called light. The study of light is sometimes called **optics.**

Propagation in straight lines.—That light travels in straight lines is a fundamental law without which there would be no such study as optics; in fact, we should not "see" anything at all; we should merely be conscious of brightness or darkness, and no object would appear to our eyes to have any definite shape. Vision in this respect would resemble hearing. The following simple experiment demonstrates the rectilinear propagation of light.

EXPT. 111.—**Rectilinear propagation of light.** Place a candle, or other source of light, behind a cardboard screen A (Fig. 486) having a small hole in it. In front of A place similar screens B and C, and adjust their positions so that the eye situated at E can see the source of light S. Then

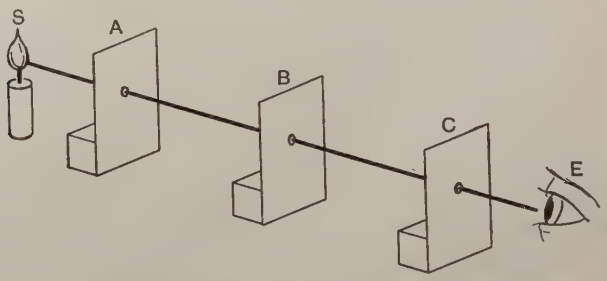

FIG. 486.—Experiment illustrating rectilinear propagation of light.

SE will be found to be a straight line. If any one of the screens be slightly moved so that the holes are no longer in line, the source can no longer be seen by the eye at E. In fact the best test that the holes are in a straight line is that light will pass through them all and that the source can be seen.

Ray and beam of light.—Light is of the nature of a wave motion, arising at the source of light and advancing forwards through the transparent medium. A line drawn in the direction in which this wave motion is travelling is called a **ray**. Thus a ray is only a useful convention and has no physical existence; for if we were to attempt to cut down a wave to an extremely narrow path, it is characteristic of wave motion that the wave would cease to be propagated in one single direction; it would spread out. For waves to be propagated unchanged in form they must be of considerable size. Such a wave would be called a **beam of light.** When all the rays defining the beam are parallel, it is called a **parallel beam**, SA (Fig. 510); when the rays approach each other, it is a **convergent beam** (Fig. 535); and when they are separating, it is **a divergent beam** (Fig. 536).

Shadows.—The simplest result of the straight-line propagation of light is the production of **shadows**, and indeed the production of sharp shadows is a proof of rectilinear propagation. Thus, a very small source of light S (Fig. 487) will illuminate all points of the screen above A and below B; but between A and B there is complete shadow,

since the light from S is stopped by the opaque body C. The boundary AB of the shadow is very sharply defined and has the shape of the contour of the opaque body as seen from S.

As a rule, the luminous source is not very small, so that the shadow has not a sharp boundary. In Fig. 488 the parts of the screen beyond A and B are fully illuminated. Within C and D there is complete shadow, but in the intermediate regions there is only half shadow, fading gradually from dense shadow at C and D to no shadow at A and B.

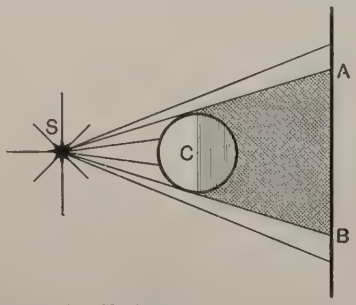

FIG. 487.—Shadow due to a point source of light.

A third case presents itself when the source of illumination is larger than the body casting the shadow (Fig. 489). The only region of complete shadow is a cone CDE, and

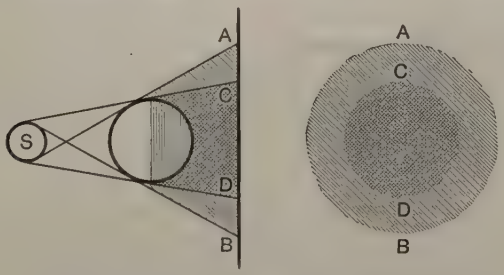

FIG. 488.—Shadow when source of light is not a point.

if a screen be placed at A, there will be a small region of dense shadow surrounded by a large region of half shadow, but if the

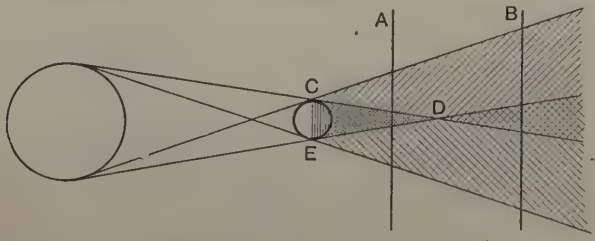

FIG. 489.—Shadow with source of considerable size.

screen be placed at B, there will be no complete shadow, merely a large, diffuse half shadow.

EXPT. 112.—**Shadows.** Cut a hole of 10 cm. diameter in a sheet of cardboard, and cover the hole with oiled paper. Behind this place the brightest source of illumination available. In front of the sheet place a circle of cardboard of 20 cm. diameter; observe the shadows produced on a neighbouring wall. Replace the 20 cm. circle by one of 5 cm. diameter and note that the shadow as at A or at B may be obtained (Fig. 489), according to whether the screen is at A or B.

Eclipses.—An example of shadows on an astronomical scale occurs in the case of eclipses. It sometimes happens at new moon that the shadow of the moon falls upon the earth, in which case the sun is wholly or partially obscured by the moon, and we get an **eclipse of the sun.** In Fig. 490, some locality A, on the earth E, falls within the shadow of the moon m_1, and there is an eclipse of the sun S. Owing to the variations in the distances of the moon

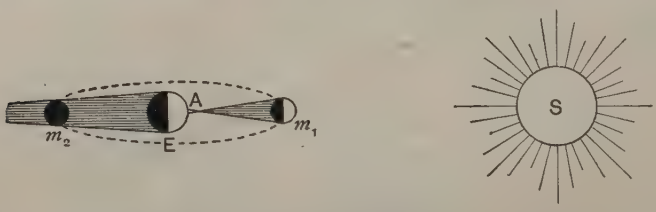

FIG. 490.—Eclipses.

and sun from the earth, the apparent diameter of the moon (the angle subtended by it at the earth) varies between 28′ 48″ and 33′ 22″, and the apparent diameter of the sun varies between 31′ 32″ and 32′ 36″. Hence the moon sometimes appears from the earth to be larger than the sun, and sometimes smaller. If it has a larger apparent diameter than the sun at the time of an eclipse, the eclipse may be **total.** If, on the other hand, the moon's apparent diameter is less than that of the sun, the sun's disc does not appear to be entirely obliterated, and the eclipse is said to be **annular.**

When the earth, moon and sun are not exactly in line a **partial eclipse** may result. Sometimes, at full moon, the moon m_2 passes through the earth's shadow, in which case there is an **eclipse of the moon.** The relative sizes and distances are not drawn to scale in Fig. 490.

Pin-hole camera.—A screen with a hole in it, placed in front of a source of light, gives an arrangement which is really the converse

of that which produces a shadow. With a candle AB and a screen having an aperture C (Fig. 491), narrow beams of light from the different points of the candle pass through the aperture and fall on a second screen at points such as *a* and *b*. The whole of these bright points build up into a representation, or **image**, of the candle, more or less perfect according to circumstances. Thus, if the pin-hole is small, each point of the candle is represented by a very small patch of light at the image, which is then sharp and well defined. If, however, the hole is large, there is considerable overlapping of the patches and the image is blurred, although bright, the amount of light passing through the hole being great. Further, the image is inverted, since all the beams cross at C. It will also be noted from the geometry of Fig. 491 that the size of the image is proportional to its distance from the pin-hole. If the screen *ab* be moved nearer to the pin-hole, the image will become smaller and brighter, but it will be noted that it becomes more blurred, owing to the increased overlapping of the patches of light.

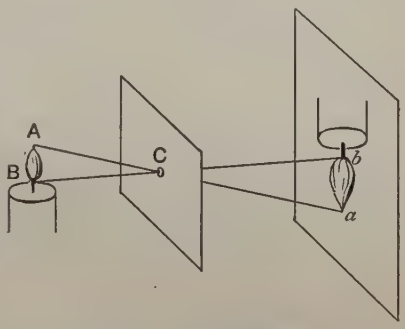

FIG. 491.—Pin-hole camera.

EXPT. 113.—**Pin-hole camera.** Obtain an ordinary cardboard box; remove one side or end and replace it by a sheet of oiled paper, or ground glass. Make a pin-hole in the side furthest from the oiled paper. On pointing this hole towards the brightly illuminated window of a room, an inverted image of the window and all bright objects near it will be seen on the oiled paper.

Transparence, opacity, translucence.—The terms 'transparent' and 'opaque,' as applied to bodies, have been used already, and they may now be defined more exactly. A **transparent** body is one which will transmit light according to the straight-line law, whereas a body is **opaque** when it will not transmit light at all. The terms are really only relative, as materials usually considered to be transparent, such as glass, absorb some light when a beam traverses them. On looking through a thick block of glass at a bright object, it will be seen that the brightness is much reduced, and generally the emergent light is of a greenish blue colour. Thus a sufficient

thickness of glass would transmit hardly any light. On the other hand, bodies so opaque as the metals will, if sufficiently thin, transmit an appreciable amount of light. Gold may be obtained in such thin layers, in the form of gold leaf, that light will pass through, and the transmitted light is of a deep green colour.

Substances such as oiled paper, ground glass, etc., transmit light, but not in straight lines. They are said to be **translucent.** Light in passing through them is scattered, and such substances may be used as screens on which to project optical images. Incandescent lamps often have frosted or translucent bulbs, to diffuse the light from the hot filament.

Vision.—The phenomena external to the eye, occurring in the process of vision, will alone be considered. If light proceeding from a luminous point enters the eye, the point is the apex of a divergent

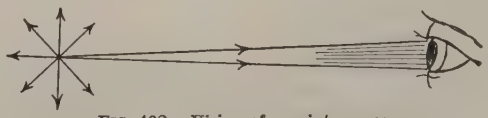

FIG. 492.—Vision of a point source.

beam, or cone of rays, the pupil of the eye being the base of the cone. Whenever such a cone of rays enters the eye (Fig. 492), the eye sees a bright point at the apex of the cone. Each point of a luminous source acts as an emitting point, and the eye sees the source as a collection of points so close together as to give the impression of uniformity.

Bodies which are not self luminous are not visible unless light falls upon them and is scattered in all directions. Each point then behaves like a luminous source. A beam of light is not visible, but may render material objects visible ; thus the path of a beam of sunlight is often rendered evident by the dust particles scattered in it. The eye sees the dust particles, not the beam of light.

EXERCISES ON CHAPTER XLI.

1. Explain, giving examples, why you believe that light travels in straight lines.

2. Describe, with a diagram, the form of the shadow of a square piece of paper, cast by an incandescent lamp on to a wall, with the square halfway between the lamp and the wall and parallel to the wall.

3. Explain the production of solar and lunar eclipses, and distinguish between the various kinds of solar eclipse that may occur.

4. If the sun's diameter be 400 times that of the moon, and if the distance of the sun from the earth is 93,000,000 miles, find whether the eclipse of the sun will be total or annular when the moon is at a distance of 238,000 miles from the point of observation on the earth; earth, moon and sun being in one straight line.

5. Describe the pin-hole camera. Find the height of the image produced by a man whose height is 6 ft., when his distance from the pin-hole is 15 ft., and the distance from the screen to the hole is $6\frac{1}{2}$ inches.

6. A candle flame 2 cm. high is placed at a distance of 15 cm. from a pin-hole in a screen. Find the size of the image when a screen is placed 25 cm. and 80 cm. respectively from the hole.

7. Explain how the brightness and definition of an image produced by a pin-hole vary with the distance of the screen from the hole.

8. A disc of diameter 4 cm. is parallel to and is situated at a distance of 7 cm. from a luminous disc of diameter 9 cm. Give a drawing of the arrangement and find the length of the cone of perfect shadow.

9. Describe carefully the formation of an image by the pin-hole camera. What is the effect of varying (*a*) the shape, (*b*) the size, of the hole?

10. A small circular object is placed near a large luminous sphere. Describe the changes in the appearance of the shadow of the small object as a screen is brought up from a distance.

CHAPTER XLII

ILLUMINATION AND PHOTOMETRY

Sensation of brightness.—Everyone is sensible of the differences of illumination at various times and places, but is quite unable to give any numerical relation between these illuminations. The reasons for this are, in the first place, that the eye is incapable of quantitative estimation; and secondly, that it unconsciously adapts itself to various illuminations, by widening or contracting the pupil to receive more or less light, according to whether the surroundings are badly or intensely illuminated. For these reasons, other methods of measuring illumination must be devised, and several preliminary definitions and laws must be considered.

Intensity of illumination.—The amount of light falling perpendicularly upon unit area of any surface in one second is taken to be the **intensity of illumination** of that surface. There is no means of determining this quantity in absolute measure, for, as has been noted above, the eye fails us; and even if the total radiant energy were measured, this would not represent illumination, as the eye might not be sensitive to this radiation. Nevertheless, this idea of intensity of illumination is useful, and intensities of illumination may be compared by several means.

Law of inverse squares.—It is common experience that at great distances from a source of light the intensity of illumination is less that at points near it. The rate at which the intensity of illumination diminishes, as the distance from the source increases, follows from the fact that light travels in straight lines.

Consider a point source of light S (Fig. 493). The rays proceed from S in all directions, and a small screen placed at A will cast a shadow. If now a screen be placed at B, of exactly the same size and shape as the shadow at B, then on removing A, the light that originally fell on A will fall on B. If the distance SB is twice SA, we

know from geometry that the area of the screen B is four times that of A, and hence the intensity of illumination at B is one-quarter of that at A, because the same amount of light falls on four times the area. Similarly, if SC is three times SA, the intensity of illumination

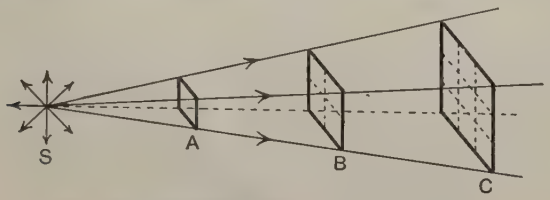

FIG. 493.—Illustration of the law of inverse squares.

at C is one-ninth of that at A, and so on. In general, since the area of any perpendicular section of the pyramid SABC is proportional to the square of the distance of the section from S, the amount of light per unit area is inversely as the square of the distance. Hence the **intensity of illumination due to a point source of light varies inversely as the square of the distance from the source.** Although ordinary sources of light are not points, yet at considerable distances they may be treated as such.

EXPT. 114.—**Law of inverse squares.** Cut three cardboard squares, the sides of which are respectively, 5, 10, and 15 cm., whose areas are therefore in the ratios 1, 4, and 9. Place one square near a candle flame and adjust the other two so that the shadow of the first falls exactly on them. Measure their distances from the candle, when they will be found to be in the ratios 1 : 2 : 3.

Illuminating power of a source.—Having defined what is meant by intensity of illumination, the illuminating power of a source may be given in terms of it. **The illuminating power of a source is the intensity of illumination it will produce at unit distance.** For a luminous point, this definition presents no difficulty, as the radiation of light takes place uniformly in all directions, but for an actual source, such as an electric lamp, the radiation is often far from uniform. However, the illumination in any given direction is quite definite, and the illuminating power in this direction is defined as above. The illuminating power due to a carbon filament lamp is greatest in a direction perpendicular to the plane of the filament and least in the 'end on' position. The metal filament lamp has usually a uniform illumination in directions at right angles to the axis of the lamp, but is very much less along the axis.

If the illuminating power of a lamp be P, this is the intensity of illumination at unit distance, so that, by the inverse square law, the intensity of illumination I at distance d, is P/d^2, or

$$I = \frac{P}{d^2}.$$

Photometers.—In order to compare the illuminating powers of two sources, the principle always made use of is that of adjusting their distances from a screen until they both produce the same intensity of illumination at the screen. Thus if P_1 and P_2 are the illuminating powers of the sources, and d_1 and d_2 their distances from the screen when they produce equal illuminations, then

$$\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2}, \quad \text{or} \quad \frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}.$$

There are many devices for carrying out the required adjustment, but they nearly all depend upon illuminating one part of the screen by one source, and another part by the other source. The eye can then detect any inequality in the two illuminations, and the adjustment to equality can be made by varying the distance of one or other of the sources.

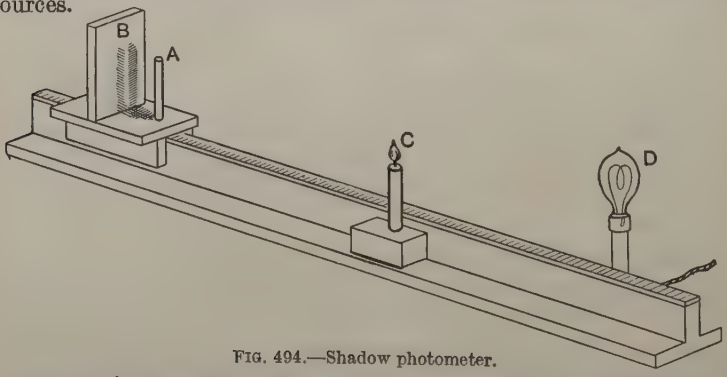

FIG. 494.—Shadow photometer.

Rumford's shadow photometer.—Two sources, such as a lamp and a candle, are placed upon an optical bench, as shown in Fig. 494, and a vertical rod A is placed in front of a white screen, so that two sharp shadows are cast, one of which is illuminated only by the lamp and the other by the candle. The positions of the lamp and candle are adjusted until the shadows are equally intense, and the distances BC and BD are then measured. From the relation

$$\frac{P(\text{lamp})}{P(\text{candle})} = \frac{BD^2}{BC^2},$$

the ratio of the illuminating powers can be found.

EXPT. 115.—**Shadow photometer.** Place a candle and a lamp in the positions shown in Fig. 494. Keep one of them fixed and move the other about until the shadows are equally intense, making six trials and taking the mean position. Repeat at four different distances; in each case, calculate the illuminating power of the lamp in terms of that of the candle, in other words, find the **candle-power** of the lamp.

Bunsen's grease-spot photometer.—If a piece of plain paper have a spot of oil or grease in the middle of it, the grease spot is more translucent than the rest of the paper. On holding the paper up to the light, the grease spot will appear brighter than the rest of the paper because more light passes through it, and reaches the eye, than passes through the rest of the paper. For exactly the same reason,

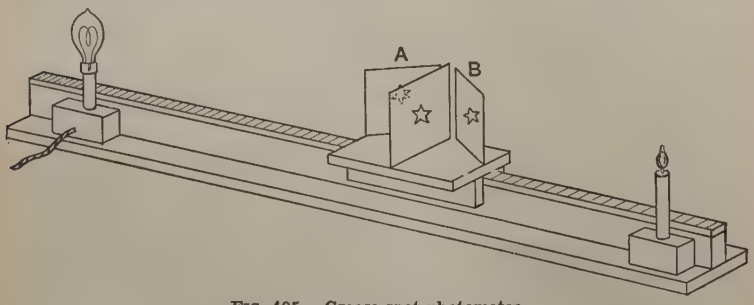

FIG. 495.—Grease-spot photometer.

on placing the eye on the same side of the paper as the source, the spot appears dark on a bright surrounding. In the grease-spot photometer, the screen, consisting of a piece of cardboard having a hole in it covered with oiled paper, is placed between the two sources of light to be compared. Its position is altered until the appearance of the screen is the same on both sides. It is then considered that both sources produce the same intensity of illumination at the screen. The distances are measured, squared, and the ratio taken as before. To enable both sides of the screen to be viewed at the same time two mirrors A and B (Fig. 495) are placed one on either side of the screen.

EXPT. 116.—**Grease-spot photometer.** Make a similar set of readings to that described in Expt. 115. In this case adjust the grease-spot screen until the appearance of both sides is the same. Reduce the readings in the same manner and compare the candle-power of the lamp found in both Expts.

Lummer-Brodhun photometer.—In the last two experiments the adjustments cannot be made with great accuracy, and the

determinations do not give constant value for the candle-power of the lamp. A much better form of photometer is the Lummer-Brodhun, shown in Fig. 496. Light from the two sources S_1 and S_2, to be compared, falls on the sides A and P of a milk-white screen. The optical arrangement is merely a device for comparing the intensities of illumination upon the two sides A and P. Light from P is reflected at Q, and passes through the glass blocks R and C where they are in contact in the middle. Light from A is reflected at B, and again at the surface of C where the two blocks R and C are not in contact. On looking through the telescope T the central part of the field of vision is illuminated by light PQR passing through the middle part, and the outer portions by light ABC reflected at C. This process will be better understood on reading Chap. XLV., where total reflection is dealt with.

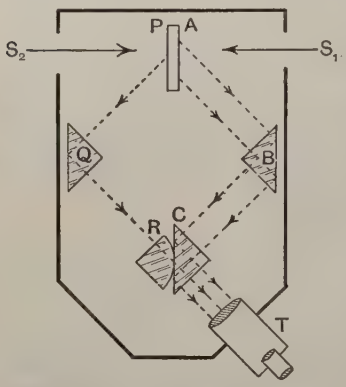

FIG. 496.—Lummer-Brodhun photometer.

It is found that this photometer is very sensitive to inequalities of illumination at A and P, and hence when the two parts of the field, as seen in the telescope, appear equally illuminated, an accurate measure of the candle-power can be obtained from the measured distances of the sources from the screen AP.

Standards of illumination.—The first light standard to be employed was the **standard candle**. This is defined by law to be a candle of spermaceti, of which six weigh one pound, the candle burning at a rate of 120 grains per hour. This standard, although useful for rough purposes, is not of sufficiently constant illuminating power to be of any scientific value. Its illuminating power varies with the condition of the atmosphere, and various candles differ slightly one from another.

A far more trustworthy standard is the **Harcourt Pentane lamp**, in which air is drawn over pentane liquid, the mixture being burnt in a standard burner, and the flame adjusted to a definite height. Corrections may be applied when the atmospheric conditions vary from the normal. The illuminating power of such a lamp is about equal to that of ten standard candles, but as its illumination is much more constant than that of a candle, it is adopted as an International Standard.

An **International Candle-power** is defined as an illuminating power equal to one-tenth of that of the Harcourt pentane lamp.

In Germany the **Hefner lamp** is used as a standard. It is a lamp constructed after a standard pattern, in which amyl-acetate is burnt. The Hefner standard is equal to 0·9 International Candle.

The French standard is the **Carcel lamp**, in which colza oil is burnt. The Carcel standard is equal to 9·62 International Candles.

Probably the most convenient standard for use in the laboratory is an **electric incandescent lamp**, properly constructed and capable of being run under constant electrical conditions. Such standards have been constructed by Prof. Fleming, by placing a carbon filament, which has been run for many hours, in a large glass bulb. This prevents blacking of the bulb. When such lamps are supplied by the instrument maker, they are accompanied by a certificate giving their candle-power when run at a definite voltage, as obtained by comparison with a pentane standard.

Practical illumination.—Although the definition of intensity of illumination given on p. 544 has served a useful purpose in enabling us to express illuminations and illuminating powers in definite terms, still it is useless for practical purposes, as we have no measure of "quantity of light." In the consideration of the illumination of rooms and public places, the intensity of illumination produced by a standard candle at a distance of one foot has been adopted, and is called the **Foot-candle.** The appropriate illumination of a book for comfortable reading is about 3 foot-candles. Hence if a 32 candle-power lamp be employed, it should be situated at such a height from the book that the intensity of illumination at the book is 3 foot-candles.

Thus $$3=\frac{32}{d^2}; \quad \therefore\ d=\sqrt{\frac{32}{3}}=3{\cdot}3 \text{ feet.}$$

This of course implies that all the light received comes directly from the lamp, as, for example, if the lamp were situated out of doors at night. In a room, a great deal of the light does not come directly to the book but is reflected from the ceiling and walls. The intensity of illumination in this case depends largely upon the nature of the walls and ceiling, whether light or dark. It may be taken as the result of experience that the reflection from the ceiling and walls amounts to something between 2 to 4 times the light received directly, according to whether they are dark or light. Thus, for a reading room having walls 15 ft. × 10 ft., each wall has an area of 150 sq. ft., and for an illumination of 3 foot-candles, 150 × 3, or 450 candle-power, would be requisite. Assuming that the walls and ceiling are light, one-fifth of the illumination is direct and four-fifths indirect (by reflection), so that the necessary candle-power of the lamp would

be $\frac{450}{5}=90$ c.p. Thus one 100 c.p. lamp, or three 30 c.p. lamps would be suitable. For a dwelling room 2 foot-candles is sufficient illumination, and the requisite candle-power in this case would be 60. This consideration points out how inadequately many houses are lighted, which in these days of high efficiency electric lamps is unfortunate.

Although the foot-candle is the unit of intensity of illumination usually employed in this country, it is desirable for many reasons to use the more scientific unit, the **candle-metre**, which is the intensity of illumination produced by a standard candle at a distance of one metre. Thus one candle-metre is equal to $0{\cdot}305^2=0{\cdot}093$ foot-candle. Hence a foot-candle is approximately equal to 10 candle-metres, and a suitable intensity of illumination of the surface for comfortable reading would be about 30 candle-metres.

Exercises on Chapter XLII.

1. Describe what is meant by 'intensity of illumination' and 'illuminating power.'

2. Describe the shadow photometer, explaining the difficulties which occur in comparing the illuminating powers of two sources of light.

3. Describe some form of photometer which may be used for the accurate comparison of the illuminating powers of two sources of light, and the measurements you would make in order to determine the average candle-power of an incandescent lamp, taking account of the fact that the light is not emitted equally in all directions.

4. If gas costs 2s. 6d. per thousand cubic feet, and electricity costs 3d. a unit, find the relative costs of lighting if a 60 c.p. gas burner uses 3 cubic feet per hour and a 50 c.p. electric lamp uses 2 units in 40 hours.

5. Describe an experiment to determine the candle-power of an electric lamp, explaining how and why the required calculation would be made. If the intensity of illumination produced on a screen 20 m. from an arc-lamp is equivalent to that of 1·2 candles at 50 cm. what is the candle-power of this lamp? L.U.

6. What is meant by candle-power? Two lamps of 32 and 16 candle-power are placed 100 cm. apart. Find the positions on a line joining them where a screen would be equally illuminated by these two lamps. How would you determine these positions experimentally? L.U.

7. Two sources of light, each 2 candle-power, are placed on the same side of a Bunsen grease-spot photometer. One is at a distance of 1 foot from the spot and the other at 2 feet. Where must a third source of 5 candle-power be placed in order that the appearance of each side of the photometer may be the same? L.U.

8. Explain the term 'foot-candle.' Find the total candle-power necessary to illuminate a room whose longest wall is 15 ft. long and 9 ft. high, whose walls and ceiling are light coloured, if the room is to be used as a reading room.

9. Explain the principle of the grease-spot photometer. If the two sides of the photometer look alike when the distances of the sources are 1 foot and 4 feet, what should be the distance of the fainter source when the brighter is 8 feet distant? L.U.

10. Describe some method of measuring the candle-power of a lamp.
A lamp produces a certain intensity of illumination at a screen when situated at a distance of 85 cm. from it. On placing a sheet of glass between the lamp and the screen the lamp must be moved 5 cm. nearer to the screen to produce the same illumination as before. What percentage of light is stopped by the glass? L.U.

11. Find the candle-power necessary to light a room whose largest wall measures 7·5 m. by 3·5 m., the intensity of illumination required being 25 candle-metres, and three-quarters of the light received being reflected light.

12. Give an account of some of the standards of illumination in practical use.

13. Describe an accurate form of photometer and explain how you would use it to prove experimentally that the illumination from a source of light falls off with increasing distance according to the inverse square law. L.U.

14. Describe a practical standard of illuminating power and a method whereby the powers of two sources of light may be compared. L.U.

CHAPTER XLIII

REFLECTION

Laws of reflection.—A beam of light in a homogeneous medium travels forwards, the disturbance progressing in one direction only; but when the beam passes from one medium to another, some of the light is **reflected**, or bent back. At the surface of polished metals nearly all the light is reflected. This reflection occurs in accordance with two definite laws:

First law of reflection.—The incident ray, the reflected ray, and the perpendicular, or normal to the surface, lie in one plane.

Second law of reflection.—The angle of incidence is equal to the angle of reflection.

In Fig. 497 if the reflecting surface is plane and perpendicular to the paper, it may be represented by the line EF. The normal DB is then in the plane of the paper, and if the incident ray AB be in this plane, the first law states that the reflected ray BC will also be in the plane of the paper.

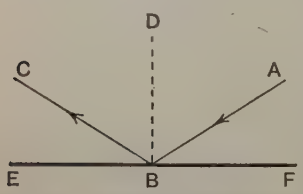

FIG. 497.—Diagram illustrating laws of reflection.

The **angle of incidence** ABD is that between the incident ray and the normal to the reflecting surface. It is convenient to consider this angle rather than the angle ABF between the ray and the reflecting surface, as will be seen when we deal with curved reflecting surfaces. According to the second law of reflection, the angle DBC, which is known as the **angle of reflection**, is equal to ABD.

There is no such thing as 'irregular reflection.' If the surface is not plane, the reflected rays will have various directions, and the light will be scattered. At each point the laws of reflection are obeyed. Thus when the surface is rough, the reflected light is **scattered or diffused**.

Demonstration of the laws of reflection.—One of the most convenient pieces of apparatus for demonstrating many of the optical laws is the **optical disc** (Fig. 498). A beam of light passing through a slit A falls upon a plane mirror B. This mirror is fixed to a graduated circle which can be rotated about an axis at B, so that the angle of incidence ABO can be varied, and may be read on the circular scale. The reflected beam traverses the path BD, and the angle of reflection OBD can be observed. It will be found that the angles of incidence and reflection are always equal.

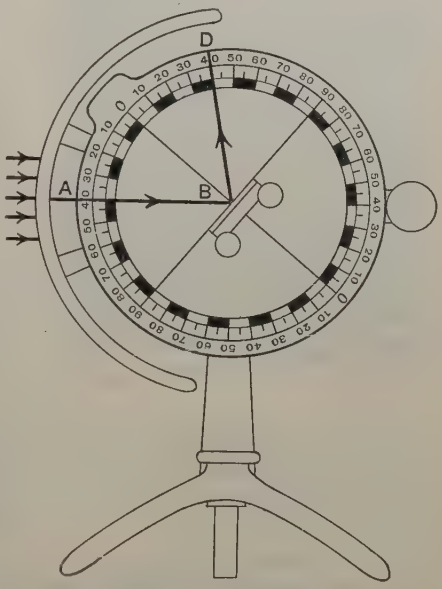

FIG. 498.—Optical disc.
(A. Gallenkamp & Co., Ltd.)

It should be noted that it is difficult to prove the first law of reflection by direct experiment. A little thought, however, will convince us of its necessity; for it is common experience that a beam of light reflected at a plane mirror is reflected as a parallel beam, that is, it does not spread out in all directions. Thus, if the reflected beam were not in the plane containing the incident ray and the normal, it would be to one side or the other of this plane, but the symmetry of the arrangement would prevent us choosing one side rather than the other. The wave theory of light also leads to the first law. And further, it may be taken as the result of experience, as the whole of our optical work depends upon its truth.

Image in a plane mirror.—Consider a luminous object situated in front of a plane mirror, such as a piece of silvered plate glass, shown in section at AB (Fig. 499). Any point S, of the source, emits light in all directions. One of the rays, such as SC, is reflected along CD, making the angle of reflection LCD equal to the angle of incidence SCL. Another ray SE is similarly reflected along EF. It will be observed that all the rays, after reflection, diverge from one and the same point S′ behind the mirror. An eye situated so as to receive some of these reflected rays will therefore see the bright point at S′. S′ is said to be the **image** of S, and every point upon the bright

source will have its corresponding image, the whole of them building up into an image of the bright body.

It may be shown, either by actual drawing to scale, or by simple geometrical reasoning, that the line SS′ is perpendicular to the mirror, and that SM = S′M. For since

$$\angle SCL = \angle LCD ;$$

$$\therefore \angle SCM = \angle DCE = \angle S'CM,$$

and consequently

$$\angle SCE = \angle S'CE.$$

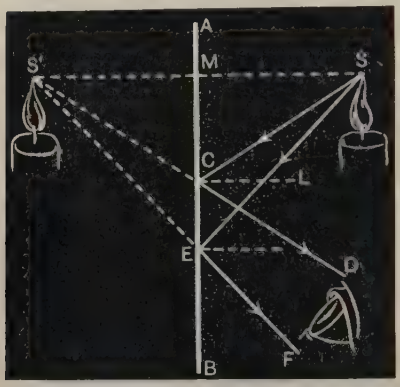

FIG. 499.—Production of image in plane mirror.

Similarly, ∠SEC = ∠S′EC, and the triangles SCE and S′CE, having the common side CE, are equal in all respects, and therefore SC = S′C. Now it follows that the triangles SCM and S′CM are equal in all respects, for SC = S′C, CM is common to them and

$$\angle SCM = \angle S'CM.$$

Hence SM = S′M, and ∠SMC = ∠S′MC, and therefore both these angles are right angles. Stating this result in words we say that: **the line joining object and image is perpendicular to the mirror, and the image is as far behind the mirror as the object is in front of it.**

EXPT. 117.—**Reflection by plane mirror.** Place a piece of silvered glass mirror vertically upon a sheet of drawing paper. In plan it is represented by AB in Fig. 500. Fix a large pin vertically into the paper at S. On looking into the mirror an image of the pin will be seen at S′.

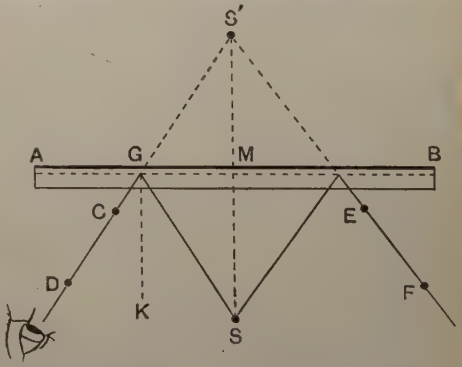

FIG. 500.—Experiment on reflection by plane mirror.

The aim is to locate this image. Put another pin at C, and, looking horizontally with C and S′ in line, place a third pin at D in line with C and S′. Repeat this procedure at E and F and at a number of other positions. Draw the line AB, remove the mirror, and join the points in pairs, forming the lines DC, FE, etc. These produced should all pass through a single point S′, the position of the image of S being thus found.

Now measure SM and S′M, and show that they are equal. Again, GCD is a reflected ray corresponding to the incident ray SG. Draw the normal GK, and measure the angles SGK and KGD with a protractor. Do the same for the other rays and make a table of the values of the angles of incidence and reflection.

It should be noted that if the mirror is a piece of thick plate glass, there is a bending of the rays on entering and leaving the glass, as will be described in Chapter XLV. This defect cannot be fully corrected, but by drawing a line parallel to AB at a distance of one-third of the thickness of the glass, in front of the silvered surface, and making all measurements from this line instead of from the silvered surface, it will be found that much more accurate results will be obtained.

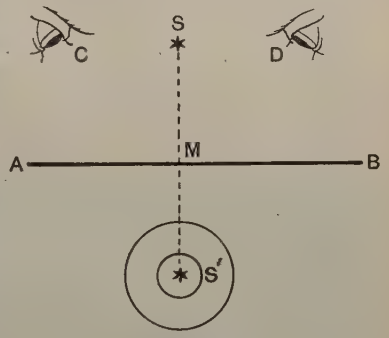

FIG. 501.—Image in plane sheet of unsilvered glass.

EXPT. 118.—**Position of the image produced by a plane sheet of unsilvered glass.** Place a piece of plane, unsilvered window glass vertically upon the table; on one side of it arrange a candle S (Fig. 501), and upon the other a flask with a narrow neck. Adjust the position of the flask until, on looking through the glass from C or D, the candle flame appears to be situated in the neck of the flask. If this appears to be so from all points of view, the image is then situated in the neck of the flask, as at S′. Measure the distances SM and S′M; they will be found to be equal.

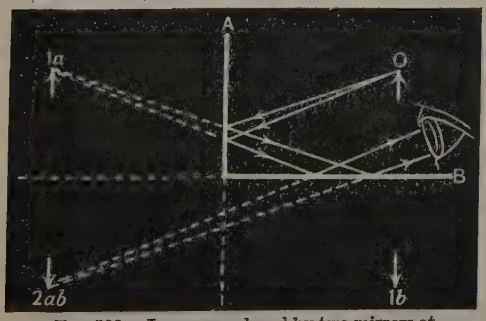

FIG. 502.—Images produced by two mirrors at right angles.

Inclined mirrors.—Two plane mirrors such as A and B, situated at right angles to each other as in Fig. 502, will of course produce two images 1_a and 1_b of the object O, 1_a being due to A and 1_b due to B. But there is a third image $2ab$, which may be looked upon as the image of 1_a in the mirror B, or of 1_b in the mirror A; this depends upon whether the

eye looks into B or A. In Fig. 502 it is shown looking into B. To find the path of the rays which produce this image, we must remember that the fact of the image being seen at $2ab$ shows that rays enter the eye as though they came from this point. But from their direction it is evident that they travel, before reflection, as though they came from 1_a. But 1_a being the image of O, these rays must originally have come from O. Hence the rays are twice reflected, first at A and then at B, and the image has therefore been called $2ab$. If the eye had looked into the mirror A, the second reflection would have occurred at A, and the image would have been called $2ba$.

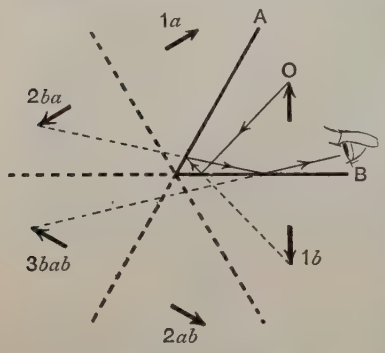

FIG. 503.—Images produced by two mirrors inclined at 60°.

Now let the mirrors be inclined at 60° to each other as in Fig. 503. The images 1_a and 1_b are each produced by one reflection, $2ab$ and $2ba$ by two reflections, as in the last case. There will now be a fifth image $3bab$ produced by three reflections; the path of the rays is given in the diagram. Note that for the position of the eye shown, the last reflection occurs at the mirror B, and the image is therefore called $3bab$. If the eye had looked into the mirror A the image would have been $3aba$.

From the last two cases it may be deduced that when two mirrors are inclined at any angle $\theta°$, the space is divided into $360/\theta$ sectors or angles, one being the real space between the mirrors. The other $(360/\theta - 1)$ sectors are image spaces, in each of which is situated one image, and there are therefore $(360/\theta - 1)$ images.

Parallel mirrors.—It will be seen from the last case discussed that when θ is small there are a great many images; for example, if $\theta = 2°$ there are 179 images. If the mirrors are parallel, θ is zero, and there will then be an infinite number of images. This might have been deduced separately from Fig. 504, in which is shown how the image $4abab$ is produced. The complete series of images cannot be drawn, as they extend to an infinite distance in both directions. This infinite train of images has doubtless been observed by the student when situated between two parallel mirrors, the only limit to the

number of images seen being fixed by the fact that some light is lost at each reflection.

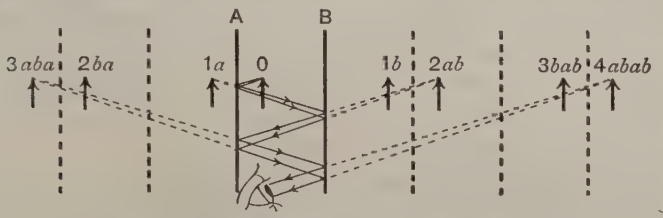

FIG. 504.—Images produced by two parallel mirrors.

Image in a plate-glass mirror.—The images seen in a plate-glass mirror are always more or less confused. When the beam SA meets the front surface of the glass (Fig. 505) a little of the light is reflected, giving rise to the faint image S_1. Most of the light enters the glass and is totally reflected at C, the greater part of it leaving along DE. This produces the bright image S_2. Some of the light is internally reflected at D, and again at F, giving the faint beam GK and the image S_3. The process is continued until the amount of light remaining is too small for any further image to be visible. A train of images is thus obtained, of which the second, at S_2, is the brightest, and it is this which is usually taken as *the* image in the mirror, the others merely producing confusion.

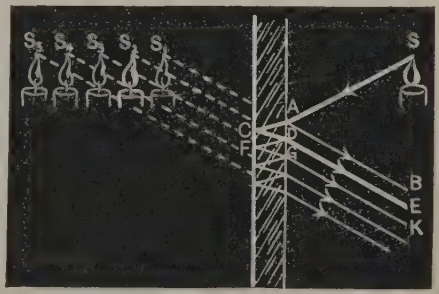

FIG. 505.—Image in plate-glass mirror.

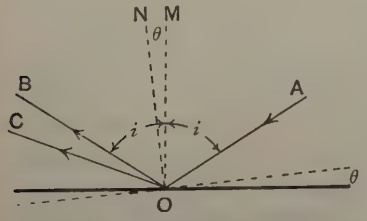

FIG. 506.—Beam reflected by rotating mirror.

Rotating mirror.—On allowing a beam of light to fall upon a mirror, the direction of the reflected beam depends upon the position of the mirror. Hence, if this be varied, the direction of the reflected beam will change. **When the mirror rotates through any angle, the reflected beam rotates through twice this angle.** For, with an angle of incidence i (Fig. 506) the angle of

reflection is also i, and the angle AOB between the incident and reflected beams is equal to $2i$. If now the mirror rotates through an angle θ, the normal OM moves to ON, the angle NOM being also θ. Thus the new angle of incidence is $(i+\theta)$, and $\angle AOC = 2(i+\theta)$; $\therefore \angle BOC = 2(i+\theta) - 2i = 2\theta$. Since AO is fixed, the angle BOC turned through by the reflected ray is 2θ; that is, it is twice the angle of rotation of the mirror.

Deviation produced by reflection.—Referring again to Fig. 506, it will be seen that the ray AO, which would have travelled straight on had there been no mirror, has been made to take the path OB. It has therefore been changed in direction by the angle

$$(\pi - AOB) = (\pi - 2i).$$

This is the **deviation** produced in any ray by reflection at a plane surface.

The deviation produced by two reflections would therefore be

$$(\pi - 2i + \pi - 2i') = 2\pi - 2(i+i'),$$

where i' is the angle of incidence at the second surface, assuming that the ray is in one plane the whole time. The relation between i and i' is not known unless the angle θ between the reflecting surfaces is known. The student will easily be able to prove that $\theta = i + i'$ (see similar proof on p. 623).

Thus, for two reflections,

$$\text{deviation} = 2\pi - 2\theta.$$

It follows that **a ray undergoing two reflections is deviated by a constant amount depending only upon the angle between the plane surfaces producing reflection.**

The sextant.—For measuring the altitudes of heavenly bodies while at sea, some instrument is required which is not affected by the motion of the ship, and by the fact that it is held in the hand. The sextant fulfils these conditions. The fixed mirror A (Fig. 507 (*a*)), called the 'horizon glass,' is only silvered on one half of its surface, so that an observer, looking through the telescope C, can observe the horizon by means of light travelling along DAC, and passing through the unsilvered half of the glass. By means of the second mirror B, called the 'index glass,' rays such as EB, also from the horizon, traverse the path EBAC, being reflected at B, and again at the silvered part of A. These also enter the telescope when A and B are parallel, and the observer sees the horizon in both halves of the field of vision. B is mounted upon the arm BG which can be made

to rotate about B, a vernier at G indicating the position relatively to the fixed circular scale MN. The vernier should read zero on the scale when the mirrors A and B are parallel, the two parts of the horizon seen through the telescope then appearing in one continuous line.

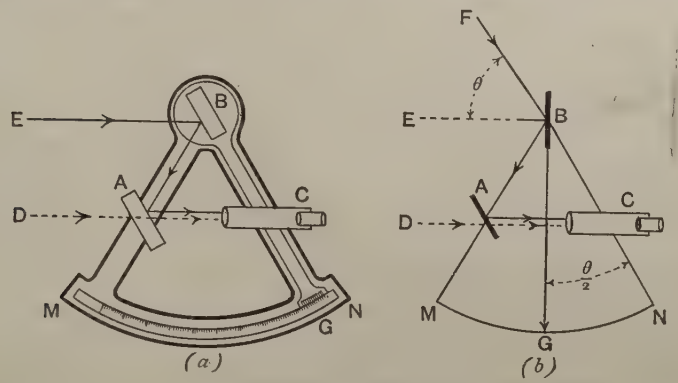

FIG. 507.—The sextant.

To find the elevation of a star, or the sun, the arm carrying the mirror B is rotated into the position shown in Fig. 507 (*b*), so that the rays FB, from the object, pass after reflection at B, along the path BAC, and the observer sees the star, or more usually, the lower edge of the sun, resting upon the horizon. From the relation for rotating mirrors on p. 558, it will be seen that the angle of elevation FBE is twice the angle rotated through by the mirror. In this case it is the reflected ray that is fixed, while the direction of the incident ray is changed. The movement of the vernier G over the circular scale MN is therefore half the altitude of the star, or lower edge of the sun. The scale is usually numbered to read altitude direct, each degree of rotation of the arm being marked 2°, the whole scale MN, which is usually 60° (hence the name 'sextant'), being therefore numbered up to 120°. For observations upon the sun, shades consisting of darkened glass are placed between A and B, and between A and C, to cut down the light to a suitable intensity.

Artificial horizon.—To determine the altitude of a celestial body from a point on land, an artificial horizon is sometimes employed, as there is then no sea horizon available. For this purpose, either a telescope mounted to rotate over a vertical circle, as in Fig. 508, or a sextant may be used. Light AB from the star is reflected at the horizontal surface of some mercury contained in a shallow vessel, and travels along BC. Light DC, also from the star, is parallel to AB owing

to the star's great distance. The angle BCD is observed, either by the sextant, or by rotating the telescope from the position in which the image of the star is seen along CB, to that in which the star is seen direct, along CD. Since the surface of the mercury is always horizontal, a horizontal line such as EC is parallel to FG, which passes along the surface of the mercury. Hence

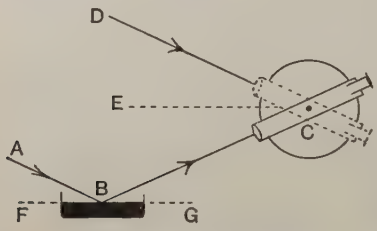

FIG. 508.—Use of artificial horizon.

$$\angle ECB = \angle CBG = \angle ABF.$$

But DC and AB are parallel, therefore $\angle ABF = \angle DCE = \angle ECB$, and it follows that the altitude ABF of the star is $\frac{1}{2}(DCB)$. Thus the sextant or telescope measures the angle DCB, which is twice the required altitude of the star.

The surface of the mercury is liable to be disturbed by ripples, so that in some cases a mirror of blackened glass is used, but it is then necessary to level the mirror carefully by means of a spirit level. Or the mercury may be fitted with a floating cover glass to prevent ripples. Yet another method is to employ a viscid liquid such as treacle, but in this case the illumination is not so good, as the light is not all reflected, as in the case with mercury.

A further advantage of an artificial horizon lies in the fact that there is no correction for the height of the observer above the sea level to be applied. When the sea horizon is employed, the apparent altitude of a heavenly body is increased if the observer is situated above the sea-level. This involves a correction, for which the height of the observer must be known.

EXERCISES ON CHAPTER XLIII.

1. State the laws of reflection, and prove that if a ray of light makes an angle θ with a plane mirror, the ray after reflection makes an angle of 2θ with the incident ray produced.

2. Prove that if a man walks towards a plane mirror the image moves at the same rate as the man; and that if the mirror moves towards the man the image moves at twice the rate of the mirror.

3. Show that if a ray of light falls on one of two mirrors situated at right angles to each other, it will, after two reflections, be parallel to its original direction.

4. A man whose eye is 5 ft. 8 in. from the ground faces a vertical plane mirror standing upon the ground. Find the height of the mirror if the man can only just see the bottoms of his feet in the mirror.

5. Draw a diagram showing how the image of a luminous object is seen in a plane mirror, and prove that the image is as far behind the mirror as the object is in front.

6. Two plane mirrors are inclined at an angle θ to each other. If a ray of light parallel to one of them travels after two reflections, parallel to the second, find the value of θ.

7. Describe the principle of the sextant and show how by means of it the angle which the sun subtends at the earth may be measured.

8. The altitude of a star is found by means of an artificial horizon to be 58°, but it is noticed that the mirror employed for the artificial horizon is inclined at an angle of 3° to the horizontal and slopes down from the observer. What is the true altitude of the star ?

9. Draw a diagram showing the positions of the images formed by two plane mirrors at an angle of 45° to each other, and trace the path of one of the rays which helps to produce one of the images formed by three reflections.

10. Explain the formation of the multiple images of an object placed between two parallel mirrors. Draw a diagram showing the pencil of rays by which an eye sees the third image in one of the mirrors. L.U.

11. Show that, when a plane mirror is rotated, a reflected beam is turned through twice the angle through which the mirror is turned. Give a general description of the sextant. Sen. Camb. Loc.

12. Find an expression for the deviation of a ray of light undergoing n reflections, the ray remaining in one plane throughout.

13. It is required to cause a reflected ray to make a constant angle of 40° with the incident ray. Find an arrangement of mirrors which will do this. (Note that the deviation is $(\pi + 40^\circ)$).

14. Prove that a ray reflected twice undergoes constant deviation, provided that it is in one plane throughout. What will be this constant deviation if the reflecting surfaces include an angle of 45° ?

15. If two plane mirrors are inclined to each other, prove that the images of an object, situated between them, are arranged in a circle whose centre is on the line of intersection of the mirrors.

CHAPTER XLIV

SPHERICAL MIRRORS

Parallel beam falling on a spherical mirror.—Imagine a hollow sphere to be silvered, or polished on the inside, and a slice then cut from the sphere. A plane drawn centrally through this slice would have a circular form, such as APB in Fig. 509. On allowing a parallel beam of light to fall on such a surface, every ray will be reflected according to the laws given on p. 552. The ray DA is reflected along AE, making $\angle DAC = \angle CAE$, the radius CA being the normal to the reflecting surface at A. The other rays are similarly reflected and will all touch a curve ANF called a **caustic** which is a curve of greatest luminosity, and has its brightest point or **cusp** at F. Such a caustic may frequently be seen on the surface of tea in a cup illuminated by a distant lamp. In Fig. 509, for the sake of clearness, the rays in one half only are shown.

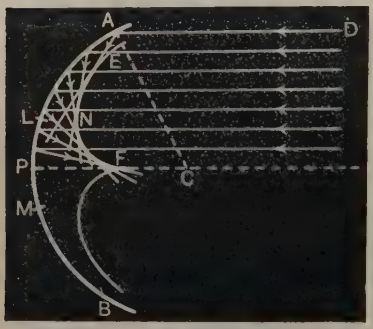

FIG. 509.—Caustic by reflection at spherical surface.

The outer parts of the caustic are faint, and it will be noted on examining Fig. 509 that the outer parts are due to rays reflected from the larger parts of the mirror lying between A and L and between M and B. If these parts be removed, the central part between L and M only remaining, the caustic is reduced to a small bright part near F. This bright point is called a **focus**; in the case of a parallel incident beam it is called the **principal focus.**

Parabolic mirror.—A spherical mirror does not bring a beam to a point focus unless it is confined to a small area. In the event of it being desired to bring a large beam to a point focus, it is necessary to use a parabolic mirror. The section of such a mirror is shown by the curve BPG in Fig. 510. The whole mirror is produced by a revolution of the diagram about PA as axis. The chief property of a parabola is that a line FB through the focus, and a line BS parallel to the axis, make equal angles with the tangent CBD of the parabola, and also with the normal BE. It follows that all rays such as SB, parallel to the axis PA, pass after reflection through the focus F, and the whole beam comes to a point focus there.

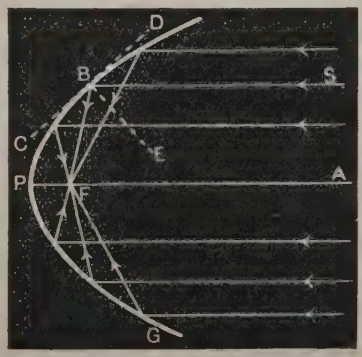

FIG. 510.—Reflection by parabolic mirror.

This principle is sometimes used in lanterns for the production of a parallel beam of considerable width; on placing a lamp at F, the rays after reflection will constitute a beam parallel to the axis. For example, in the case of a search-light, the luminous part of the arc lamp is placed at F. The reflected beam is then parallel to the axis, and since there is no spreading out it retains great intensity for a considerable distance. Any spreading out which actually occurs is due to the fact that the luminous source is not confined to a point at F.

Spherical mirrors.—Parabolic mirrors, sufficiently good for optical purposes, are difficult to construct, and moreover they would be of use only in connection with parallel beams. Hence spherical mirrors are used, care being taken that only a comparatively small part of the complete spherical surface is employed, so that the outer parts of the caustic are absent, and the point at the cusp only is present. It must be remembered that in all the diagrams given in this book the curvature will be very much exaggerated, as well as the dimensions perpendicular to the axis; if these were drawn to true scale, the rays would be so close together as to be indistinguishable.

The **centre of curvature** C (Fig. 511) is the centre of the sphere of which the mirror is a part. The middle point of the mirror P is called the **pole**. The line passing through the pole and the centre is

called the **principal axis**, PC. Thus, a ray AB, parallel to the principal axis, passes after reflection through the principal focus F, and by the laws of reflection $\angle ABC = \angle CBF$. But $\angle BCF = \angle ABC$, hence $\angle CBF = \angle BCF$, and $FB = FC$. Now, remembering that in practice all the rays are very near to the principal axis, we see that to all intents and purposes $FP = FB$, and therefore $PF = FC$, or **the principal focus is midway between the centre and pole.**

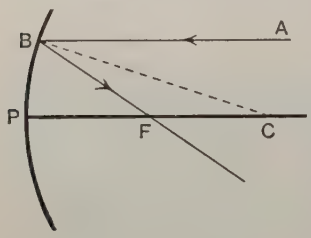

FIG. 511.—Concave spherical mirror.

The distance PF from the pole to the principal focus is called the **focal length** f, of the mirror; calling its radius of curvature r, we have

$$r = 2f.$$

EXPT. 119.—**Reflection by a concave surface.** Place a strip of glass mirror, or of silvered metal bent into a circle, upon a piece of drawing paper on a drawing board. Place two pins upright at A and B, on a line parallel

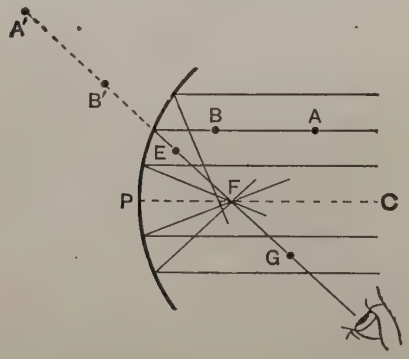

FIG. 512.—Reflection by concave surface.

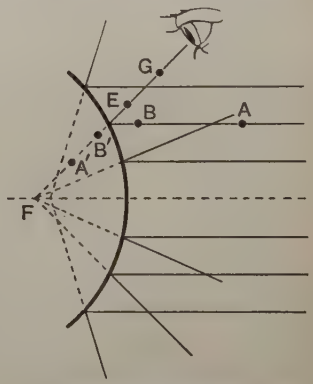

FIG. 513.—Reflection by convex surface.

to the axis PC. On looking horizontally into the mirror the images of A and B can be seen at A′ and B′. Place two more pins at E and G in line with A′ and B′, and draw a line through E and G. Repeat this for the other rays, when it will be found that the reflected rays near the axis pass through a point F, but those far from it do not.

EXPT. 120.—**Reflection by a convex surface.** Repeat the last experiment, using a strip with its convex surface reflecting; it will be found that

the reflected rays appear to come from some point F behind the mirror (Fig. 513).

Point object and image.—The case of a luminous point situated on the principal axis of a concave mirror will now be considered. Rays from O (Fig. 514) falling upon the mirror are each reflected according to the ordinary laws, and if OAI be such a ray,

$$\angle OAC = \angle CAI = i.$$

Denoting by θ the angle AIP, we have

$$\angle ACP = \theta - i, \quad \text{and} \quad \angle AOP = \theta - 2i.$$

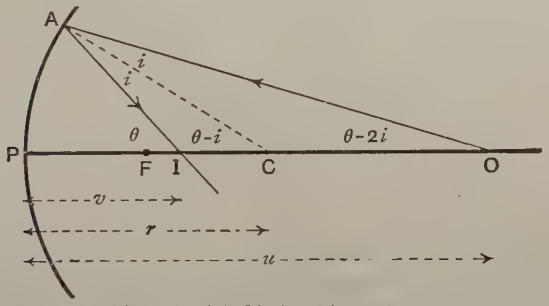

FIG. 514.—Positions of point object and image for a concave mirror.

Now the circular measure of the angle ACP is $\frac{\text{arc AP}}{\text{PC}}$ or $\frac{\text{AP}}{r}$. Remembering that, in practice, the rays are all very near the axis and the angles are all small, we may without sensible error write

$$\theta = \frac{\text{AP}}{v}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\theta - i = \frac{\text{AP}}{r}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\theta - 2i = \frac{\text{AP}}{u}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where r is the radius of curvature of the mirror, u the distance of O from the mirror, and v the distance of I.

Adding equations (1) and (3), we get

$$2\theta - 2i = \frac{\text{AP}}{v} + \frac{\text{AP}}{u},$$

and multiplying (2) by 2,

$$2\theta - 2i = \frac{2\text{AP}}{r};$$

$$\therefore \frac{\text{AP}}{v} + \frac{\text{AP}}{u} = \frac{2\text{AP}}{r}, \quad \text{or} \quad \frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

We may, if we wish, write f for $\frac{r}{2}$, when

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

This is a most important equation, giving, as it does, the relation between the distances of object and image from the mirror in terms of the focal length.

EXAMPLE.—Given that an object is situated at a distance of 75 cm. from a concave mirror of radius of curvature 50 cm.; find the position of the image.

Here, $u=75$, $f=\frac{50}{2}=25$, to find v,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}; \quad \therefore \frac{1}{v}+\frac{1}{75}=\frac{1}{25},$$

$$\therefore \frac{1}{v}=\frac{1}{25}-\frac{1}{75}=\frac{3-1}{75}=\frac{2}{75}; \quad \therefore v=\frac{75}{2}=37{\cdot}5.$$

Hence the image is 37·5 cm. from the mirror.

Convex mirrors.—In Expt. 120 it was seen that parallel rays were reflected so that they diverged from some point behind the mirror.

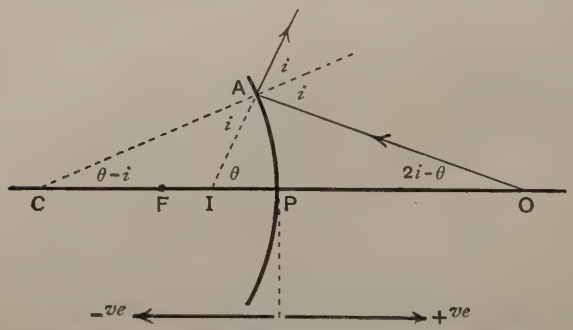

FIG. 515.—Positions of point object and image for convex mirror.

These reflected rays will never pass through a real focus, and hence can never produce a bright point upon a screen. But if the eye receives them, a bright point will be seen at the point F (Fig. 513) from which they are diverging, as already explained on pp. 554 and 555. This is called a **virtual focus.**

When the source is a bright point O, upon the principal axis, the rays after reflection diverge from some virtual focus I (Fig. 515).

This is said to be a **virtual image.** From the figure it will be seen, exactly as in the case of the concave mirror (p. 565), that

$$\theta = \frac{AP}{v}, \quad \text{.....(1)}$$

$$\theta - i = \frac{AP}{r}, \quad \text{.....(2)}$$

$$2i - \theta = \frac{AP}{u}. \quad \text{.....(3)}$$

From (1) and (3) $\quad 2i - 2\theta = \frac{AP}{u} - \frac{AP}{v},$

from (2), $\quad 2i - 2\theta = -\frac{2AP}{r};$

$$\therefore \ -\frac{1}{v} + \frac{1}{u} = -\frac{2}{r} = -\frac{1}{f}. \quad \text{.....(4)}$$

Convention of signs.—In order to understand and use the mirror equations properly, some convention must be adopted regarding the signs of the quantities used. The convention usually employed is as follows: **All quantities measured from the mirror in the direction in which the incident light is travelling are taken as negative (−ve) and those in the opposite direction as positive (+ve).**

Thus in Fig. 515 the incident light is travelling from **right to left**; hence PO, which is measured from **left to right** from the mirror is positive, and thus u is positive. PI and PC, measured from **right to left**, are negative, and v and r are therefore negative, as is also f.

Hence in equation (4), v and f are essentially negative, and if we remember this, when putting numbers into the equations, we may take the following as a universal equation for all spherical mirrors:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}. \quad \text{.....(5)}$$

EXAMPLE.—A bright point is situated at a distance of 40 cm. from a convex spherical mirror of radius 60 cm. Find the position of the image.

From Figs. 513 and 515 we see that for a convex mirror f and r are always negative; $\therefore f = \frac{r}{2} = -30.$

Now, $\quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad$ or $\quad \frac{1}{v} + \frac{1}{40} = -\frac{1}{30},$

$$\frac{1}{v} = -\frac{1}{30} - \frac{1}{40} = \frac{-4-3}{120} = -\frac{7}{120};$$

$$\therefore \ v = -\frac{120}{7} = -17\tfrac{1}{7} \text{ cm.}$$

That is, the image is at a distance of 17·14 cm. behind the mirror.

Object and image not points.—When the object is not a point it is not entirely on the axis, and we must therefore consider how the image of a point that is not situated on the axis is found. Consider an object AB (Fig. 516). To find the image of A, remember that an

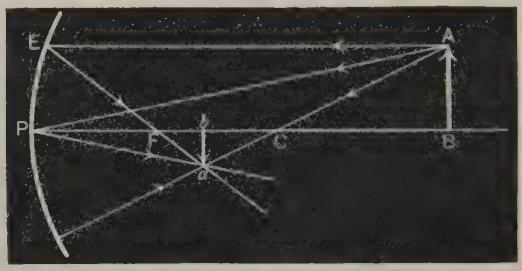

FIG. 516.—Production of image by concave mirror.

infinite number of rays leave A, fall upon the mirror, and after reflection pass through some focus; from these, any rays may be chosen. Consider a ray AE, parallel to the principal axis; it passes, after reflection, through F, the principal focus. Another through C falls normally upon the mirror and is reflected back upon its own direction. The intersection a of these two serves to determine the position of

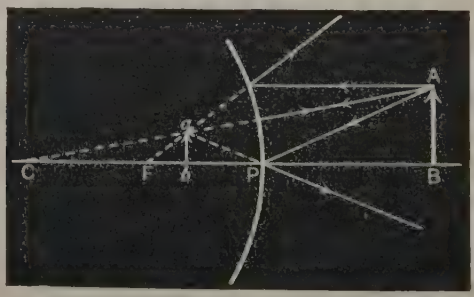

FIG. 517.—Production of image by convex mirror.

the image of A. Consider also a third ray, AP, arriving at P, which will be reflected so that $\angle APB = \angle bPa$. These three rays meet at a, as will all others, provided that the mirror is not too large a portion of the spherical surface, as discussed on p. 562. Then a is the image of A, and point by point it may be shown that ab is the image of AB.

If the mirror were convex (Fig. 517) instead of concave, the position of the image would be obtained in exactly the same way, but we should find that it is situated behind the mirror and that it is virtual.

Size of object and image.—In the case of either the concave or convex mirror, there is a simple relation between the sizes of the object and image. In Fig. 516 or Fig. 517 the triangles ABC and abC are equiangular and therefore similar; therefore $ab/bC = AB/BC$.

Hence $$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Distance of image from centre}}{\text{Distance of object from centre}}.$$

Or, from the triangles aPb and APB, $ab/Pb = AB/PB$;

$$\therefore \frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Distance of image from mirror}}{\text{Distance of object from mirror}} = \frac{v}{u}.$$

In Fig. 516 the image is inverted, and v and u are both positive; hence **when v/u is positive the image is inverted.**

Again, in Fig. 517 the image is erect; also v is negative and u positive. Hence, **when v/u is negative the image is erect.**

EXAMPLE.—Find the position and size of the image of a body 3 cm. high when placed 60 cm. from a mirror of focal length 15 cm., (*a*) when the mirror is concave, (*b*) when it is convex.

(*a*) $$u = 60,\ f = 15;\quad \therefore \frac{1}{v} + \frac{1}{60} = \frac{1}{15},$$

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{60} = \frac{3}{60};\quad \therefore v = +20 \text{ cm.}$$

and the image is in front of the mirror.

Also $$\frac{v}{u} = \frac{20}{60} = +\frac{1}{3};$$

$$\therefore \text{ size of image} = +3 \times \frac{1}{3} = 1 \text{ cm.},$$

and the image is inverted.

(*b*) $$u = 60,\ f = -15;\quad \therefore \frac{1}{v} + \frac{1}{60} = -\frac{1}{15},$$

$$\frac{1}{v} = -\frac{1}{15} - \frac{1}{60} = -\frac{5}{60};\quad \therefore v = -12,$$

and the image is behind the mirror.

Also $$\frac{v}{u} = \frac{-12}{+60} = -\frac{1}{5};$$

$$\therefore \text{ size of image} = -\frac{3}{5} \text{ cm.},$$

and the image is erect.

EXPT. 121.—**Focal-length of a concave mirror.** Place a concave mirror vertically, holding it by a clip or retort stand. Place horizontally a pin at some distance from the mirror, holding the pin also by a clip with the

point A of the pin near the axis of the mirror (Fig. 518). A real image of A is produced at A′, and may be seen by an eye placed to receive the rays after they have passed their focus at A′, and are in the act of diverging. Now place a second pin B so that its point appears to touch the point of the image A′. The position of B must be adjusted, until, on moving the

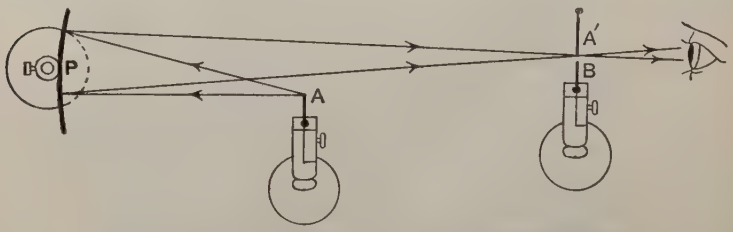

FIG. 518.—Focal length of concave mirror by method of parallax.

eye up and down, A′ and B do not separate, and parallax is thus avoided. If they do separate it means that one is nearer the eye than the other and there is then said to be parallax. Measure the distances of A and B from the mirror and calculate f from equation (5) (p. 567). Now change the distance of A from the mirror, and repeat the experiment at various distances. Make a table of the results as follows:

u	v	$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

EXPT. 122.—Graphical method of finding an image. With compasses and set-square, solve the example on p. 569 by drawing the figure to scale, taking one millimetre to the centimetre for the scale parallel to the axis and 1 cm. to the cm. for that perpendicular to the axis.

Conjugate foci.—From the principle of reversibility of the rays, it will be noted that object and image can change places; that is, if the object be moved to the position originally occupied by the image, the image takes the position originally occupied by the object. We may, however, with advantage, follow in detail the several cases that present themselves in the case of a concave mirror.

(i) Object at infinity; $u=\infty$, $\frac{1}{v}+\frac{1}{\infty}=\frac{1}{f}$; $\therefore\ v=f$,

and the image is at the principal focus (see Fig. 511).

(ii) Object at a considerable distance, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; $\therefore\ v=\frac{uf}{u-f}$.

Image is at a distance from the mirror greater than f, since $\frac{u}{u-f}>1$.

(iii) Object at the centre of curvature, $u=2f$.

$$\therefore\ v=\frac{2f^2}{f}=2f;\quad \therefore\ v=u.$$

Both the image and object are at the centre, a result which might have been explained otherwise ; for the rays now fall normally upon the mirror and are reflected back upon their incident paths.

(iv) Object at a small distance beyond the principal focus.

This is the conjugate case to (ii) and may be understood by imagining ab to be the object in Fig. 516. AB will then be the image.

(v) Object at the principal focus. $u=f$.

$v=\infty$. The image is at infinity. This is the conjugate case to (i). If the object is at F (Fig. 511) the rays after reflection are parallel.

(vi) Object between the principal focus and the mirror.

This case presents certain novel features. Here $u<f$. Since $v=\frac{uf}{u-f}$, v is negative and the image is behind the mirror.

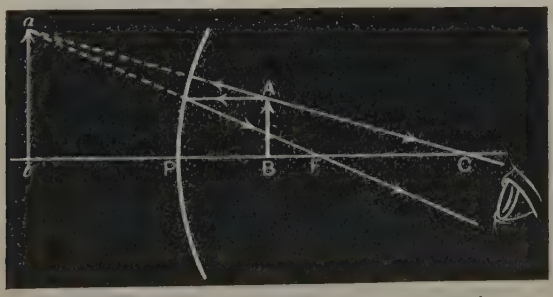

FIG. 519.—Production of virtual image by concave mirror.

On performing the geometrical construction given on p. 568, it will be found that the rays after reflection are diverging ; the image is therefore virtual at ab (Fig. 519). In this case $b\mathrm{C}>\mathrm{BC}$, and hence the image is enlarged. Also since v/u is negative, the image is erect.

EXERCISES ON CHAPTER XLIV.

1. Describe the effect produced when a parallel beam of light falls on a concave mirror of considerable extent, and point out the condition necessary for the beam to come to a point focus.

2. A luminous point is situated on the principal axis of a concave mirror of radius of curvature 65 cm. Find the position of the image when the point is 55 cm. from the mirror.

3. Describe a method of measuring the focal length of a concave mirror.

4. How may a parallel beam of light of considerable extent be produced, a source of light of small size being available ?

5. An incandescent lamp having a filament 6 cm. long is placed at a distance of 72 cm. from a concave mirror of radius 60 cm. Find the position and size of the image of the filament.

6. Concave mirrors are sometimes used instead of plane mirrors as looking glasses. Explain this.

7. A concave mirror has radius of curvature 40 cm. Find the position and size of the image when a disc of $2\frac{1}{2}$ cm. diameter is held at a distance (*a*) of 25 cm. from it, and (*b*) at a distance of 15 cm. from it.

8. Find by construction and by calculation the size and position of the image when an object 5 cm. high is held at a distance of 45 cm. from a concave mirror whose radius of curvature is 30 cm.

9. A rod 4 cm. long is held in front of a concave mirror of radius of curvature 30 cm., and the length of the image is 2 cm. Find the positions of the object and image.

10. A concave spherical mirror has a radius 18 cm. An object is placed in such positions that its real image is (*a*) half the size of the object, (*b*) twice its size. Find how far the object is in each case from the mirror.

11. An object 2 inches high is set up 16 inches to the right of a concave mirror of 4 inches focal length. Find by a construction to scale the position and magnitude of the image.

Verify from your drawing the following rule: If the object is n focal lengths to the right of the focus, the image will be $1/n$th of a focal length to the right of the focus, and the magnification will be $1/n$th.

Sen. Camb. Loc.

12. Prove the formula for the relation between the radius of curvature of a concave mirror, and the distance of a point source from the mirror, and the distance of the image; and hence prove that the focal length of the mirror is half its radius of curvature. L.U.

13. A concave mirror is to be used to give an image of an illuminated slit in a paper screen, on the screen 15 cm. from the slit, 1·25 times the length of the slit, and parallel to it. Find the focal length of the mirror, and show by a diagram how it must be placed. L.U.

14. A concave mirror of radius of curvature 20 cm. and a convex mirror of radius 30 cm. are placed 40 cm. apart, facing each other, and having a common axis. A luminous body of length 5 cm. is placed perpendicular to the axis of the mirrors and 15 cm. from the concave mirror. Find the position and nature of the image formed by two reflections, the first occurring at the concave and the second at the convex mirror.

15. A convex mirror and a plane mirror are placed facing each other and 28 cm. apart. Midway between them, and on the principal axis of the

convex mirror, a small luminous object is situated. On looking into the plane mirror, two images of the luminous object are seen. Draw a diagram showing how the two images are formed, and calculate the radius of curvature of the convex mirror if the image formed by two reflections is situated at a distance of 38 cm. behind the plane mirror.

16. If the sun's disc subtends an angle of half-a-degree at the earth's surface, find the focal length of the concave mirror which would produce a real image of the sun upon a screen, the diameter of the image being 4·5 cm.

17. An object of length 4 cm. is placed 14 cm. from a convex mirror, and at right angles to the principal axis. If the size of the virtual image is 2·5 cm., find the radius of curvature of the mirror.

CHAPTER XLV

REFRACTION

General considerations.—In a homogeneous medium light travels in straight lines, and it has been seen that reflection occurs at the surface of separation of two media. But some of the light passes from the first medium into the second. For many substances the amount of light which penetrates is very small; but for others, which have been called 'transparent,' a considerable fraction of the light enters. The behaviour of the light in passing from one medium to another must now be studied, and it is well to remember throughout that the proportion of light reflected or transmitted depends to a great extent upon the angle of incidence of the light, as well as upon the nature of the media concerned. The case of light passing from air to some other substance, such as water or glass, will first be considered. The ray is always bent towards the normal on entering a more dense medium. This bending is called **refraction.**

Laws of refraction.—Refraction takes place according to two laws, analogous to the laws of reflection.

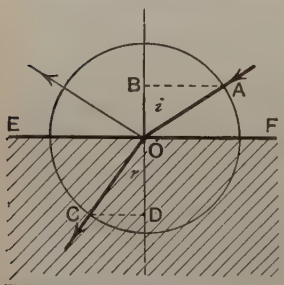

FIG. 520.—Illustration of laws of refraction.

First law of refraction.—The incident ray, the refracted ray, and the normal to the surface at the point of incidence, lie in one plane.

Second law of refraction.—The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant quantity for any two given media.

In Fig. 520 let EF be the surface of separation, air being above EF and water or glass below. The ray AO meets the surface at O, at the angle of incidence AOB. Part of the light is reflected, but part enters and travels along OC, the angle of refraction being DOC.

Then the second law of refraction states that

$$\frac{\sin \text{AOB}}{\sin \text{COD}} = \text{a constant.}$$

This constant is called the **index of refraction** for the two media, and is usually represented by the Greek letter μ.

Thus, $$\frac{\sin i}{\sin r} = \mu.$$

From Fig. 520 it will be seen that $\sin i = \frac{\text{AB}}{\text{OA}}$ and $\sin r = \frac{\text{CD}}{\text{OC}}$;

$$\therefore \ \mu = \frac{\sin i}{\sin r} = \frac{\text{AB}}{\text{OA}} \cdot \frac{\text{OC}}{\text{CD}} = \frac{\text{AB}}{\text{CD}},$$

since OA and OC are equal, being the radii of a circle which has its centre at O.

This proof provides a useful geometrical construction for finding the path of the refracted ray.

EXAMPLE.—A beam of light meets the surface of water at an angle of incidence of 60°. Find the path of the refracted ray.

Make the angle AOB (Fig. 521), equal to 60°. Draw any circle with O as centre and drop a perpendicular AB to the normal. Now the index of refraction of water is approximately $\frac{4}{3}$. Thus AB should be divided into 4 equal parts, and a length OE taken equal to 3 of these parts. Draw EC parallel to the normal; then CD is also equal to these 3 parts. OC is then the refracted ray, for obviously $\frac{\text{AB}}{\text{CD}} = \frac{4}{3}$.

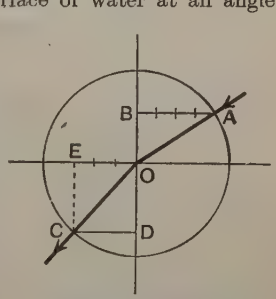

FIG. 521.—Construction for finding the position of the refracted ray.

Demonstration of the laws of refraction.—The optical disc, described on p. 553, may be usefully employed to demonstrate the laws of refraction. For this purpose the mirror is replaced by a semicircular plate of glass. The incident beam AB (Fig. 522) falls on the plane surface of the plate and is refracted on entering. It is not bent on leaving the plate because it travels along a radius, and therefore falls normally upon the curved surface of the plate. The path BD therefore indicates by its position, the angle of refraction. By turning the disc into various positions, the corresponding angles of incidence and refraction may be observed, and the relation $\sin i / \sin r = \text{a constant}$, can be proved.

Further, by turning the disc so that the beam enters by the curved and leaves by the plane surface, it may be shown that a beam passing from glass to air is bent away from the normal. Thus in Fig. 521, if the beam travels along CO in the glass, it will emerge along OA. The index of refraction in this case will be

$$\frac{\sin \text{COD}}{\sin \text{AOB}} = \mu' = \frac{1}{\mu}.$$

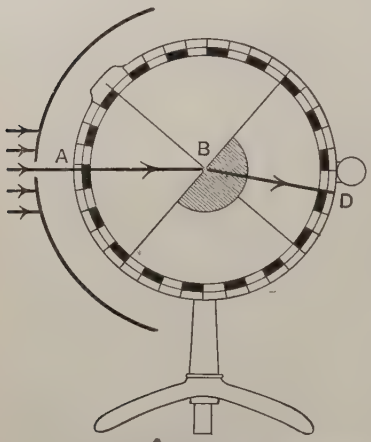

FIG. 522.—Optical disc used to demonstrate the laws of refraction.

Thus **the index of refraction for an emergent ray is the reciprocal of that for an entering ray.**

As regards the first law of refraction, see the note on p. 553 concerning reflection, which applies equally well here.

EXPT. 123.—**Refraction by a rectangular plate of glass.** Place a rectangular block of glass on a sheet of drawing paper with its longest edges horizontal, and fix two upright pins at A and B (Fig. 523). On looking through the block in the direction DC, the pins will appear to be at A′ and B′. Place two more pins at C and D in a line with the images A′ and B′. Draw the outline of the block on the paper; remove the block and pins and draw a straight line through A and B, and another through C and D. AB is the path of a ray entering the glass at E and leaving at F. EF is therefore the path through the block. Notice that CD is parallel to AB. At E and F draw normals to the respective surfaces and measure i and r, i' and r'. Find the ratio $\sin i/\sin r$, or

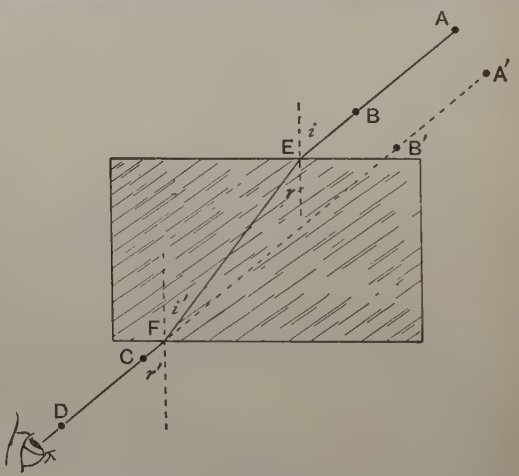

FIG. 523.—Index of refraction of a block of glass.

obtain it by drawing a circle with E as centre, as shown in Fig. 521. Repeat the above process for six different angles of incidence and record the results in the form of a table.

EXPT. 124.—**Measurement of the refractive index of a block of glass.** Place the block of glass, used in the last experiment, on its edge as shown in plan and elevation in Fig. 524. Let the block rest upon a line A drawn on the paper. On looking into the glass, the line A will appear to be broken, the part B seen through the glass being displaced from its original position A. Move the eye up and down until the part B and the edge of the block C appear to coincide, and mark this position G on the paper. It follows, as shown in the elevation, that the path of the ray is ACE, and by drawing the elevation on the paper, the angles ACD and BCD may be found. Find the ratio sin BCD/sin ACD. This is the index of refraction of the material.

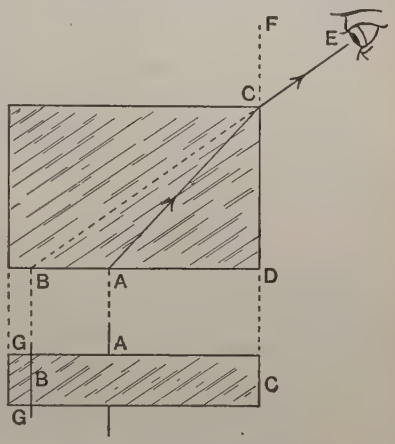

FIG. 524.—μ by displacement.

Apparent thickness of a transparent body.—On looking through a plate of transparent material, the body observed appears to be nearer

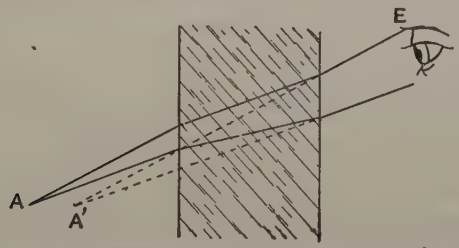

FIG. 525.—Displacement of rays traversing plate glass.

to the observer than is actually the case, and it may also be displaced to one side. Thus, in Fig. 525, rays from A, on passing through a thick plate of glass, emerge, each parallel to its original direction. But the rays are now, owing to their refraction, diverging from some point A′, which is now the apparent position of the body.

When the eye looks normally through the surface, it perceives the object A (Fig. 526) apparently raised to the point D, the point

from which the emergent rays diverge. From the figure we see that $\angle FCG = \angle DCE = \angle BDC$, and $\angle ACE = \angle BAC$.

Now, $$\mu = \frac{\sin FCG}{\sin ACE} = \frac{\sin BDC}{\sin BAC} = \frac{BC}{DC} \cdot \frac{AC}{BC} = \frac{AC}{DC}.$$

The eye only makes use of rays very near the normal, so that without sensible error, we may write AB for AC, and DB for DC.

Thus, $$\mu = \frac{\text{Real thickness of medium (BA)}}{\text{Apparent thickness of medium (BD)}}.$$

EXPT. 125.—**Index of refraction by means of the microscope.** Place a few grains of lycopodium powder, or any other very small particles, on a sheet of glass placed upon the stage of a microscope, and focus upon them. The microscope must be provided either with a micrometer movement, or with a scale and vernier to measure its vertical travel. The position when focussed as described above is observed. Place a thick sheet of glass upon the lycopodium powder, and raise the microscope by means of the micrometer screw until the powder is again in focus, and note its position. The vertical travel being thus known, this gives the distance AD in Fig. 526. Focus in a similar manner on the top of the plate, and, by reading the position again, obtain the distance DB. Hence we can find the real thickness of the plate AB and the apparent thickness DB, and can therefore calculate μ.

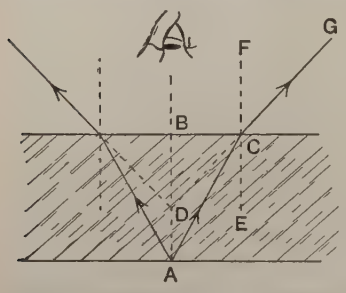

FIG. 526.—Apparent raising by transparent medium.

If the microscope is vertical, the same method may be used for a liquid; first focus on the bottom of an empty vessel, then pour in the liquid and refocus on the bottom. Then focus on the surface of the liquid, thus obtaining as before the real and apparent thicknesses.

TABLE OF INDICES OF REFRACTION.

Substance.	μ.	Substance.	μ.
Alcohol	1·36	Glycerine	1·47
Crown glass (mean)	1·51	Ice	1·31
Diamond	2·42	Turpentine	1·47
Flint glass (mean)	1·65	Water	1·333

These values of μ are for the sodium D line, for light travelling from air to the substance at 15° C.

Passage of a ray from water to glass.—In the preceding pages the passage of a ray from air to a medium such as water or glass has been dealt with. When the ray passes from one transparent medium to another, the index of refraction may be found as follows. Let the index of refraction for a ray passing from air to, say, water be μ_1, and from air to, say, glass be μ_2. Consider the ray ABCD passing from air, through a layer of water with parallel faces, and into glass (Fig. 527).

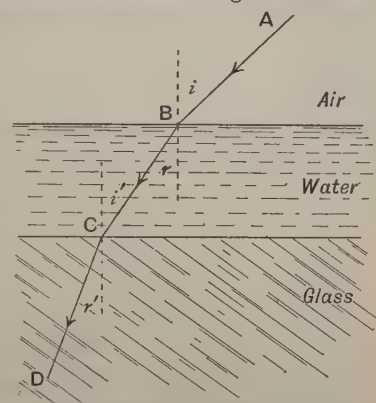

FIG. 527.—Index of refraction for any two media.

At the first surface,

$$\frac{\sin i}{\sin r} = \mu_1.$$

For a ray passing from air to glass, $\sin i/\sin r' = \mu_2$, the parallel layer of water between them not producing any deviation (see Expt. 123).

$$\therefore \frac{\sin i}{\sin r'} \cdot \frac{\sin r}{\sin i} = \frac{\mu_2}{\mu_1};$$

$$\therefore \frac{\sin r}{\sin r'} = \frac{\mu_2}{\mu_1}.$$

But $r = i'$; $\therefore \dfrac{\mu_2}{\mu_1} = \dfrac{\sin i'}{\sin r'}$,

and this is the index of refraction of a ray travelling from water to glass.

Hence, Index for water and glass $= \dfrac{\text{Index for air and glass}}{\text{Index for air and water}}$

Glass and water have been chosen as examples, but the rule applies whatever the substances may be.

Visibility of transparent bodies.—Reflection only occurs when there is a discontinuity of refractive index, such as occurs at the surface of separation of two media. If the two media have the same refractive index, no reflection occurs at their surface of separation. Thus cedar-wood oil has very nearly the same refractive index as glass, and fragments of glass immersed in it cannot be seen, as there is then no reflection at the surfaces of the fragments.

The index of refraction of materials in small quantities, such as precious stones, may be found by immersing them in mixtures of various oils, changing the proportion of the oils until the stones are invisible. Their refractive index is then the same as that of the

liquid, which may be found in the usual manner, as described on p. 627. Thus, olive oil has a refractive index of 1·47, and carbon bisulphide 1·63, and mixtures may have any value of μ between these. For high refractive indices a solution in water of barium-mercuric iodide may be used. The greatest value of μ for this solution is 1·79.

Total reflection. — In the demonstration with the optical disc (Fig. 522), when the light enters the curved surface and leaves by the plane surface, it may be observed that some of the light is internally reflected. As the angle of incidence increases, the intensity of the reflected beam also increases, until for a certain angle of incidence, and for all angles above it, the whole beam is reflected, none of it emerging. The angle of incidence at which this phenomenon first occurs is called the **critical angle**, and the beam is said to be **totally reflected.**

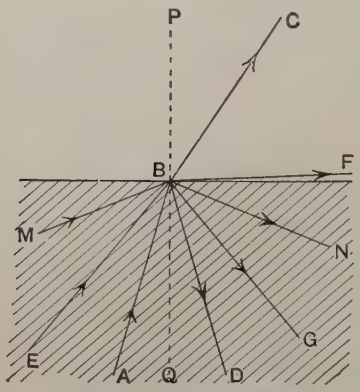

FIG. 528.—Internal reflection.

Thus, in Fig. 528 the ray AB emerges in part along BC, some being reflected along BD. The ray EB gives rise to a very feeble emergent beam BF very nearly parallel to the surface, and an intense reflected beam BG. A beam such as MB is totally reflected along BN. The critical angle is EBQ. For this critical angle, the angle of refraction is 90°; hence

$$\frac{\sin \text{EBQ}}{\sin 90^\circ} = \mu' = \frac{1}{\mu},$$

and since $\sin 90^\circ = 1$;

$$\therefore \text{ sine of critical angle} = \frac{1}{\mu}.$$

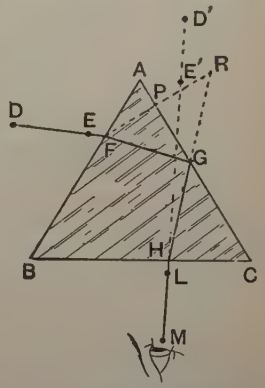

FIG. 529.—Internal reflection in glass prism.

EXPT. 126.—**Total reflection by a glass prism.** Place a glass prism upright upon a piece of drawing paper, and draw in its outline ABC (Fig. 529). Place vertical pins at D and E so that DE is nearly normal to the face AB. On

looking through the face BC the images of the pins will be seen at E′ and D′. Place two other pins L and M in line with E′D′, and draw the lines DEF and MLH. The ray DE entered the prism at F and left it at H, being totally reflected at G. To find the point G, draw FR perpendicular to AC and make PR = FP. Join RH, thus determining G by the intersection of RH and AC. This follows from the laws of reflection in a manner similar to that given on p. 554.

Measurement of μ by the method of total reflection.—Several methods of measuring μ by making use of the relation $\sin i = 1/\mu$ have been devised. One of the best methods is to immerse two glass plates with a small air space between them, in a vessel containing the liquid whose refractive index is required. The pair of plates, shown in plan at AB (Fig. 530), are carried by an axle, not shown, which also carries the circular scale EF. The plates, which are situated below the scale, are immersed in the liquid contained in the glass vessel CD. The light from the sodium burner passes through a slit in the screen L, and an observer looking through the telescope T can see the slit by means of light which has passed through the liquid and the plates AB. On rotating AB a position will be found, for which the light is suddenly cut off, being totally reflected in the plates. The position of AB is known from the circular scale and the fixed pointers at E and F. From Fig. 531 it will be seen that the light is just cut off when BCE is the critical angle for glass and air. Hence $\sin BCE = 1/\mu_g$.

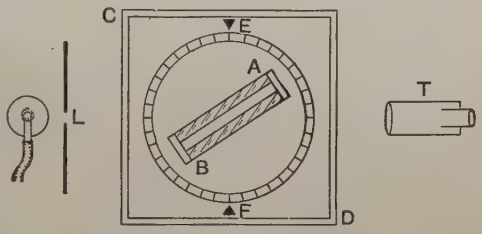

FIG. 530.—μ by total reflection.

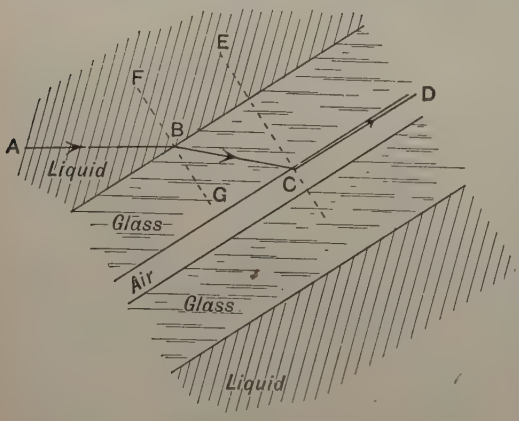

FIG. 531.—Total reflection at surface of glass plate.

Now at B the ray passes from the liquid to glass. From p. 579 we see that

$$\frac{\mu_g}{\mu_w} = \frac{\sin \text{ABF}}{\sin \text{GBC}} = \frac{\sin \text{ABF}}{\sin \text{BCE}} = \mu_g \cdot \sin \text{ABF}.$$

$$\therefore \sin \text{ABF} = \frac{1}{\mu_w}.$$

Hence the index of refraction for the liquid is the sine of the angle ABF between the incident ray and the normal when the light is just cut off. It is determined by finding two symmetrical positions for extinction and taking half the angle between the two positions.

The Pulfrich refractometer.—For the rapid determination of the refractive index of liquids of which only a small quantity is available, the principle of total reflection is made use of in the refractometer of Pulfrich. The upper plane face of a glass cylinder is polished and on to this face is cemented a ring of glass to contain the liquid under examination.

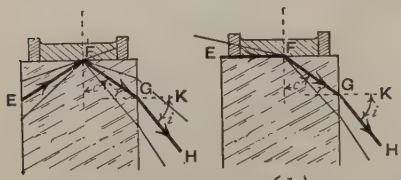

FIG. 532.—The Pulfrich refractometer.

The refractive index (μ) of the liquid must be less than that of the glass cylinder (μ_1). The light entering at E (Fig. 532 (*a*)) is reflected at the surface of separation of the glass and the liquid. If EF is the ray making the critical angle with the normal, rays above this are totally reflected, but rays below this are only partially reflected. If then a telescope, having a cross wire, be directed to receive the beam GH, the lower half of the field of vision is not so brightly illuminated as the upper half. The halves of the field of vision are separated by a sharp line corresponding to the direction GH ; the cross wire is made to coincide with this line. By means of a circular scale over which the telescope moves the angle HGK = i is measured.

If the light entering the side of the glass ring be employed as in Fig. 532 (*b*), the upper half of the field of vision in the telescope is dark and the lower half illuminated ; the angle HGK is measured as before. Calling this angle i, the critical angle c, and the internal angle of incidence r,

$$\frac{\sin i}{\sin r} = \mu_1.$$

But $$r + c = 90^\circ ; \quad \therefore \sin r = \cos c,$$

and $$\cos c = \frac{\sin i}{\mu_1}.$$

Again, for the critical angle c,

$$\sin c = \frac{\mu}{\mu_1},$$

since the index of refraction for a ray passing from the liquid to the glass is μ_1/μ.

Also,
$$\sin^2 c + \cos^2 c = 1;$$
$$\therefore \frac{\mu^2}{\mu_1^2} + \frac{\sin^2 i}{\mu_1^2} = 1,$$
$$\mu^2 + \sin^2 i = \mu_1^2,$$
$$\mu = \sqrt{\mu_1^2 - \sin^2 i}.$$

From which μ can be found after i has been measured, μ_1 having been previously determined.

Right-angled prisms as reflectors.—For some purposes it is desirable to employ plane mirrors, but the confusion produced by multiple

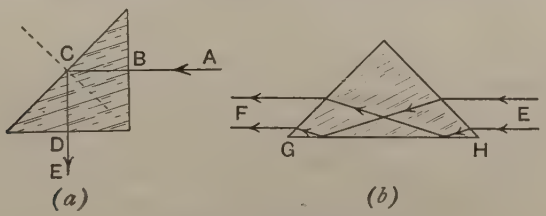

FIG. 533.—Prismatic reflectors.

reflections (p. 557) must be avoided, and the tarnishing which an unprotected metallic mirror rapidly undergoes would render its use impossible. A right-angled prism is then used, as it has neither of the above disadvantages. The beam AB, on entering one face normally at B (Fig. 533 (*a*)) is not bent, but travels towards the hypothenuse and meets it at C, the angle of incidence being 45°. For most glass μ is about 1·5, and since $\sin i = 1/\mu = 0{\cdot}667$, this gives a value 41° 50′ for the critical angle. Hence the light is totally reflected at C, and leaves the prism normally by the face D.

Erecting prisms are also designed on a similar plan. The beam E (Fig. 533 (*b*)) enters the prism and is refracted, meeting the face GH and being there totally reflected. It emerges eventually along F, and it will be seen that the upper parts of the beam at E are the lower at F. The function of the erecting prism will be described further on p. 605.

Atmospheric refraction.—Light is refracted in passing from a vacuum to air, and in the preceding discussions all the indices of refraction should have been calculated on the assumption that the rays passed from a vacuum to the transparent substance. Since,

however, the index of refraction of air at standard pressure and temperature is 1·00029, the index can be corrected by the relation

$$\frac{\text{Index of refraction, referred to vacuum}}{\text{Index of refraction, referred to air}} = 1{\cdot}00029.$$

Air, like all other gases, changes considerably in density as the pressure and temperature vary (p. 411), and thus, as we ascend from the earth's surface the index of refraction of the air becomes less owing to the diminution in density. It follows that the light from any heavenly body, such as a star, is refracted on passing through the earth's atmosphere. Thus the light from B (Fig. 534) on reaching an observer situated at O, arrives in the direction B′O, and a telescope

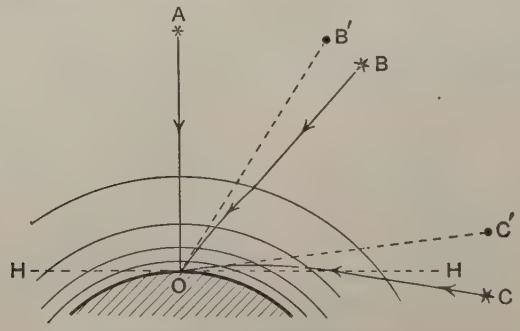

FIG. 534.—Atmospheric refraction.

must be directed along this line in order to observe the body. For this reason the altitude of any heavenly body always appears too great, and the observed altitude HOB′ must be corrected to obtain the true altitude. The correction is not easy to calculate, on account of the irregularity of the atmosphere and the gradual change in its density. Tables of corrections have been constructed, and the following gives an idea of the magnitude of the atmospheric refraction.

TABLE OF ATMOSPHERIC REFRACTION.

Altitude.	Refraction.	Altitude.	Refraction.
10°	5′ 19·2″	60°	0′ 33·6″
20°	2′ 38·6″	70°	0′ 22·2″
30°	1′ 40·6″	80°	0′ 10·3″
40°	1′ 9·4″	90°	0′ 0″
50°	0′ 48·9″		

Further corrections for temperature and for the height of barometer are also applied.

It will be noticed that a star in the zenith, as at A, is not displaced by refraction. Also the refraction at low altitudes is so great, and varies so rapidly with the altitude, that no accurate correction can be applied. A body at C, which is really below the horizon, appears to be at C′, and it is on account of the rapid variation in refraction that the setting sun or moon appears elliptical. For the horizontal diameter is unchanged by refraction, but the lower edge is raised more than the upper, the apparent vertical diameter of the setting sun being about 27′, while the horizontal diameter is 32′.

Air is sometimes visible, as may be observed when it is heated in contact with the ground on a hot summer day. The hot air rising irregularly has a different refractive index from that of the cooler air into which it rises. Hence the shimmering with which everyone is familiar, which is erroneously put down to 'heat rising.'

Exercises on Chapter XLV.

1. State the laws of refraction and describe some method of demonstrating the truth of the second law.

2. Describe how you would trace the path of a beam of light through a block of glass having plane faces, and how the index of refraction of the glass may be measured.

3. A beam of light falls on a rectangular block of glass at an angle of incidence of 45°. Find by construction the path of the beam in the glass whose refractive index is supposed to be 1·5.

4. Why is it that a pond of clear water appears less deep than it really is? Find an expression for the apparent depth of a pond for perpendicular incidence, and use it to find the apparent depth of a pond whose real depth is six feet. (μ for water = 1·333.) Sen. Camb. Loc.

5. Describe carefully how you would determine the refractive index of a liquid of which only a small quantity is available.

6. Give the laws of refraction of light, and describe an experiment to illustrate the second law. What is the critical angle, and how is it related to the refractive index of the medium?

7. Show by means of a sketch the path of a ray of light through a prism. Describe how you would trace the path of a ray, and so determine the refractive index of the glass of the prism.

8. A microscope is focussed upon a small object, and on covering the object with a sheet of transparent material the microscope must be raised a distance of 2·1 mm. to refocus the small object, and a further distance of 4·5 mm. to focus upon a scratch on the upper surface of the sheet. What is the refractive index of the material of the sheet?

9. The index of refraction of a prism is 1·5 and the angle of the prism 50°. Construct a diagram to scale showing the path of a ray of light through the prism, if the angle of incidence upon the first face is 20°.

10. Prove that a beam of light which enters at one face of a slab of material with parallel faces, will emerge from the opposite face. Also show that if the faces are not parallel it may be impossible for the beam to emerge.

11. What is meant by total internal reflection ? Under what circumstances does it occur ?

If the refractive index of glass is 1·6, show how to construct the smallest angle of incidence at which total reflection in the glass occurs. L.U.

12. What is meant by the term 'critical angle' ? Explain how you would measure the critical angle for a given specimen of glass and so obtain the refractive index of the glass. L.U.

13. A block of glass rests on a piece of paper. Explain why the paper appears to be nearer than it really is to an observer viewing it from above.

If the thickness of the block is 5 cm., and its refractive index is $\frac{3}{2}$, find the apparent displacement in the position of the paper. L.U.

14. Describe how the apparent position of a heavenly body is affected by refraction due to the atmosphere. Account for the peculiar appearance of the setting sun.

CHAPTER XLVI

LENSES

General considerations.—**Lenses** are bodies made of transparent material and bounded by faces having a spherical form. Although there is a great variety of form in lenses, they may be divided into two classes according to their behaviour towards a parallel beam of light. Consider the parallel beam shown in Fig. 535. The path

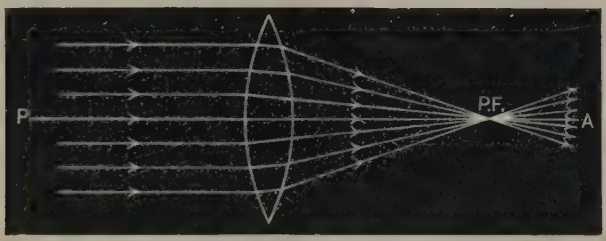

FIG. 535.—Refraction by converging lens.

of every ray may be found from the laws of refraction (p. 574). Each ray is bent towards the normal on entering, and away from the normal on leaving the lens, the normal at each point being the radius of the spherical surface. The line passing through the centres of curvature of the two faces of the lens is called the **principal axis**, PA. If the incident beam is parallel to the principal axis, the rays, after passing through the lens, are brought to a focus at PF, the **principal focus.** Since the parallel rays converge to a focus after passing through the lens, it is said to be a **converging lens.** The same limitations as to the size of the beam hold for lenses as for mirrors (p. 562). The more the beam is restricted to a narrow part near the principal axis, the more accurately will the refracted rays pass through a point focus. For a wide beam, the outer rays will not pass through the same focus as those near the axis.

With the arrangement of faces shown in Fig. 536, the beam after passing through the lens is divergent, each ray appearing to come from PF, the **principal focus**, which in this case is virtual. Such a lens is said to be a **diverging lens.**

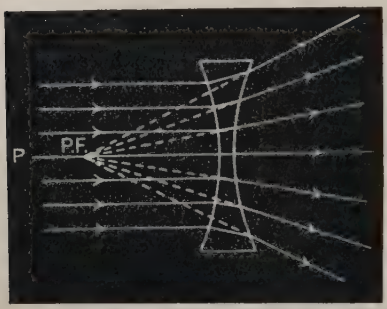

FIG. 536.—Refraction by diverging lens.

Referring to the convention as regards sign given on p. 567, we see that **the focal length of a converging lens is negative,** while **that of a diverging lens is positive.**

Classification of lenses.—Names are sometimes adopted which are descriptive of the shapes of the lenses. The first three (illustrated in Fig. 537) are all converging, and as shown, would all have the same focal length. A similar remark applies to the three diverging forms. We shall only use the terms 'converging' and 'diverging.'

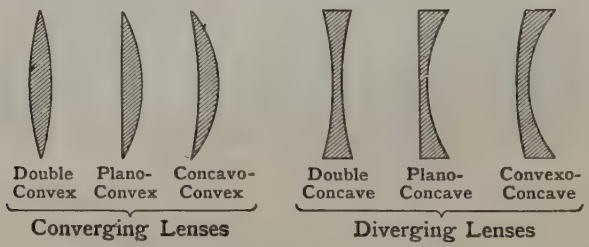

FIG. 537.—Classification of lenses.

Refraction at single spherical surface.—In order to obtain the lens formula, the production of an image by a single refracting surface will be treated first, and the result then applied to an actual lens, which has, of course, two refracting surfaces.

In Fig. 538 let A, in either diagram, be a point source of light situated on the principal axis. A ray such as AB will, on entering the glass, be refracted along BD. In Fig. 538 (*a*) we have

$$\frac{\sin \text{ABE}}{\sin \text{CBD}} = \frac{\sin i}{\sin r} = \mu,$$

and in Fig. 538 (*b*) we have

$$\frac{\sin \text{ABC}}{\sin \text{EBD}} = \frac{\sin i}{\sin r} = \mu.$$

Draw BP perpendicular to the axis; then from either diagram, since all the rays are much nearer the axis than is shown in the diagrams,

$$\left.\begin{aligned}\theta &= \frac{BP}{u}\\ \theta - i &= \frac{BP}{r_1}\\ \theta - i + r &= \frac{BP}{v}\end{aligned}\right\} \quad \therefore\ i = \frac{BP}{u} - \frac{BP}{r_1}$$

and $$r = \frac{BP}{v} - \frac{BP}{r_1}.$$

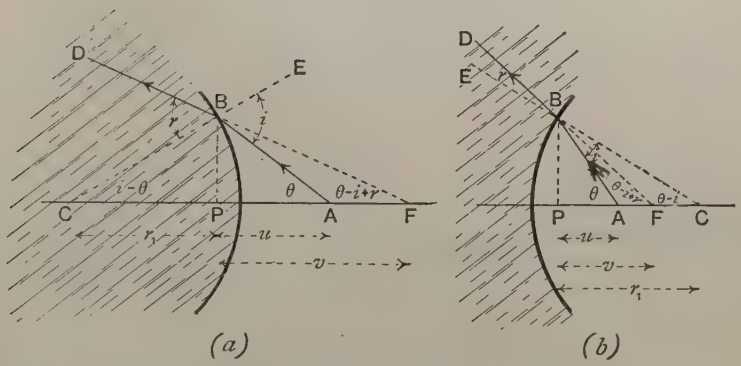

FIG. 538.—Spherical refracting surface.

It must be remembered that in Fig. 538 (*a*), r_1 is essentially negative, in accordance with the convention of signs. Again, when i and r are small, we may, without sensible error, write

$$\frac{\sin i}{\sin r} = \frac{i}{r} = \mu, \quad \text{or} \quad i = \mu r.$$

Hence,
$$\frac{BP}{u} - \frac{BP}{r_1} = \mu\left(\frac{BP}{v} - \frac{BP}{r_1}\right),$$

or
$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r_1}.$$

This gives the relation between u and v for a single spherical refracting surface. The equation is more complicated than for a spherical mirror, and involves the refractive index of the medium, which it obviously must do.

EXPT. 127.—**Refraction at a curved surface.** Place a pin A (Fig. 539) upright inside a glass crystallising basin containing water, fixing it by means of soft wax. Let the basin rest upon a sheet of drawing paper.

Fix two other pins at B and C, in line with the image I as seen by the eye. Now fix other pairs of pins at DE, FG, etc. Mark the outline of the

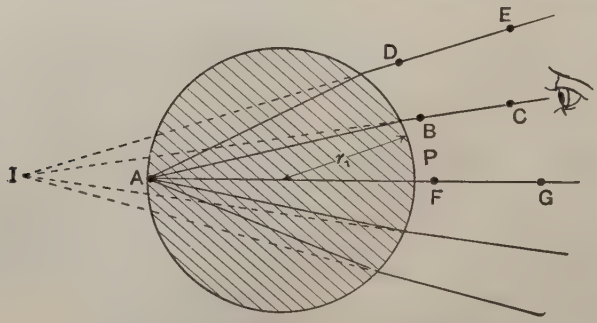

FIG. 539.—Refraction at a curved surface.

basin on the paper and then remove it. Produce ED, CB, GF, etc., backwards to meet at I. Taking IP as v, AP as u, and r_1 as the radius of the basin, calculate μ from the equation on p. 589. Note that rays far from the axis AG must not be taken, and further, that the thickness of the glass walls will prevent any great accuracy being attained.

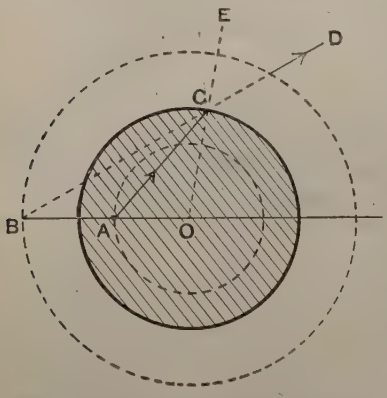

FIG. 540.—Refraction at aplanatic surface.

Aplanatic surface.—There is one important case in which a spherical surface produces a true point image however large may be the beam employed. A surface which does this is called an **aplanatic surface.** Spherical surfaces as a rule are not aplanatic. Consider a sphere of glass of radius $r = OC$, having centre at O (Fig. 540). Draw also two circles having their centres at O, one having radius μr and the other r/μ. Thus $BO = \mu r$ and $AO = r/\mu$, or $BO/OC = \mu$, and $OC/AO = \mu$. Hence $BO/OC = OC/OA$, and the triangles BOC and COA, having the common angle at O, are similar, and $\angle OBC = \angle OCA$.

Again, from trigonometry we have

$$\frac{BO}{OC} = \frac{\sin BCO}{\sin OBC}.$$

But $\frac{BO}{OC}=\mu$, $\angle BCO=\angle ECD$, and $\angle OBC=\angle OCA$;

$$\therefore \frac{\sin ECD}{\sin OCA}=\mu.$$

It follows that for an incident ray AC, the refracted ray is CD ; and all other rays that leave A and emerge from the sphere, in a manner similar to CD, will travel after refraction as though they came from B. Thus the sphere is an aplanatic surface with respect to two conjugate points such as A and B. This fact is used in oil immersion microscopes (p. 611).

Refraction at two spherical surfaces.—Lenses.—The result on p. 589 may be used to find the lens formula by applying it to the two refracting surfaces in turn. In Fig. 541 the radii r_1 and r_2 are taken

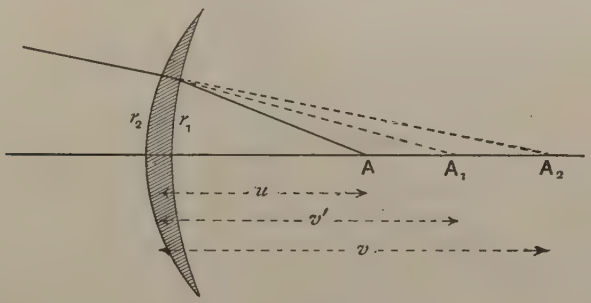

FIG. 541.—Refraction by a lens.

as both positive to avoid confusion of signs, but the result is quite general. A is the object, and refraction at the first surface of radius r_1 would produce an image at A_1, according to the relation

$$\frac{\mu}{v'}-\frac{1}{u}=\frac{\mu-1}{r_1}. \quad \text{.....(1)}$$

A′ is now the object with respect to the second surface r_2, which by refraction gives rise to the final image A_2. For this we have

$$\frac{\frac{1}{\mu}}{v}-\frac{1}{v'}=\frac{\frac{1}{\mu}-1}{r_2},$$

remembering that the index of refraction for a ray passing from glass to air is $1/\mu$;

$$\therefore \frac{1}{v}-\frac{\mu}{v'}=-\frac{\mu-1}{r_2}. \quad \text{.....(2)}$$

Adding equations (1) and (2) for the two surfaces, we get

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right). \quad \text{.....(3)}$$

If the object A be at infinity, the rays from it are parallel, and $1/u=0$, but in this case the image is at the principal focus, so that $v=f$.

Hence

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right). \quad \text{.....(4)}$$

Further, if this value is substituted for $(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ in the lens equation (3), we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}. \quad \text{.....(5)}$$

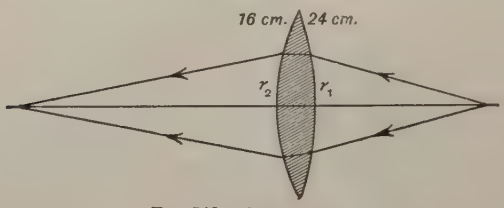

FIG. 542.—Convex lens.

EXAMPLE.—A double convex lens has radii of curvature 16 and 24 cm. respectively, and the glass has index of refraction 1·5. Find the focal length of the lens, and the position of the image when the object is 32 cm. from it.

In Fig. 542, $r_1=-24$ and $r_2=+16$;

$$\therefore \frac{1}{f}=(1\cdot5-1)\left(-\frac{1}{24}-\frac{1}{16}\right)=-\frac{1}{2}\left(\frac{5}{48}\right); \quad \text{(from equation (4))}$$

$$\therefore f=-\frac{96}{5}=-19\cdot2 \text{ cm.}$$

Again,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f};$$

$$\therefore \frac{1}{v}-\frac{1}{32}=-\frac{5}{96},$$

$$\frac{1}{v}=\frac{1}{32}-\frac{5}{96}=-\frac{2}{96}.$$

$v=-48$; that is, the image is 48 cm. from the lens on the side remote from the object.

EXPT. 128.—**Focal length of a converging lens.** Place a converging lens upright by fixing it in a slit in a cork, as shown at L (Fig. 543). At some

distance from L place a blanket pin A. The lens produces an image of A at A′, and if the eye be placed at E to receive the rays, the inverted image of the pin will be seen. Place another pin B, so that B and A′ do not separate when the eye is moved about. B and A′ are then coincident

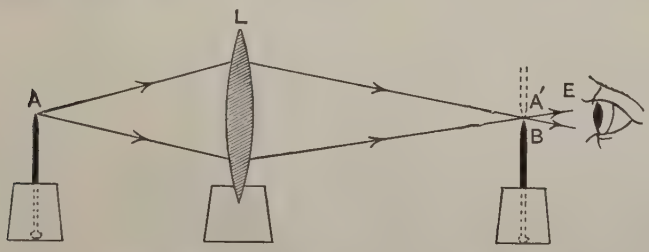

FIG. 543.—Measurement of focal length of a converging lens.

in position. Measure AL and BL, and calling them u and v, calculate f. Repeat for a number of distances, and tabulate the results as described on p. 570.

Optical centre of a lens.—For every lens there is some point for which the rays passing through it are not deviated by the lens. Let A and B be the centres of curvature of the faces of the lens in either (*a*) or (*b*) (Fig. 544). Draw AC and BD parallel to each other. These

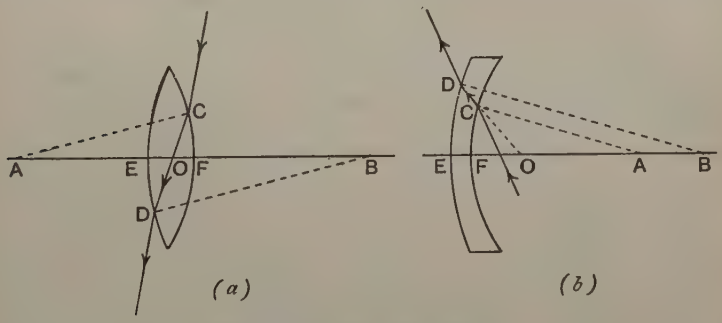

FIG. 544.—Optical centre of a lens.

being the normals to the surfaces, the surfaces are parallel to each other at C and D. The ray which passes along the path COD in the lens has thus the same direction before entering as it has after emerging (p. 576), and is therefore undeviated. It is laterally displaced, but in the case of thin lenses this displacement is small.

Now since AC and BD are parallel, the triangles ACO and BDO are similar;

$$\therefore \frac{AC}{BD} = \frac{AO}{BO};$$

but AC = AF, and BD = BE.

$$\therefore \frac{AF}{BE} = \frac{AO}{BO},$$

and by a well-known proposition in proportion,

$$\frac{AF}{BE} = \frac{AF - AO}{BE - BO}, \quad i.e. \quad \frac{AF}{BE} = \frac{OF}{OE}.$$

Hence, all other rays which are undeviated pass through the same point O, which is called the **optical centre** of the lens. It will be seen that O may be within the lens as in (*a*), or outside it, as in (*b*), but in all cases its distance from the face is proportional to the radius of that face.

Object and image not points.—In order to obtain the image of an object of definite size, we follow the principles given for concave

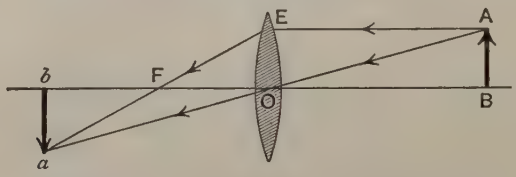

FIG. 545.—Production of image by converging lens.

mirrors (p. 568). Take a point A upon the object AB (Fig. 545). Of the rays given out by A, choose the one, AE, which is parallel to the principal axis. This ray, after passing through the lens, travels through the principal focus F. Another ray, AO, which passes through the optical centre O, is undeviated. These two meet at *a*, as will also all others originating at A. *a* is thus the image of A. The images of other points upon AB can be found in a similar manner, and will build up into the image *ab*.

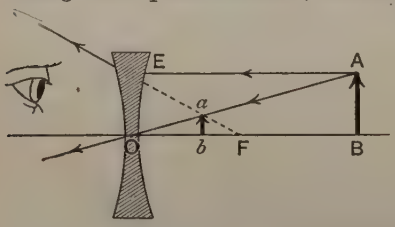

FIG. 546.—Production of image by diverging lens.

When dealing with a diverging lens, the same construction applies, but the image in this case is virtual. The ray AE, Fig. 546, emerges as though it came from the principal focus F. The ray AO, through the optical centre is undeviated, so that the beam, after passing through the lens, is diverging from the point *a*. *ab* is then the image of AB.

Size of object and image.—From either Fig. 545 or 546, it may be seen that the triangles AOB and aOb are similar.

Hence $$\frac{ab}{AB}=\frac{Ob}{OB}.$$

$$\therefore \frac{\text{Size of image}}{\text{Size of object}}=\frac{\text{Distance of image from lens}}{\text{Distance of object from lens}}=\frac{v}{u}.$$

In Fig. 545 the image is inverted, and u and v have opposite signs; while in Fig. 546 the image is erect, and u and v have the same sign.

Thus, **when v/u is positive, the image is erect,** and **when v/u is negative, the image is inverted.**

It will be noticed that in this respect mirrors and lenses differ, the reason being that the rays pass through a lens but are reflected from a mirror. The same difference can be observed in the relations $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ for a lens, and $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ for a mirror.

EXPT. 129.—**Size of object and image.** Cut a slot, in the form of an arrow if desired, in a cardboard screen; cover it with tracing paper, and illuminate it by a bright source of light. On the opposite side to the source,

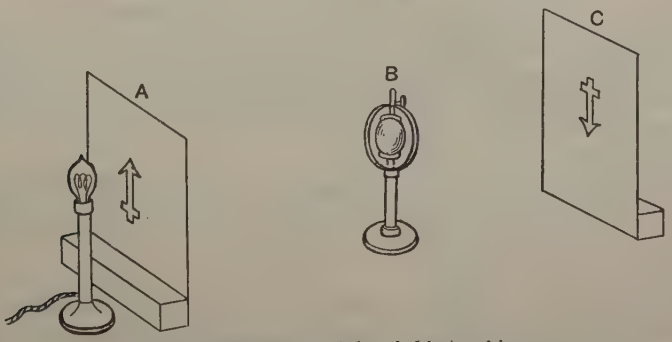

FIG. 547.—Measurement of size of object and image.

place a lens, as at B in Fig. 547. Place a screen at C. Then, the room being dark, the screen C is moved about until a sharply defined image is produced. Measure AB and BC, and also the lengths of the object and image, for six positions, and tabulate as follows:

AB	BC	Length of image.	$\frac{BC}{AB}$	$\frac{\text{Length of image}}{\text{Length of object}}$

Various positions of object and image.—The equation for a lens may be written $v=uf/(u+f)$, and since f is negative for a converging lens, it follows that v is negative so long as u is greater than f. The image is then on the opposite side of the lens to the object, as in Fig. 545, and is real. As in the case of mirrors (p. 570), it follows from the principle of reversibility that the object and image are interchangeable ; that is, their positions are **conjugate.**

When, however, u is less than f, v is positive. This is a case we have not yet discussed, for the image is obviously **virtual.** On performing the usual construction, as in Fig. 548, we see that the rays

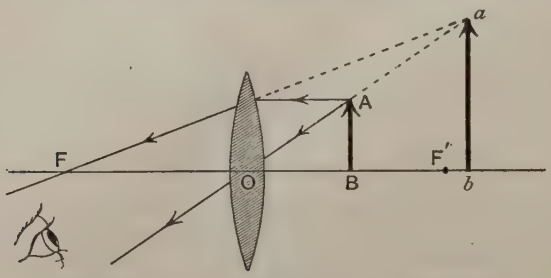

FIG. 548.—Virtual image produced by converging lens.

after passing through the lens are diverging ; although not so much as they were before entering it. The rays diverge from a, and an eye placed to receive them will see the image at a. Since the rays are diverging they never cross, so that it is no longer possible to obtain an image upon a screen.

In the case of the diverging lens, f is positive, and since $v=uf/(u+f)$, v is always positive, and the image is always virtual. There is therefore only one case to consider, and this is illustrated in Fig. 546.

Optical bench.—For the accurate measurement of focal lengths, the **optical bench** is employed. It consists of a horizontal beam provided with a linear scale, usually two metres long (Fig. 549). At one end is a screen A, in which is an aperture having two cross wires, illuminated by a lamp L, which should have a frosted bulb. A lens-carrier B, holding the lens, and a vertical white screen C, ride upon the bench so that their distances from A are adjustable and measurable. On obtaining a well-defined image of the cross wires upon the screen C, u and v can be measured upon the scale, and f can be calculated. The chief objection to the optical bench is that it must be used in a dark room.

EXPT. 130.—**Focal length of a converging lens (1st method).** By means of the optical bench measure the focal lengths of several converging lens, by finding u and v in each case.

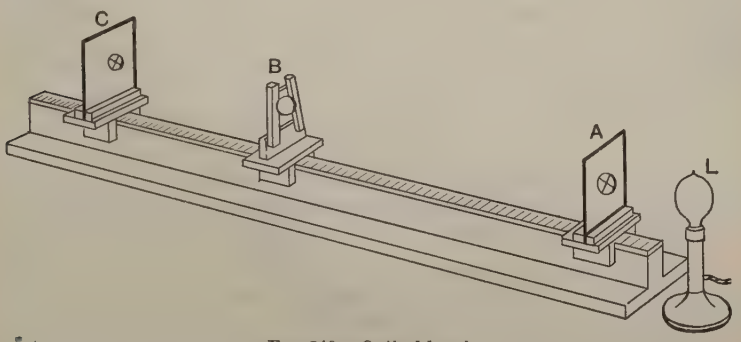

FIG. 549.—Optical bench.

EXPT. 131.—**Focal length of a converging lens (2nd method).** Replace the screen C, Fig. 549, by a vertical plane mirror, and adjust the distance of the lens from the cross wires until a sharp image is produced alongside of the object. The rays must have struck the mirror normally, since they are reflected back very nearly along their own paths (Fig. 550). Thus the beam on first emergence from the lens is parallel, and the distance from the lens to the cross wires is the focal length of the lens. This method is particularly applicable to lenses of great focal length.

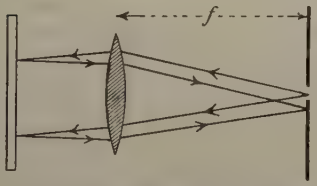

FIG. 550.—Measurement of focal length of converging lens.

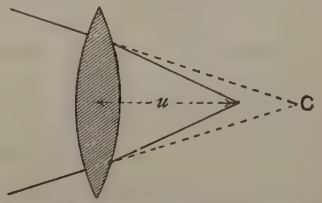

FIG. 551.—Measurement of radii of curvature of lens faces.

EXPT. 132.—**Radii of curvature of the faces of a lens.** With the lens alone, an image of the cross wire can be produced alongside of the cross wires. This image is produced by means of light which has entered the lens, but is reflected from its back face. Since the light returns along its previous path the rays must have struck the back face normally; hence, within the lens, they are diverging from the centre of curvature of the face C (Fig. 551). C may be considered to be the virtual focus for the rays, some of which instead of being reflected internally, leave the lens. Calling

u the distance of the lens from the cross wires, and r_1 its distance from the centre C, we have

$$\frac{1}{r_1} - \frac{1}{u} = \frac{1}{f}.$$

Since f can be determined separately, as in Expt. 130 or 131, r_1 may be found.

Now turn the lens round and in the same way find r_2, the radius of the other face, and from the equation

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{(p. 592)}$$

calculate μ, the refractive index of the material of the lens.

When the face of the lens is concave, the radius may be measured directly, by obtaining an image beside the object (p. 571).

EXPT. 133.—**Index of refraction of a liquid.** Place a large concave mirror on the table and adjust the position of a pin A, situated above it, until the image and object coincide. The distance of the pin from the mirror is then the radius of curvature, AC, Fig. 552. Now put a thin layer of liquid in the mirror and again perform the experiment. B is now the position of the pin. The rays producing the image coincident with B have struck the mirror normally, and it may be shown from the angles in Fig. 552 that,

$$\mu = \frac{\sin i}{\sin r} = \frac{DC}{DB} \times \frac{AD}{DC} = \frac{AC}{BC}.$$

Hence the refractive index of the liquid is found.

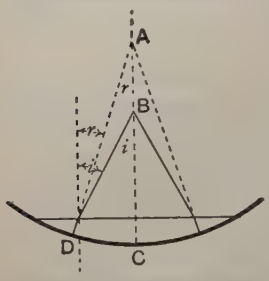

FIG. 552.—μ for liquid by means of concave mirror.

Note that the layer of liquid **must be thin**, or the above approximations are not valid.

Thin lenses in contact.—When two thin lenses are in contact the focal length of the combination may be found by a method

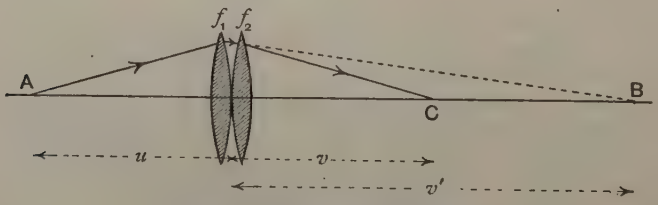

FIG. 553.—Thin lenses in contact.

similar to that employed on p. 591. Let A be the object (Fig. 553), and let the first lens f_1 produce an image at B, at a distance v'.

Then $$\frac{1}{v'}-\frac{1}{u}=\frac{1}{f_1}. \quad \text{.....(1)}$$

B is now the object with respect to the second lens f_2, which therefore produces an image at C.

$$\frac{1}{v}-\frac{1}{v'}=\frac{1}{f_2}. \quad \text{.....(2)}$$

On adding the equations (1) and (2),

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}.$$

But considering the two lenses together as equivalent to a single lens of focal length F,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{\mathrm{F}};$$

$$\therefore \frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}. \quad \text{.....(3)}$$

EXPT. 134.—**Combination of thin lenses in contact.** Measure the focal lengths of two thin converging lenses f_1 and f_2 by means of the optical bench. Place them in contact and find the equivalent focal length F. Show that

$$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}.$$

EXPT. 135.—**Focal length of a diverging lens (1st method).** Measure the focal length f_1 of a converging lens, and then place a diverging lens, f_2, in contact with it. Measure the focal length F of the combination, and calculate f_2 from equation (3) above, taking great care to give F and f_1 their correct signs.

Measure the radii of curvature, r_1 and r_2, of the diverging lens by the method given in Expt. 132, in which the image is obtained alongside of the object.

From equation (4) (p. 592), calculate μ.

EXPT. 136.—**Focal length of a diverging lens (2nd method).** The difficulty met with in determining the focal length of a diverging lens is that the image is virtual. Its position is therefore not easy to find. One method of considerable accuracy consists in placing the concave mirror M upon the optical bench, behind the lens L (Fig. 554). Adjust their positions until there is a sharp image beside the object at S. It is evident that the rays meet the mirror normally and return along their original paths. Hence on leaving the lens they must have been

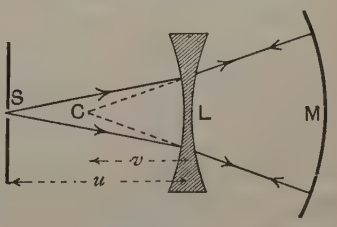

FIG. 554.—Measurement of focal length of diverging lens.

travelling from the centre of curvature C of the mirror. Measure the distance SL $= u$. To find CL $= v$, remove the lens and move the mirror towards the screen until the image formed by reflection at the mirror falls beside S. The centre of curvature of the mirror is now at S, and the distance through which the mirror has moved is equal to CS, and must be noted. Now CL = SL − CS.

From equation (5) (p. 592), calculate f.

The Dioptre.—Opticians have a method for treating lenses somewhat different from those described above. A lens of 1 metre focal length is said to have a power of **one dioptre**. Converging lenses are considered to be positive and diverging lenses negative. Then the power of a lens in dioptres is

$$\frac{1}{\text{focal length in metres}}, \quad \text{or} \quad \frac{100}{f \text{ in centimetres}}.$$

Thus, taking p as the power in dioptres, and f the focal length in centimetres, $p = 100/f$. This method is convenient for some purposes, for on placing two thin lenses of powers p_1 and p_2 in contact, the power P of the combination is given by $\mathrm{P} = p_1 + p_2$. This may be proved as follows:

Since $$\mathrm{P} = \frac{100}{\mathrm{F}}, \quad p_1 = \frac{100}{f_1}, \quad p_2 = \frac{100}{f_2}, \quad \text{and} \quad \frac{1}{\mathrm{F}} = \frac{1}{f_1} + \frac{1}{f_2};$$

$$\therefore \frac{100}{\mathrm{F}} = \frac{100}{f_1} + \frac{100}{f_2},$$

that is, $$\mathrm{P} = p_1 + p_2.$$

Example.—What must be the power of a lens which, when combined with a converging lens of focal length 20 cm., will produce a combination of power 3 dioptres?

The power of the converging lens is $\frac{100}{20} = 5$ dioptres.

Now $3 = 5 + p$; hence $p = -2$ dioptres. That is, a lens of power -2 dioptres must be used; this is a diverging lens of 50 cm. focal length.

Example.—A converging lens of 6 dioptres is combined with a diverging lens of -2 dioptres; find the power and focal length of the combination.

$$\mathrm{P} = +6 - 2 = +4 \text{ dioptres};$$

therefore the power of the combination is $+4$ dioptres, and this is equivalent to a converging lens of 25 cm. focal length.

Exercises on Chapter XLVI.

1. Find by a graphical construction the size and position of the image of an object 4 inches long, placed perpendicularly on the axis of a thin convex lens of focal length 7 inches, at a distance of 17 inches from the lens. Sen. Camb. Loc.

2. The focal length of a convex lens is 20 cm.; where must an object be placed so as to obtain a real image five times as long as the object?

3. An upright image is formed of an object by means of two convergent lenses of equal focal length. Show that the distance between the two lenses must be at least equal to double the focal length of one lens; and draw a diagram illustrating the course of the rays through the lenses. L.U.

4. Define the focus of a lens. A source of light and a screen are placed 150 cm. apart. Where should a convex lens of 20 cm. focal length be placed in order to form a real image of the source upon the screen? L.U.

5. An arrow 5 centimetres in length is laid along the axis of a convergent lens of which the focal length is numerically equal to 10 cm. If the arrow points towards the lens, its head being at a distance of 20 cm. from the lens, what will be the position and character of its image? L.U.

6. Two convex lenses each of focal length f are placed at a distance $3f$ apart. For what positions of the object will a real image be formed by this combination of lenses? L.U.

7. Find the length of the image of a straight luminous filament 3 cm. long placed along the axis of a thin lens of 12 cm. focal length with its near end 21 cm. from the lens. What would be the length of the image if the filament was perpendicular to the axis of the lens and 21 cm. from the lens? L.U.

8. Calculate the curvature necessary for the faces of an equi-convex lens of 6 inches focal length made of glass if refractive index is 1·55.

Find the focal length of a lens which magnifies three times an object placed 2 inches from it. L.U.

9. A beam of light diverges from a point P on the axis of a convex lens and after passing through the lens is reflected from the surface of a convex mirror. The reflected beam is brought to a focus by the lens at a point coinciding with P. Find the radius of curvature of the mirror, being given that the distance of the lens from the mirror is 10 cm., the distance of P from the mirror 30 cm., and the focal length of the lens 12 cm. L.U.

10. Two convex lenses, each of focal length 20 cm., are situated 10 cm. apart, and have a common axis. An object 2 cm. in height is placed at a distance of 15 cm. from the first lens. Find the size and position of the final image. Give diagram. L.U.

11. Light is converging towards a point situated 15 cm. behind a diverging lens of focal length 20 cm. Find the position of the point at which the light will actually come to a focus.

12. An object 1 foot in size is situated at a distance of 10 feet from a converging lens of focal length 2 feet. After passing through this lens, the light falls upon a diverging lens of focal length 6 inches, situated 2 feet from the first lens. Find the position and size of the final image.

13. At a distance of 9 inches in front of a convex lens of 6 inches focus is placed a convex lens of 3 inches focus; 6 inches in front of this is placed

an object. Make a sketch, approximately to scale, showing the paths of the rays through the system. What is the nature of the final image ?
Allahabad University.

14. Show how the focal length of a convex lens depends on the curvature of its surfaces. A small object is enclosed in a sphere of solid glass of 7 cm. radius. It is situated 1 cm. from the centre, and is viewed from the side to which it is nearest. Where will it appear to be if the refractive index of the glass be 1·4 ? Madras University.

15. Explain in general terms why a glass lens which is thicker in the middle than at the edges deviates parallel rays of light in such a manner as to bring them to a focus.

A convex lens, of focal length 10 cm., is used to form an *erect* image of an object, the image being twice as large as the object. Find the position of the object. L.U.

16. Derive a relation connecting the distances of object and image from a converging lens with the focal length of the lens.

A converging lens of 4 inches focal length is employed to form an image magnifying the object three times. Show that there are two possible positions of the object. Distinguish between the images formed in the two cases. L.U.

17. A convex lens casts an image equal in size to the object which is placed 20 cm. away from the lens. If another lens is placed in contact with the first, the image is found to be reduced to one quarter of its previous linear dimensions. What are the focal lengths of the two lenses used ?
L.U.

CHAPTER XLVII

OPTICAL INSTRUMENTS

Photographic camera.—Perhaps the simplest application of lenses for optical purposes is found in the photographic camera, in which a lens, or lens system, produces a real image of an external object. The lens system L (Fig. 555) may be a single converging lens, or a combination of lenses, but in any case, for purposes of the simple theory, it may be considered as equivalent to a single lens, and it produces a real image upon the screen P. This screen may be a sheet of ground-glass, for the purpose of focussing the image, the lens being moved backwards or forwards by means of the rack and pinion R. P is then replaced by the sensitive plate for the production of the photograph. The bellows B serve to exclude all light that does not contribute to the formation of the image.

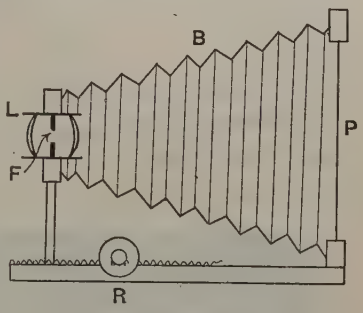

FIG. 555.—Photographic camera.

The **definition** or sharpness of the image can be varied by means of the **diaphragm** or **stop** at F, which is a screen having a circular aperture, and cuts off any light that does not pass through the central parts of the lens. The smaller the diameter of this stop the better is the definition, as each point on the object then produces a better 'point focus' (pp. 562 and 587).

In photography it is necessary that the image should be as bright as possible, but a reduction in the size of the stop cuts off light, and hence diminishes the brightness of the image. The choice of stop is always a matter of compromise. The intensity of illumination of the image depends upon the focal length of the lens, and if we take an image at the principal focus, the area of any piece of it

is directly proportional to f^2 (p. 595). Hence the intensity of illumination is inversely proportional to f^2. Now the amount of light passing through the lens is proportional to the area of the stop, and therefore to d^2, if d be the diameter of the stop. Hence, for constant brightness for all lenses, under the same external conditions of illumination, d^2/f^2 must be the same, since the brightness of the image is proportional to this quantity. For this reason the stops supplied with a lens are usually such that d^2/f^2 is proportional to the numbers 1, 2, 4, 8, 16, 32, and 64.

These stops are usually marked

$$f/64,\ f/45,\ f/32,\ f/22,\ f/16,\ f/11,\ \text{and}\ f/8,$$

the diameters being

$$\tfrac{1}{64},\ \tfrac{1}{45},\ \tfrac{1}{32}\ \ldots,\ \text{etc., of the focal length of the lens,}$$

because 64, 45, 32, 22, 16, 11, and 8 are the nearest whole numbers that would make d^2/f^2, and therefore the brightness of the image, proportional to, 1, 2, 4, 8, 16, 32, and 64.

On this system, exposure tables, properly determined, apply to all lenses, whatever the focal length.

Projection lantern.—Optically, this lantern is similar to the photographic camera, because a single lens, or lens system, produces a

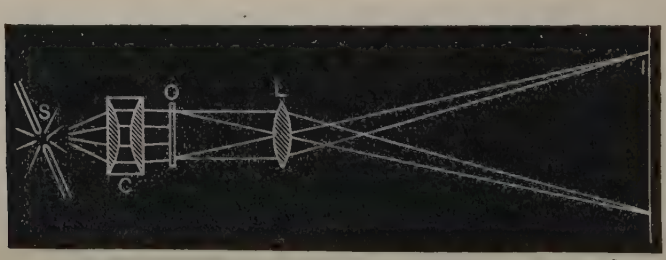

FIG. 556.—Arrangement of lenses in a projection lantern.

real image. Thus the lens L (Fig. 556) produces an image of the transparency O upon the screen I. Since O is near the principal focus of L, the image is large (p. 595), and if the screen be of suitable white material, the image may be exhibited to a large audience. The transparency O is illuminated by a bright source of light S, and as much of the light as possible is made to take the direction necessary to help in forming the image, by means of the condenser C. This condenser usually consists of two plano-convex lenses with their plane faces outwards.

The **magnification** is v/u, and hence from the relation $1/v - 1/u = 1/f$, the requisite focal length for any size of screen at a given distance may be found.

EXAMPLE.—Find the requisite focal length for a lens which is to produce an image of a slide 8 cm. square, upon a screen 3 metres square, at a distance of 10 metres from the lantern, the image to occupy the whole of the screen.

Here, $$\text{magnification} = \frac{v}{u} = -\frac{300}{8}.$$

Again, $$v = -1000 \text{ cm.}; \quad \therefore \ u = \frac{8000}{300}.$$

Hence, from $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have, $$-\frac{1}{1000} - \frac{300}{8000} = \frac{1}{f};$$

$$\therefore f = -\frac{8000}{308} = -25{\cdot}9 \text{ cm.}$$

Thus a converging lens of focal length nearly 26 cm. is required.

Erecting prism.—Since, in the case of the projection lantern, the image is inverted, the slide is usually inverted in putting it into

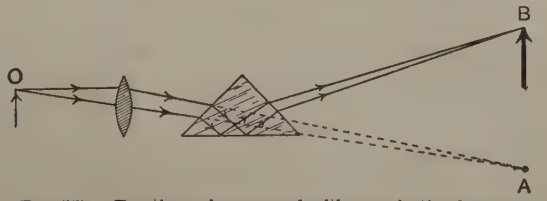

FIG. 557.—Erecting prism as used with a projection lantern.

the lantern, so that the picture upon the screen is erect. If it is desired to project upon the screen an image of any object, such as a liquid in a flat glass cell, the cell must not be inverted, and the **erecting prism** (p. 583) may then be employed. Fig. 557 shows how the right-angled prism totally reflects the light, so that the image, which would fall on the screen at A when the erecting prism is absent, will now fall at B ; the images are thus rendered erect.

The eye.—Like the camera, the eye may be looked upon as a light-tight enclosure with a lens at one end and a screen at the other, the lens producing on the screen real images of external objects. The eye is really a most complex piece of apparatus, but the complexity arises from the necessity for various adjustments. The optical

principles involved may, for elementary purposes, be resolved into the above simple statement.

A horizontal section through the eye is given in Fig. 558. The thick tough coating *s*, called the **sclerotica**, is maintained distended, and of a form roughly spherical, by means of the transparent gelatinous **vitreous humour** *v*. The **crystalline lens** *o*, the **aqueous humour** *a*, and the **cornea** or transparent portion of the sclerotica *c*, form a lens system. The inner wall of the posterior portion of the sclerotica is coated with a tissue containing an immense number of cells, in each of which is the end of a ramification of the **optic nerve** *n*. This tissue is called the **retina**, and is the sensitive layer upon which the luminous images fall. The manner in which the brain becomes sensible of light falling upon the retina does not here concern us.

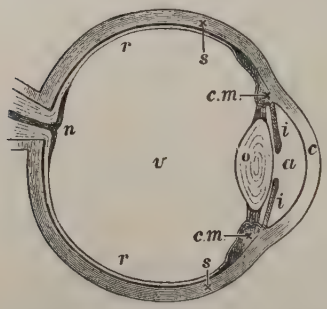

FIG. 558.—Horizontal section through the eye.

The **iris** *i* is a diaphragm which plays a part similar to that of the stop of the camera lens (p. 603). In the centre there is a circular aperture *a* called the pupil, which is the perfectly black spot usually seen in the middle of the eye. The reason for its blackness is that the inner layers of the eye contain a black pigment, thus preventing the light which falls on the retina from being internally reflected and thus disturbing vision. When the external illumination is bright, the pupil becomes smaller, by reason of the contraction of a set of circular muscles of the iris. When the external illumination is weak, the circular muscles relax and certain radial muscles contract, thus enlarging the pupil and admitting enough light to effect vision. Hence changes of illumination are not consciously perceived until the amount of light is too great or too small for the adjustment of the iris to regulate the amount entering to a suitable quantity.

It is clear that, with a fixed lens system, only those objects at one particular distance from the eye will produce a sharply focussed image upon the retina. For any other distance of the object, the image is blurred, and for the formation of a good image in this new position, refocussing is necessary. In the camera and lantern the refocussing is done by altering the position of the lens, but in the eye, the focal length of the lens itself is changed. This is performed by the circular muscle c.m. (Fig. 558) called the **ciliary muscle.** The crystalline lens is elastic, and when the eye is at rest, the ciliary

muscle is relaxed and the lens is too convex for acute vision. In order to focus any object, the ciliary muscle contracts until the lens has the right curvature to produce a sharply defined image upon the retina. This power of adjustment of the eye is called **accommodation**; it is involuntary, and is what physiologists call a reflex action.

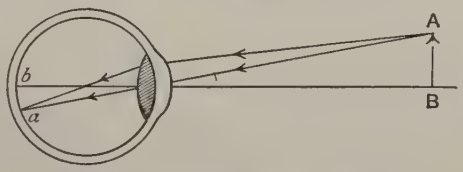

FIG. 559.—Inversion of image in the eye.

There is another point that deserves attention. Representing the eye diagrammatically in Fig. 559, we see that the object AB is represented by an inverted image *ab* upon the retina. The mind, however, is conscious of the presence of the object in the erect position AB. This should not cause surprise, for it must be remembered that we have no experience whatever of the processes occurring inside our own eyes, and the mind has learnt by touch and muscular sense that the image *ab* means the presence of an object as shown at AB.

Defects of vision.—There are certain common defects of vision which may easily be corrected by means of spectacles.

Myopia, or short-sight. The lens of the eye may be of too short a focal length, or the eye-ball may be too long. The image of a point A (Fig. 560 (*a*)) then falls in front of the retina at *a'*. In order to bring the image on to the retina at *a* (Fig. 560 (*b*)), either the object A must be moved nearer to the eye, or a diverging spectacle lens L may be placed in front of the eye, in order to increase the equivalent focal length. Thus, near objects can be clearly seen but distant objects cannot, and hence the name 'short-sight.'

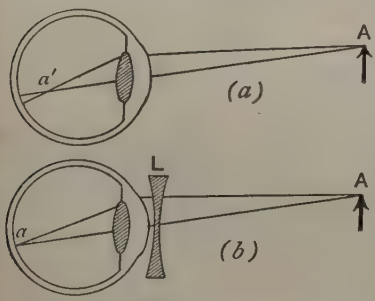

FIG. 560.—Short-sight and its correction.

In order to find the requisite focal length for the lens L, let an object be slowly carried away from the eye. At a certain point clear vision ceases; this is called the **far point.** Let its distance from the eye be *d*. If the eye were normal, the far point would be at infinity, and the distance *v* from the lens to the retina would be *f*, the focal length of the eye lens. But for the far point we have

$$\frac{1}{v}-\frac{1}{d}=\frac{1}{f}.$$

Now let f_1 be the focal length of the lens L required to make v the focal length of the combination (Fig, 560 (b)), so that the eye with the lens in front can see clearly points at infinity. Then

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{f_1} \text{ (p. 599).}$$

On comparing these two equations, we see that $f_1=d$, or, **the focal length of lens required to correct this eye, is equal to the distance of the far point from the eye.**

Hypermetropia, or long-sight. In this case the focal length of the lens of the eye is too great, or the eye-ball is too short, so that the rays, for near objects, cannot be brought to a focus sufficiently forward to give a clear image on the retina. Here A (Fig. 561 (a)) must be moved further away, or a converging lens L (Fig. 561 (b)) interposed, in order to make vision clear.

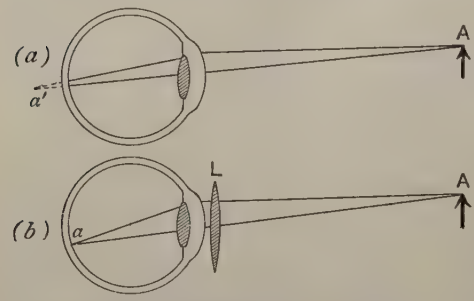

FIG. 561.—Long-sight and its correction.

For the normal eye, there is a position, nearer than which acute vision cannot be obtained without straining the eye. The distance of this position from the eye is about 25 cm., and is called the **least distance of distinct vision.** In reading and working, the eye should be at this distance from the object viewed.

It is now required to find the focal length of lens L (Fig. 561 (b)) which will render a long-sighted eye normal. If the object A be gradually brought up to the eye from a distance, distinct vision ceases at some point further off than 25 cm. This is called the **near point.** As before, let its distance from the eye be d; then $1/v-1/d=1/f$. If the eye is to be rendered normal, the eye combined with the lens L should have a near point at 25 cm.; $\therefore \frac{1}{v}-\frac{1}{25}=\frac{1}{f}+\frac{1}{f_1}$.

Comparing this with $1/v-1/d=1/f$, we see that

$$\frac{1}{d}-\frac{1}{25}=\frac{1}{f_1},$$

from which f_1 may be calculated.

There is a form of long-sight which is due to age, and is called **presbyopia.** The crystalline lens loses elasticity gradually all through life and consequently its power of recovery on relaxing of the ciliary

muscle decreases. Thus an eye which is short-sighted in childhood tends to become normal in after years, but an eye which is long-sighted is sure to become more so.

Astigmatism. A very common defect of vision known as **astigmatism** may be noticed. Unless the surfaces of the lens system of the eye are surfaces of revolution about the axis of the eye, their curvature will not be the same in different planes containing the axis. It follows that the focal length of the system is different in different planes, and a point upon the object can never be reproduced by a point on the image. This defect may be remedied by using lenses with cylindrical surfaces, in order to vary the focal length in one plane only.

Simple microscope.—On p. 596 the virtual image produced by a converging lens was studied, and it was seen that the image is larger than the object. A lens used in this way constitutes a **simple microscope.** In order to obtain the greatest advantage, the eye should be placed as near to the lens as possible. Not only is the field then as large as possible, but the eye is at the minimum distance from the virtual image *ab* (Fig. 562). The student should bear clearly in mind that, for the greatest advantage in vision, the angle subtended at the eye by the object should be as great as possible; the actual size of the object is of no consequence. Thus in Fig. 562 the angles subtended by the object AB and by the image *ab* are the same. But O*b* is the least distance of distinct vision, and therefore if the lens were absent the eye could not see clearly the object situated at AB. The object would have to be removed to A′*b*; hence the advantage in using the lens.

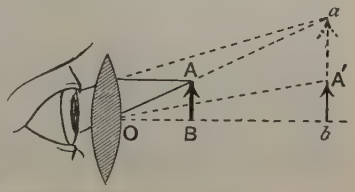

FIG. 562.—Single lens used as a simple microscope.

The **magnification** is $\frac{ab}{AB}=\frac{v}{u}$.

But $1/v-1/u=1/f$, and for the greatest advantage, $v=25$ cm., *i.e.* the least distance of distinct vision;

$$\therefore\ 1-\frac{v}{u}=\frac{v}{f},\quad \text{or}\quad \frac{v}{u}=1-\frac{25}{f}.$$

Thus, for a converging lens of focal length $-2{\cdot}5$ cm., the greatest magnification obtainable is $1+25/2{\cdot}5=11$.

EXPT. 137.—**Simple microscope.** Place a sheet of square-ruled paper at a distance of 25 cm. from a short focus converging lens, and place the

eye nearly in contact with the lens. Bring up a small piece of the ruled paper towards the lens until the eye looking through the lens can see an image of the rulings, at the same time as the other eye sees the sheet. Obtain as near a coincidence as possible, and read the number of divisions on the sheet, as seen by direct vision, which coincide with one division as seen through the lens. This is the magnification produced by the lens.

Compound microscope.—A converging lens of short focal length produces a real image of a small object, the image being viewed by another lens, used as a simple microscope. The arrangement is called a **compound microscope.** The lens O (Fig. 563) which produces

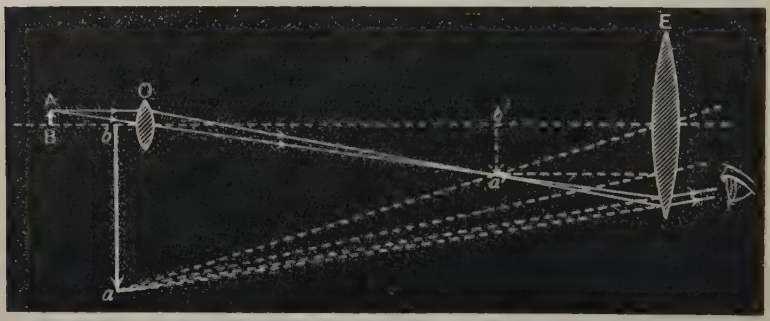

FIG. 563.—Two lenses used as a compound microscope.

the real image $a'b'$, of the object AB, is called the **object-glass,** or **objective.** The lens E which acts as a simple magnifier to produce the virtual image ab is called the **eyepiece.**

The lines which are dotted in Fig. 563 are mere construction lines for finding the position of the image ab. The full lines represent actual rays which pass from A to the eye in helping to form the final image a.

The magnification depends upon the focal lengths of both objective and eyepiece. Thus, in a practical case, with an objective of focal length 0·5 cm., the image $a'b'$ may be at a distance of 20 cm. from the objective, giving a magnification of nearly $20/0{\cdot}5 = 40$. If the eyepiece has a focal length of 2·5 cm., the magnification it alone produces is $1 + 25/2{\cdot}5 = 11$ (p. 609). Hence the total magnification is $40 \times 11 = 440$.

In practice the objective and eyepiece are not single lenses, but combinations of lenses, in order to correct for several defects. In Fig. 564 is shown a section of this form of microscope. The objective O, consisting of two lenses, would produce a real image at $a'b'$. The first lens E_1 of the eyepiece, called the **field lens,** however interposes and the real image is produced at S, where a finely divided scale is

situated. The second lens E_2 of the eyepiece, called the **eye lens**, produces a virtual image at *ab*. If the scale at S be calibrated (p. 20 and Expt. 138) the real size of the object AB may be read off

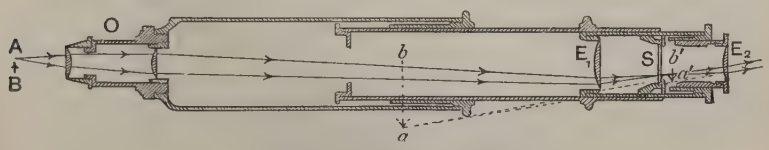

FIG. 564.—Compound microscope.

upon it, care being taken that the real image is in focus in the plane of the scale. This may be assured by moving the eye laterally, when the images of the object and the scale should not move relatively to each other. The scale in the eyepiece should be fixed in such a position that the final image *ab* is 25 cm. from the eye.

EXPT. 138.—**Magnifying power of a microscope.** Place a test object of known size, such as is supplied by instrument makers, upon the stage of a microscope, and place a scale graduated in millimetres, alongside of the tube of the instrument. Let the scale be 25 cm. from the eye, and arrange it so that it can be seen by the eye that is looking through the microscope. Focus until the image in the microscope can be seen simultaneously with the scale, and read off its apparent size upon the scale. Knowing its true size, the magnification can be calculated.

Immersion objective.—For very high-power microscopes, the illumination of the field of vision becomes very small owing to the magnification, unless precautions are taken to get as much light as possible to pass through the objective and so contribute to the formation of the image. With lenses as usually employed, only the central parts may be used (p. 587), since lenses are not as a rule aplanatic. There is one case, however, in which a spherical surface is aplanatic (p. 590), and this type has been made use of for high-power microscopes. AB (Fig. 565) is the lowest lens of

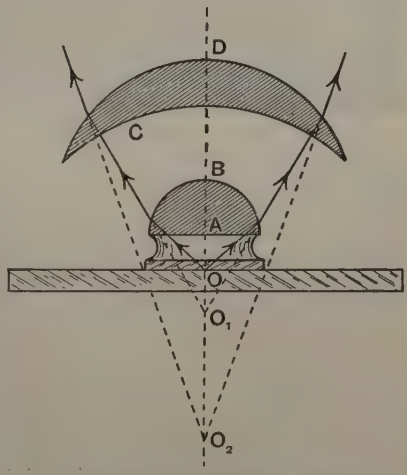

FIG. 565.—Immersion objective.

the objective. It is plane on its under side A, and its upper surface B is spherical. The space between A and the microscope slide is occupied with cedar-wood oil, which has the same refractive index as glass, so that the object O may, for optical purposes, be considered to be within the sphere B. O and O_1 are aplanatic points with respect to B, so that the wide cone of rays, after refraction at O, diverges from O_1. The surface C of the next lens is spherical and has O_1 as centre, so that the rays are not bent on entering; but the surface D is spherical with O_1 and O_2 as aplanatic points. On leaving D the rays are not diverging too much to be made use of in the microscope, and they are all diverging from the point O_2. Evidently more light is made use of than if the object O were placed at O_2 and the oil immersion and lens AB were absent.

Astronomical telescope.—The principle of the telescope is similar to that of the microscope, but modifications are made to meet the

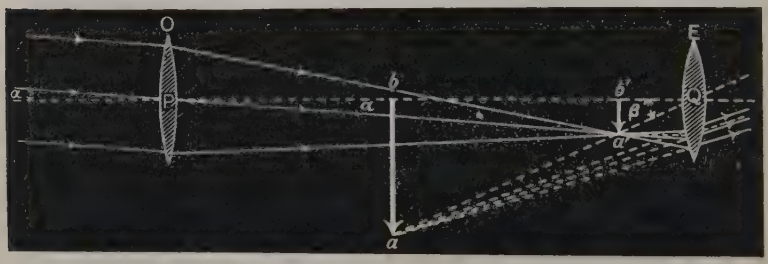

FIG. 566.—Two lenses used as an astronomical telescope.

change in purpose, viz. the viewing of distant objects. A lens, again known as the **objective**, produces a real image of the object to be examined, and this image is observed by means of an **eyepiece**.

In Fig. 566 O represents the objective which produces the real image $a'b'$ of the distant object. This is viewed by means of the eyepiece E, giving the virtual image ab.

Owing to the great distance of the object, the rays from any point of it are sensibly parallel on reaching the instrument; hence $a'b'$ is at the principal focus of the objective. It follows that Pb' is the focal length of the objective. In focussing the telescope, the observer unconsciously arranges the virtual image ab to be also at infinity, so that Qb' is the focal length of the eyepiece. In observing distant objects, it is their apparent size only that concerns us, and this is determined by the angle they subtend at the eye. Without the telescope the angle subtended by the object would be α; with the

telescope the angle subtended by the image is β, or $\angle a'Qb'$. Hence the magnification is

$$\frac{\beta}{\alpha}=\frac{a'Qb'}{a'Pb'}=\frac{\dfrac{a'b'}{Qb'}}{\dfrac{a'b'}{Pb'}}=\frac{Pb'}{Qb'}.$$

Hence $$\text{Magnification}=\frac{\text{Focal length of objective}}{\text{Focal length of eyepiece}}.$$

Thus, for great magnification, the objective must have a long focal length and the eyepiece a short one.

Surveying telescope.—When the telescope is to be used for the measurement of altitude (p. 560), or for the angle subtended by a base line at a distance, the eyepiece must be provided with cross wires which meet on the axis of the instrument. These must be placed at the position of the real image produced by the objective.

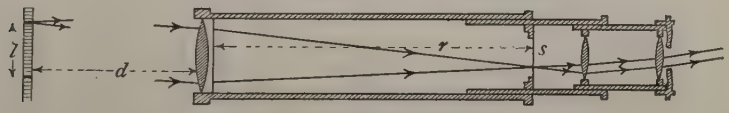

FIG. 567.—Surveying telescope.

If the instrument is to be used to measure distances merely, a staff, or scale is set up at the point whose distance away is, say, d. Then the image falls upon the cross wires s (Fig. 567), and the distance apart of two parallel cross wires being s, and a length l of the staff coinciding with s, as seen through the telescope, we have $d/r=l/s$ (p. 595), where r is the distance of the cross wires from the objective. Thus r, s, and l being known, d, the distance of the staff, can be calculated. A correction must be made for the fact that d is the distance of the staff from the objective and is not measured from the axis of the stand of the telescope, and also that the position of the cross wires must be varied in order to focus for different distances of the staff.

Annallatic telescope.—The above correction is avoided in some cases by the addition of an extra lens, known as the **annallatic lens.** In Fig. 568, L is the annallatic lens. To understand the principle, notice that $a'b'$ is the real image of the staff AB, and two cross wires are placed there **at a fixed distance apart.** Thus a' is at a fixed distance from the axis of the telescope. For the sake of clearness, only half of the staff AB and its image $a'b'$ are shown in the diagram. Consider a ray AgfCa' which is parallel to the axis on leaving the lens L. Whatever the distance of the staff, this ray follows the path gfCa', and hence before meeting the objective O it is travelling towards

some fixed point P of the telescope. Thus the triangle APB has fixed angles, and for two positions of the staff, such as AB and A′B′, we have AB/BP = A′B′/B′P. Hence the reading of the image of the portion of the staff intercepted by the cross wires at $a'b'$, being proportional

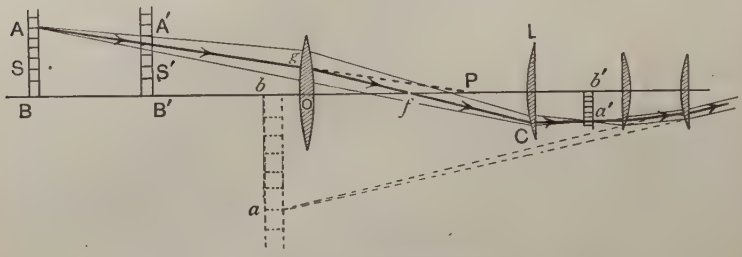

FIG. 568.—Annallatic telescope.

to AB or A′B′, is proportional to the distance of the staff from the fixed point P. If the point P is situated at the axis of the theodolite or stand of the instrument, the staff reading intercepted by the cross wires is proportional to the distance of the staff. The constant required in order to convert the intercept by the cross wires into the distance of the staff may be determined by taking a reading at a known distance, or it may be calculated from the focal lengths and positions of the lenses.

Terrestrial telescope.—In the above types of telescope, the image is inverted. This is objectionable for many purposes, and hence

FIG. 569.—Use of erecting lenses in terrestrial telescope.

most terrestrial telescopes are provided with a pair of lenses whose function is to invert the already inverted image produced by the objective. In Fig. 569, $a'b'$ is the real image produced by the objective which is not shown. This falls at the focal distance from C, the rays thus passing as parallel beams to D, which brings them to a real focus at $a''b''$. The eyepiece E produces a virtual image ab of $a''b''$, which is then erect. The pair of lenses C and D have equal

focal lengths and are placed at twice their focal length apart, their only function being to invert the image and so act as erecting lenses.

Galilean telescope, or opera-glass.—The pair of erecting lenses C and D in the terrestrial telescope cause loss of light by reflection at their surfaces, and also add to the length of the instrument. In the case of the opera-glass, these defects are eliminated, but the magnification produced is not so great as with the terrestrial telescope.

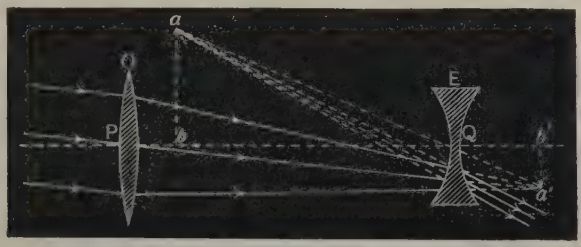

FIG. 570.—Two lenses used as a Galilean telescope.

The objective O converges the rays towards the point a' (Fig. 570); but before they come to a focus, the strong diverging lens E is interposed. This causes the rays to diverge, giving a virtual image at ab, which will be seen to be erect.

The eye lens E must be so placed that the final virtual image ab is at infinity, which means that the rays emerging from E are practically parallel. Hence $a'b'$ is at the focal distance from E, or Qb' is the focal length of the eye lens. Similarly, Pb' is the focal length of the objective. Also the magnification produced is $aQb/a'Pb'$, as on p. 613, that is $a'Qb'/a'Pb'$. Since all the angles are small, $a'Pb' = a'b'/Pb'$, and $a'Qb' = a'b'/Qb'$;

$$\therefore \frac{a'Qb'}{a'Pb'} = \frac{Pb'}{Qb'}.$$

Thus, $$\text{Magnification} = \frac{\text{Focal length of objective}}{\text{Focal length of eye lens}}.$$

Prism binoculars.—For producing the second inversion in the terrestrial telescope, total reflection prisms are sometimes employed in place of the erecting lenses. The rays from the objective are reflected by the prism P (Fig. 571) and retraverse the length of the instrument. They are then again reflected by the prism Q, eventually passing through the eyepiece E. Thus the optical length of the instrument is three times its actual length, and hence a longer focus objective can be used and higher magnification obtained than for the same length of instrument without prisms.

From the fact that these instruments are usually made in pairs, one for each eye, they are generally termed **prism binoculars.**

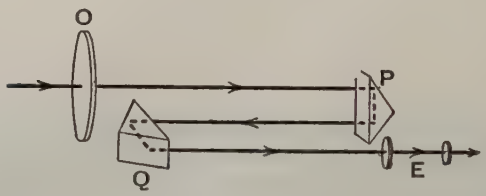

FIG. 571.—System of lenses and prisms in prism binoculars.

EXPT. 139.—**Magnifying power of a telescope.** Focus a telescope on a vertical scale placed at a distance of at least 10 metres, and with the other eye observe the scale direct. Move the head until the direct scale and the image are in the same plane, parallax being thus avoided. Observe the number of scale divisions seen by direct vision which coincide with one image division. This is the magnification produced by the telescope.

The periscope.—A pair of plane mirrors parallel to each other and one vertically above the other, constitute a simple form of **periscope**; but unless the mirrors are close together the range of vision would be limited to a small angle. In the submarine boat it is necessary to have the upper mirror about 20 feet above the lower, and since the tube between them has a diameter of about six inches, the angle of vision would be limited to that obtained by looking along a tube 20 feet long and six inches wide, unless some lens system were employed.

The method adopted for getting over this difficulty and increasing the angle of vision, is to employ a number of telescopic systems. In Fig. 572 is represented a periscope consisting of two systems EO and O′E′; the former is inverted, that is, has its short focus lens E towards the object. To understand its use, reference may be made to Fig. 566, in which it will be seen that the rays entering the objective are much less inclined to the axis than the rays leaving the eyepiece. On applying the eye to the objective and directing the eyepiece towards the object, that is, on looking through the wrong end of a telescope, the object appears much smaller than usual, but the **range of vision is great.**

Thus in Fig. 572 the rays from the object fall on the 45° prism P_1 and are reflected downwards; the lens E then produces a real image

$a'b'$. This image is at the principal focus of O, so that the rays emerge from O parallel, and also are very nearly parallel to the axis of the tube. The function of the reversed telescopic system EO is thus to render the rays, coming from a wide field, nearly parallel to the axis of the tube. If the eye were to look through O, a very much diminished image would be seen. The function of the telescopic system O′E′ at the lower end of the tube is to produce a magnification, which in practice a little more than compensates for the diminution produced by the upper telescope. The total magnification of the two systems is about $1\frac{1}{4}$.

Before the rays from O′ come to a focus to form a real image, the 45° prism P_2 is interposed, which renders the rays horizontal and also produces a final rectification of the image $a'b'$, so that the image ab seen on looking through E′ is erect. There is always some arrangement of gearing to enable the tube with the upper prism P_1 to be rotated in order to enable the observer to look out in any desired direction.

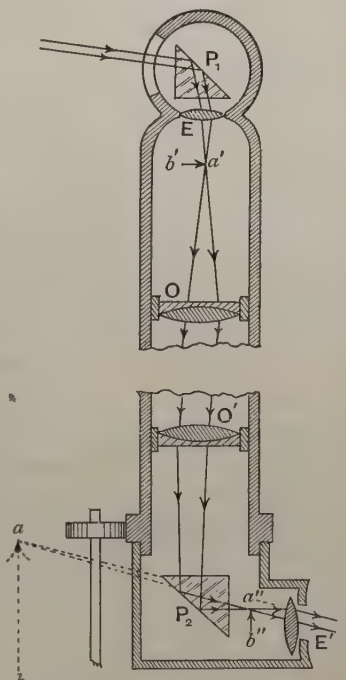

FIG. 572.—Submarine periscope.

Range-finders.—In order to measure the distance of an object from the observer, the method of the staff and telescope may be employed (p. 613), but this involves the placing of a standard of length in the position occupied by the object. This is for many purposes inconvenient, and various range-finders have been devised in which the standard of length is at the observing station. An instrument devised for this purpose is called a **range-finder.**

If the object be at infinity, rays from it such as AB and CD (Fig. 573) are parallel, and if reflected from two mirrors, as shown at B and D, the rays travel along BE and DE respectively. Two mirrors, m_1 and m_2, one above the other, now reflect the rays to G, and the eye will then see the two images of the object superimposed. If, however, the object be at a finite distance, say at A, the mirror D must

be rotated in order to reflect the ray AD along the path DE, so that the two images of A may be superimposed. The amount of rotation of the mirror D determines the angle CDA or BAD, and thus the distance AB becomes known in terms of this angle and the fixed distance BD.

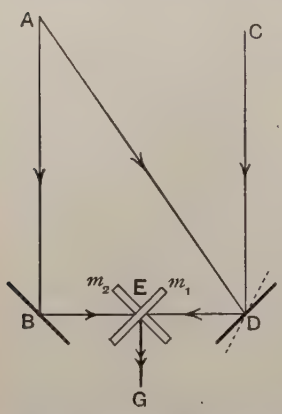

FIG. 573.—Principle of the range-finder.

When the distance AB is great compared with BD, the angle of rotation of the mirror D, to effect the adjustment, is so small that it cannot be measured with sufficient accuracy to afford a useful measure of the distance AB. To get over this difficulty the range-finder of Barr & Stroud makes use of another device. Instead of rotating the mirror D (Fig. 574) an achromatic prism P of small angle is interposed between D and the pair of mirrors m_1 and m_2. These mirrors are situated one above the other so that the upper part of the object occupies one half of the field of vision and the lower half of the object the other. The mirror D is not quite at right angles to the mirror B, so that the two halves of the image of an object at infinity form one continuous image when the prism occupies a position such as P. The prism is rigidly attached

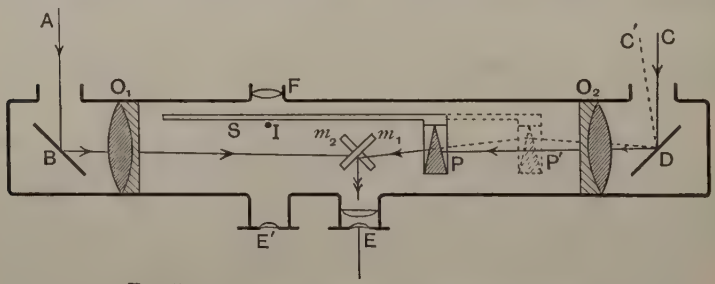

FIG. 574.—Principle of the Barr & Stroud range-finder.

to a carrier provided with a scale S, and the scale is at the infinity mark as seen by the left eye looking through E′ at the fixed mark I, when AB and CD are parallel.

When the object is not at infinity, the rays AB and C′D from it are inclined to each other, and the prism must be moved to P′ in order that the two halves of the image shall again be continuous. The scale is so calibrated that its new position indicates directly

the distance of the object under observation. Owing to the small angle of the prism, the deviation it produces is slight, and a considerable travel of the prism and scale is required to compensate for the want of parallelism of the rays AB and C′D. This allows of a scale sufficiently open to enable distances up to 1000 yards to be measured with an accuracy of about 1 per cent. The objectives O_1 and O_2 together with the eyepiece E really form a pair of telescopes. The mirrors B and D are of speculum metal, and in the later forms of instrument m_1 and m_2 consist of a suitable set of 45° prisms. The optical and mechanical refinements cannot be described here, but it may be noted that F is a lens used to illuminate the translucent scale S. The lens E′ is in two parts, the upper for the observation of the scale, while the lower half is a diverging lens which, with F, forms a Galilean telescope which acts as a wide range low-power finder to enable the observer to sight the instrument rapidly on any desired object. The distance BD is $4\frac{1}{2}$ feet in the larger instruments and 2 feet in the portable instrument.

Constant deviation reflecting prism.—The mirrors at B and D (Fig. 574) are required to produce a constant deviation (90°) of the incident beam AB or CD. For this reason the mirrors B and D must be rigidly fixed, since any movement of one or the other will vitiate the observations of the range of the distant object. To get over this difficulty, prisms have been employed as reflectors in the more recent range-finders. The incident ray PQ (Fig. 575) passes normally through the face AB and is internally reflected at the face ED. Its path is then QR, and it is again reflected at the face BC, and finally emerges normally through the face AE. Since the angle between the faces BC and ED is constant, the deviation of the ray undergoing two reflections is likewise constant (p. 558), and is given by

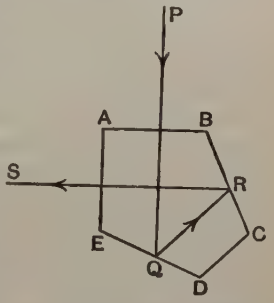

FIG. 575.—Constant deviation reflecting prism.

$$\text{deviation} = 2\pi - 2\theta,$$

where θ is the angle between the two reflecting faces. In this case the deviation is 270°;

$$\therefore\ 270° = 360° - 2\theta,$$
$$\theta = 45°.$$

Thus the faces BC and ED are inclined at an angle of 45° to each other. The total deviation of the ray is then 270° or 90°, as is required, and if the prism be displaced or rotated, this deviation is unchanged. If the prism be rotated slightly, the angles of incidence and emergence

are no longer 90°, and refractions will occur. It can easily be seen, however, that the refractions at incidence and emergence are equal and opposite so that the total deviation is not affected. Prismatic reflectors have the further advantage over metallic mirrors, that the faces do not tarnish, so that no repolishing is necessary.

Exercises on Chapter XLVII.

1. Describe some form of telescope and explain its action, illustrating your answer with a diagram.

2. Describe the optical arrangements of the eye.
A short-sighted person sees distinctly at a distance of $4\frac{1}{2}$ inches. What kind of lens is necessary to enable him to see clearly at a distance of 10 inches and what must be its focal length ?

3. State the meaning of the magnifying power of a telescope.
Draw a diagram showing how a convex lens and a concave lens may be arranged to form a telescope.

4. Describe the use of (*a*) a microscope, (*b*) a telescope. With a telescope the final image is smaller than the object. Explain how this is reconcileable with the use of the telescope.

5. What do you mean by the magnifying power of a telescope ?
Two convex lenses, of 6 cm. and 12 cm. focal length respectively, are combined to form a telescope. Draw a diagram showing the paths of the rays by which the image of a distant object is formed. Calculate the magnifying power.

6. What is accommodation and how is it effected ?
What spectacles are required by a person, whose minimum distance of distinct vision is 8 ft., to enable him to read a book at a distance of 18 inches ? Sen. Camb. Loc.

7. Give a diagram illustrating the arrangement of two converging lenses to form a compound microscope, showing the paths of some of the rays which form the image. Upon what does the magnifying power of the arrangement depend ? L.U.

8. Compare and contrast the optical arrangements of the human eye and those of the photographic camera.
A short-sighted person can only see objects distinctly if they lie between 8 cm. and 100 cm. from the eye. What spectacles would he require in order to see a star distinctly ? With these spectacles what would be the least distance of distinct vision ? L.U.

9. Explain, with the aid of a diagram, the formation of an image by means of a compound microscope comprising two single lenses. If the observer sees a distinct image at a distance of 25 cm., find the position at which the object must be placed, the focal lengths of the lenses being 5 cm. and 1 cm., and the distance between them being 20 cm. Calculate also the magnifying power. L.U.

10. Calculate the focal length of a lens which would produce an image 6 feet square of a lantern slide 3 inches square, the screen on which the

image is to be formed being 20 feet from the lantern. Show by means of a diagram how the slide would have to be placed to give an erect image on the screen. L.U.

11. A person whose nearest distance of distinct vision is 15 cm. uses a lens of 5 cm. focal length to magnify a small object. What is the distance of the object when in focus, and what magnification is obtained? L.U.

12. When does total reflexion of light occur? Give examples of the use of total reflexion in the construction of optical instruments. L.U.

13. A convergent lens of focal length 30 cm. and a divergent lens of focal length 5 cm. are used to form a Galilean telescope. Draw the arrangement, showing two of the rays which go to form the final image. What is the magnifying power of the instrument? L.U.

14. You are given two convex lenses of focal lengths 16 cm. and 4 cm. respectively. How will you arrange the lenses to form a telescope?
Give a diagram to show the paths of the rays of light through the arrangement, and determine its magnifying power. The Presidency College.

15. In a simple form of astronomical telescope the focal length of the object glass is 30 inches, that of the eye-piece is 2 inches. Calculate the magnifying power when the final image of a distant object is seen (i) a long way off, (ii) at a distance of 12 inches.
Find the distance between the two lenses in each case. Give illustrative sketches to show the formation of the images. Bombay University.

16. Describe the eye as an optical instrument and compare its action with that of the photographic camera.
What lens would be required to enable an eye that cannot focus objects nearer than six feet to read a book at ten inches distance? L.U.

17. Describe diagrammatically the two commonest defects of vision.
A long-sighted person whose nearest distance of distinct vision is 50 cm. finds that this distance is reduced to 20 cm. by using spectacles. Find the nature and focal length of the lenses used. L.U.

CHAPTER XLVIII

PRISMS AND DISPERSION

Refraction by prism.—In the case of a piece of plate-glass with parallel faces, a ray passing through it is undeviated (p. 576). This, however, is no longer the case when the faces are not parallel. The path of a ray through such a piece of glass, which is, for optical

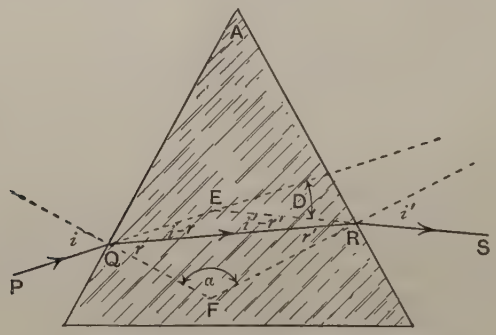

FIG. 576.— Deviation produced by a prism.

purposes, called a **prism**, must be found from a knowledge of the refractive index of the material. Thus for a ray PQRS (Fig. 576) $\sin i/\sin r = \mu$ and $\sin i'/\sin r' = \mu$. The emergent ray RS is not parallel to the incident ray PQ, since at each surface there has been a bending towards the base of the prism. The angle D between the incident and emergent directions is the **deviation** produced by the prism.

It will be seen from Fig. 576 that at the first surface, the amount of bending of the ray is $(i-r)$, and at the second surface $(i'-r')$. Hence the total bending is $(i-r)+(i'-r')$,

or $$D = (i+i') - (r+r'). \quad \text{.....(1)}$$

Again, since the sum of the four angles of a quadrilateral is equal to 360°, the four angles of the quadrilateral AQFR are together 360°. But ∠AQF and ∠ARF are each right angles, and therefore $A+\alpha=180°$. Also the three angles of a triangle are together equal to 180°, so that $r+r'+\alpha=180°$. It follows that

$$A=r+r'. \quad \text{.....(2)}$$

This, together with the equation (1) gives

$$A+D=i+i'. \quad \text{.....(3)}$$

The deviation produced by a prism may be demonstrated by means of the **optical disc** (p. 576), by replacing the semicircular glass plate in Fig. 522 by a prism.

EXPT. 140.—**Deviation by a prism.** Place a prism upon a sheet of drawing paper and fix two upright pins at E and F (Fig. 577). On looking through

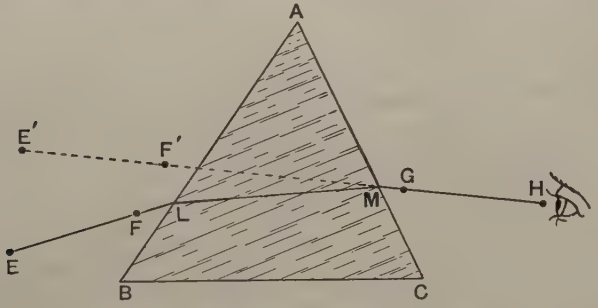

FIG. 577.— Experiment on deviation by a prism.

the face AC, the pins will be seen at E′ and F′. Two other pins are then placed at G and H, in a line with E′ and F′. Draw the outline of the prism on the paper, then remove and draw the lines EF and HG. These cut AB and AC at L and M. LM is then the path of the ray through the prism. Draw circles with centres at L and M, and also normals; find the refractive index by means of the construction given on p. 575. Repeat the experiment, with a different angle of incidence, and again with another prism of different angle.

Minimum deviation.—Either by means of the optical disc, or by means of the prism and pins in Expt. 140, it may be shown that a deviation of less than a certain amount cannot be obtained. Starting with a great deviation and slowly rotating the prism, it will be seen that for a time the deviation gets less. But on continuing the rotation the deviation will, after a time, get greater

again. The smallest deviation that can be obtained with any given prism is called the **minimum deviation.**

The position for which the deviation is a minimum is that for which the ray passes symmetrically through the prism, that is, $i = i'$ and $r = r'$ in Fig. 576.

Thus, in this case, the equations (3) and (2) (p. 623) reduce to $i = \frac{1}{2}(A + D)$ and $r = \frac{1}{2}A$, and since $\mu = \sin i / \sin r$, we have

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}, \quad \text{.....(4)}$$

This equation gives rise to the best of all methods for finding refractive index, since the angle of the prism, and the minimum deviation, can be measured with great precision by means of the spectrometer, as we shall see on p. 625.

A consideration of the principle of reversibility of the rays leads us to the conclusion that when the deviation is a minimum, then $i = i'$ and $r = r'$. Consider a ray such as EFGH (Fig. 578) passing unsymmetrically through the prism, the entrant angle of incidence being i. Then a ray HGFE in the reverse direction would undergo the same deviation, although its entrant angle of incidence i' is not equal to i. The deviation undergone by this ray cannot be a **minimum** since it occurs for **two** different values of i, and there is only **one** position of minimum deviation. Hence the only position for which the deviation is a minimum is that for which $i = i'$, the ray in this case passing symmetrically through the prism.

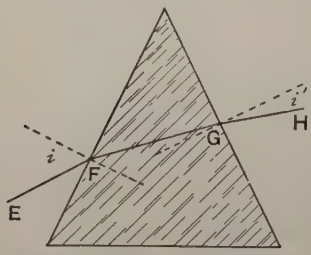

FIG. 578.—Deviation produced by a prism.

EXPT. 141.—**Minimum deviation.** Place a prism and two pins E and F as shown in Fig. 577. Rotate the prism so that the images E′ and F′ approach as near as possible to the positions E and F. In this position the deviation is a minimum. Then place the pins at G and H in line with E′ and F′. Draw in the prism and the rays as before and measure the angle D between EF and GH, and also A, the angle of the prism, and from equation (4) above, calculate μ.

Limiting angles of prism.—When the prism is very thin, that is, the angle A is small, the deviation A is small, as are also i and r.

Then for the position of minimum deviation

$$\mu = \frac{i}{r} = \frac{\frac{1}{2}(A+D)}{\frac{1}{2}A},$$

and $$A+D = \mu A, \quad \text{or} \quad D = (\mu - 1)A.$$

Of course when $A = 0$, D is also zero, in which case the prism resolves itself into a piece of parallel plate-glass, for which we have already seen that there is no deviation (p. 576).

On the other hand, there is a certain angle of prism for every substance, for greater values than which no ray can traverse the prism. For, consider a ray making an entrant angle of incidence very nearly 90° as EF in Fig. 579. If the angle of the prism is such that ∠ FGP is the critical angle for the substance, the ray will emerge along GH. For any angle of prism greater than this, ∠ FGP would be greater than the critical angle, and the ray would be totally reflected. Also, a ray such as E′F′G′H′ would be totally reflected at G′. For this limiting angle of prism, EFGH is evidently the position for minimum deviation, so that $r = A/2$ (p. 624), also $\sin r = 1/\mu$, so that for glass for which $\mu = 1{\cdot}5$, $r = 41° 50'$, and A will be 83° 40′. For angles greater than this, no ray can pass through the prism.

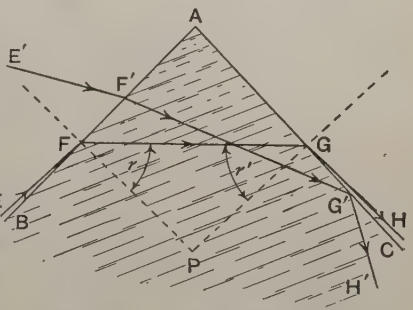

FIG. 579.—Limiting angle of prism.

Spectrometer.—For the measurement of refractive index, the angles of deviation and of the prism must be measured with great accuracy. For this purpose the **spectrometer** has been devised. The prism rests upon a table T (Fig. 580) which is provided with three levelling screws, and can be rotated about the axis O of the instrument. The circular scale S′ and the vernier V′ attached to the table enable its position to be observed. A slit A is illuminated by a source of light (usually a sodium burner) and is situated at the principal focus of a lens L, so that the rays emerge from L in the form of a parallel beam. A and L are fixed in a tube, the whole being called the **collimator**. The parallel beam falls on the prism, is deviated, and then reaches the objective M of the telescope, being brought to a focus at the cross wires N, suitably placed with respect

to the eyepiece E, so that the observer sees an image of the slit in the plane of the cross wires. The telescope is carried by an arm

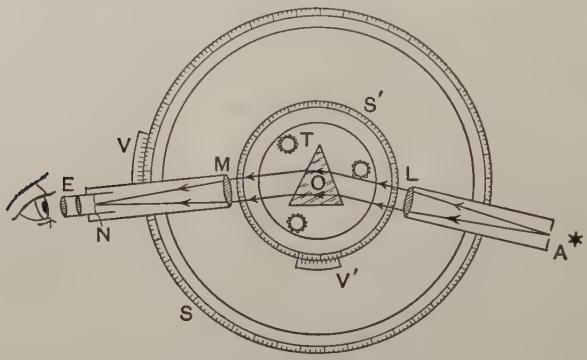

FIG. 580.—Plan of the spectrometer.

so that it can be rotated about the axis O of the spectrometer. Its position with respect to the scale S can be observed by means of the vernier V.

EXPT. 142.—**Determination of the angle of a prism by means of the spectrometer.** Focus the cross-wires by drawing the eyepiece of the telescope in or out. Then focus the telescope on infinity, or on some distant

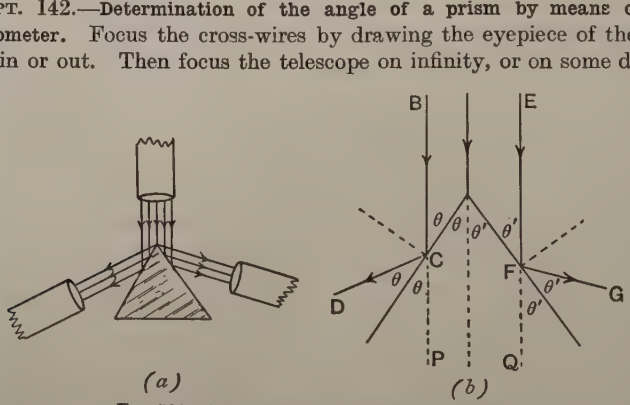

FIG. 581.—Measurement of the angle of a prism.

object. Next turn it to face the collimator, the slit being illuminated by a bunsen burner, in which some common salt is held on a piece of iron gauze or asbestos board. Draw the slit in or out until its image in the eyepiece is focussed. The collimator and telescope are then both in focus for parallel rays. There is a better method of performing this focussing for parallel rays, but this must be left to the more advanced student.

Place the prism on the table so that the parallel beam from the collimator falls on the angle to be measured. Part of the beam is reflected to the left and the remainder to the right, and in each case the telescope is turned until the image of the slit as seen by reflected light coincides with the cross-wire (Fig. 581 (*a*)). The venier is then read. The angle between these two positions of the telescope is twice the angle of the prism. The reason for this is seen on examining Fig. 581 (*b*); for $\angle DCP = 2\theta$, and $\angle GFQ = 2\theta'$. But, $\theta + \theta'$ is the angle of the prism, and CP is parallel to FQ, so that the angle between DC and FG is twice the angle of the prism.

There is an alternative method of measuring the angle of the prism, which is more advantageous when the angle is large. The image produced by reflection at the face AB (Fig. 582) is obtained as in the previous case. Keeping the telescope fixed, the table carrying the prism is rotated until the face AC comes into a position parallel to that previously occupied by AB, when the image seen by light reflected by AC will be obtained. The angle θ, through which the prism has been rotated, will be seen to be the supplement of the angle of the prism:

$$A = 180° - \theta.$$

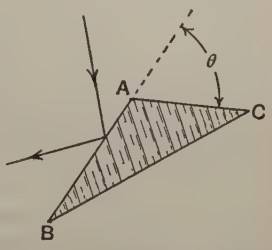

FIG. 582.—Alternative method for measuring angle of prism.

This is the principle of the **goniometer**, which is an instrument for measuring the angles of crystals. The design of the instrument differs from that of the spectrometer on account of its different use, but the principle is as described above. Measure the three angles of a prism by both these methods and show that in each case their sum is 180°.

EXPT. 143.—**Determination of minimum deviation.** With the spectrometer set up as in the last experiment, obtain an image of the slit by means of light which has passed through the prism. Rotate the table carrying the prism until the deviation is a minimum. Bring the telescope to such a position that the image of the slit is on the cross-wire, and read the vernier. Remove the prism and take the direct reading with the telescope facing the collimator. The difference between these two readings is the angle of minimum deviation. Knowing the angle of the prism from the previous experiment, calculate μ from equation (4) (p. 624).

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}.$$

EXPT. 144.—**Determination of the refractive index of a liquid.** Place the liquid in a bottle having two plane faces inclined to each other. Find the angle between the plane faces by the method of Expt. 142. Find the minimum deviation by the method of Expt. 143, and calculate μ. Empty

the bottle and make sure that the faces themselves do not produce any deviation.

Dispersion.—On allowing a narrow beam of light to fall on a prism, the emergent beam will not be deviated merely, but will, in general, exhibit a variety of colours.

The screen B (Fig. 583) placed in a beam of sunlight or in front of an electric arc, allows a narrow beam of white light to fall upon the

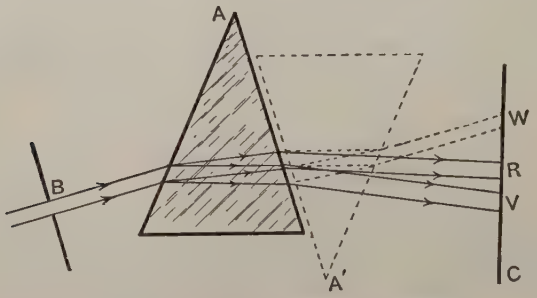

FIG. 583.—Dispersion produced by a prism.

prism A. On placing the white screen C to receive the emergent beam, it will be seen that this is spread out as well as deviated, giving a band of colour, red at R, passing through yellow, green, and blue to violet at V. This band of colour is called a **spectrum**, and the phenomenon of the splitting up of light in this manner is called **dispersion.**

If by any means the differently coloured beams, into which the original white beam is decomposed, be recombined, they again form a white beam. Thus, if a second prism A′ with its vertex to the base of A (Fig. 583) be placed to receive the emergent beams, a white spot is produced at W, because of the recombination of the differently coloured beams into a white beam. This result makes it appear that the original white light consists of a mixture of an infinite number of differently coloured portions, and that these have different refractive indices and are hence separated in passing through the prism. Thus the glass has the greatest index of refraction for the violet light and the least for the red. The beam of violet light is therefore bent most, on passing through the prism, and the red least. Between these extremes there is an infinite gradation of refractive index, each variety of tint having its own refractive index for each refracting medium.

If a narrow slit be made in the screen C, and the light of any one colour be allowed to pass through, a second prism will not analyse this into fresh colours, but will spread out the beam to a small extent.

Pure spectrum.—The arrangement of Fig. 583 is very crude, the different beams giving coloured patches of light on the screen, with considerable overlapping. The spectrum is said to be **impure.** A great improvement may be effected by the arrangement shown in Fig. 584.

A bright source of light illuminates the slit S, and a lens L is so situated that it would, in the absence of the prism, produce a real image of the slit at I. On interposing the prism at A, a number of images of the slit, R to V, are produced on the screen, on account of

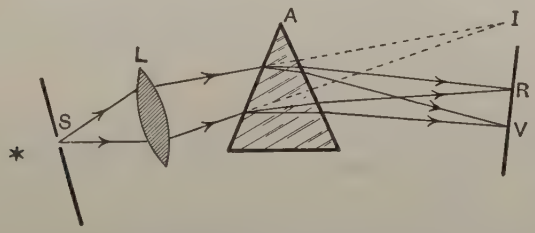

FIG. 584.—Arrangement for producing coloured images of a slit.

the varying refractive index of the differently coloured beams. If there is no overlapping of the variously coloured images the spectrum is said to be a **pure spectrum.** The pure spectrum is an ideal which can never be attained, owing to the infinite gradation in refractive index in the light of various colours, but for all practical purposes a close approximation to purity may be obtained when the optical arrangement is sufficiently good.

There is a defect in the arrangement shown in Fig. 584 owing to the fact that the rays from different parts of the lens make different angles of incidence at the prism and are therefore differently deviated, the focussing being thereby spoilt. This difficulty may be avoided by the arrangement shown in Fig. 585. The slit S is at the principal focus of the lens L. Hence the rays falling on the prism A are parallel, and each coloured beam emerges as a parallel beam, and is brought to a focus by means of the lens M, at some point of the screen RV. By using a narrow slit, a good spectrum may be obtained. It will be noticed that this is exactly the arrangement of the spectrometer (Fig. 580), with the exception that the real images fall upon a screen instead of being observed by means of the eyepiece E. By rotating the prism into the minimum deviation position for any particularly

coloured ray, the index of refraction for this ray may be found. Hence the use of the sodium burner flame in Expt. 143 (p. 627). The

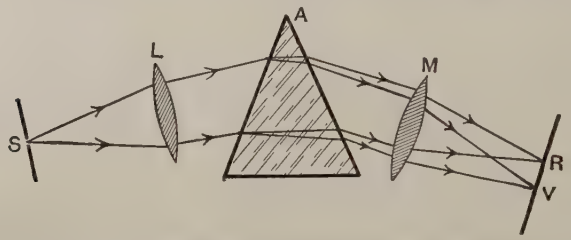

FIG. 585.—Arrangement for producing a pure spectrum.

spectrum of this light from the sodium flame will be seen to consist of one yellow line only, unless the dispersion is very great, and therefore it is commonly used in optical work where homogeneous light is required.

EXPT. 145.—**Refractive index for different colours.** Use a source of white light such as an incandescent lamp, and determine the refractive index of a prism for the extreme red end of the spectrum and also for the extreme violet end. As the spectrum fades off gradually at both ends, these values are not of a definite nature, and different observers would find slightly different values, but they will indicate in a general manner the variation of refractive index in the spectrum.

Dispersive power.—For the purposes of measurement, a particular red part of the spectrum and a particular blue are chosen, the corresponding refractive indices being called μ_r and μ_b. The choice of colour will be discussed in Chap. XLIX.

For a thin prism we have seen (p. 625) that the deviation is given by the relation, $\mathsf{D} = (\mu - 1)\mathsf{A}$, where A is the angle of the prism. Hence the deviation for the blue ray is $\mathsf{D}_b = (\mu_b - 1)\mathsf{A}$, and for the red ray, $\mathsf{D}_r = (\mu_r - 1)\mathsf{A}$; therefore, $\mathsf{D}_b - \mathsf{D}_r = (\mu_b - \mu_r)\mathsf{A}$. The quantity $\mathsf{D}_b - \mathsf{D}_r$ is the angle between the blue and red rays on emergence from the prism, and measures the **dispersion** produced. If the average $\frac{1}{2}(\mu_b + \mu_r)$ be taken as μ, we have for the deviation of this mean ray,

$$\mathsf{D} = (\mu - 1)\mathsf{A},$$

and

$$\mathsf{D}_b - \mathsf{D}_r = \frac{\mu_b - \mu_r}{\mu - 1}(\mu - 1)\mathsf{A}$$

$$= \frac{\mu_b - \mu_r}{\mu - 1}\mathsf{D}.$$

The quantity $\frac{\mu_b - \mu_r}{\mu - 1}$ is called the **dispersive power** (ω) of the material of which the prism is made.

Hence, $$D_b - D_r = \omega D.$$

This equation applies to thin prisms only.

Achromatic prism.—It is possible to construct a prism which shall produce deviation without dispersion, by constructing it of

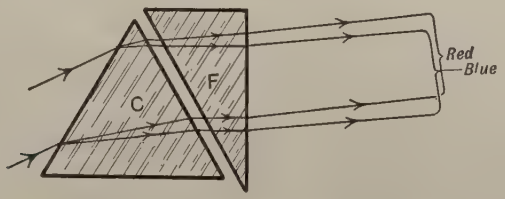

FIG. 586.—Achromatic combination of prisms.

two parts made of different kinds of glass. If the prism C (Fig. 586) consist of **crown glass** and have an angle of 60°, and the prism F of **flint glass**, with angle 29° 17′, the dispersion produced by each is found to be the same, since the dispersive power of the flint glass is much greater than that of the crown glass. Hence, as the prisms are placed so that the dispersions they produce are in opposite directions, the total dispersion is zero, and all the variously coloured rays emerge parallel to each other. But the refractive index of the flint glass is only slightly greater than that of the crown glass, so that the former cannot undo the deviation produced by the latter. Thus the combination produces deviation, but not dispersion.

The emergent beam is thus white throughout its central part; it will, however, be coloured at the edges, since the displacements of the red and blue beams are not quite the same.

Direct-vision spectroscope.—If the angle of the flint glass prism in Fig. 586 be imagined to increase until the flint glass produces equal and opposite deviation to that produced by the crown glass prism, it will, of course, now produce a greater dispersion. The emergent beam will then be dispersed, but its mean position will be parallel to that of the incident beam. Thus we get dispersion without deviation.

This principle is employed in the construction of the **direct vision or pocket spectroscope.** A train of prisms consisting alternately of crown glass, C (Fig. 587) and flint glass, F, produce dispersion, but the resultant deviation is zero. Light enters by the slit S, and

the lens E produces a virtual image of the slit. When white light is used, a spectrum R–B is seen, consisting of the variously coloured images of the slit.

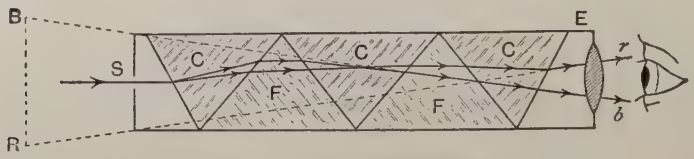

FIG. 587.—Direct vision spectroscope.

Chromatic dispersion.—From the above considerations we should expect that the images produced by lenses would not be free from the effects of dispersion. If the source S (Fig. 588) emits white light, the blue rays will come to a focus at B and the red rays at R, the glass having a greater refractive index for the blue rays than for the red. The other colours will have foci at intermediate points.

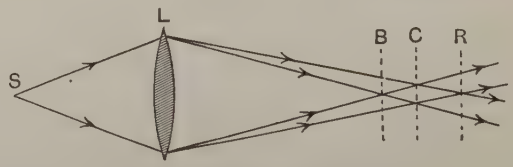

FIG. 588.—Dispersion produced by a lens.

Thus a screen placed at B will exhibit an image with a red border and at R an image with a blue border. There is some point C for which the image is less coloured than at others, but it is obvious that there is no position for which a point focus for all the rays can be found. Hence, for optical instruments, single lenses can never produce sharp images when white light is used, as there is no material known which has zero dispersive power.

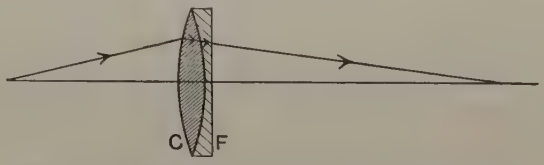

FIG. 589.—Two lenses forming an achromatic combination.

Achromatic lens.—A device somewhat similar to that used for prisms (p. 631) may be adopted for lenses, in order to correct for chromatic dispersion. Thus it is possible to choose a combination

of a converging crown glass lens C (Fig. 589) and a diverging flint glass lens F, which shall be converging as a whole, but such that the dispersion due to F corrects that due to C.

Consider equation

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right),$$

which applies for each separate value of μ. Then for any lens where f_b, f_r, μ_b, μ_r are the corresponding focal lengths and refractive indices for the blue and red rays respectively, and f and μ their mean values, we have

$$\frac{1}{f_b}=(\mu_b-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right), \quad \text{and} \quad \frac{1}{f_r}=(\mu_r-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right);$$

$$\therefore \frac{1}{f_b}-\frac{1}{f_r}=(\mu_b-\mu_r)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$=\frac{\mu_b-\mu_r}{\mu-1}(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$=\frac{\omega}{f},$$

since $$\frac{\mu_b-\mu_r}{\mu-1}=\omega \quad \text{(p. 631.)}$$

Now the focal length, F, of two thin lenses in contact, is given by $1/\mathrm{F}=1/f_1+1/f_2$ (p. 599), and for the separate lenses,

$$\frac{1}{f_{1b}}-\frac{1}{f_{1r}}=\frac{\omega_1}{f_1}, \quad \text{and} \quad \frac{1}{f_{2b}}-\frac{1}{f_{2r}}=\frac{\omega_2}{f_2};$$

$$\therefore \frac{1}{f_{1b}}+\frac{1}{f_{2b}}-\frac{1}{f_{1r}}-\frac{1}{f_{2r}}=\frac{\omega_1}{f_1}+\frac{\omega_1}{f_2},$$

or

$$\frac{1}{\mathrm{F}_b}-\frac{1}{\mathrm{F}_r}=\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}.$$

Now if the combination of the two lenses is to bring all the variously coloured rays to the same focus, it must have the same focal length for all the rays; thus, $\mathrm{F}_b=\mathrm{F}_r$;

$$\therefore \frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0,$$

or

$$\frac{f_1}{f_2}=-\frac{\omega_1}{\omega_2}.$$

From the negative sign we see that if one is a converging lens the other must be a diverging lens. Further, the focal lengths must be in the direct ratio of the dispersive powers of the materials. Taking the dispersive powers of crown and flint

glass as 0·21 and 0·45 respectively, we see that the diverging lens F must have $0{\cdot}45/0{\cdot}21 = 2{\cdot}1$ times the focal length of the converging lens C.

With lenses not in contact, achromatic arrangements can be obtained with glass of one kind; thus the Huygens eyepiece (Fig. 564) and the Ramsden eyepiece (Fig. 567) are both achromatic, but the treatment of such combinations is beyond the scope of the present work.

Exercises on Chapter XLVIII.

1. How would you arrange apparatus in order to procure a well-defined spectrum upon a screen? Upon what fact does this resolution of white light into various colours depend? L.U.

2. Describe some method of measuring the angle of a prism, proving any relation you use.

3. Show that a ray undergoes minimum deviation by a prism when it makes equal angles with the faces on entrance and emergence.

4. Give an expression for the amount of deviation which a beam of light undergoes in passing through a prism of small angle, and find a value for the angle of dispersion produced in such a case.

5. Prove that no ray of light can pass through a prism if its angle is greater than a certain amount. What is the relation of this limiting angle of prism to the refractive index of the material?

6. Describe how the refractive index of a liquid may be determined accurately.

7. Define 'dispersive power,' and find the angle of a prism of dispersive power 0·42 and refractive index 1·6, which will render achromatic a prism of 5° angle of material having index of refraction 1·5 and dispersive power 0·21.

8. Explain the terms 'deviation' and 'dispersion' as applied to the path of light through a prism.

A glass prism, with a very small refracting angle α, is arranged so that an incident ray of light falls normally on one of its faces. If the refractive index of the glass is equal to μ, find an expression for the deviation of the ray that emerges from the prism. L.U.

9. What is meant by the minimum deviation of a prism? Establish the relation between the angle of minimum deviation, the angle of the prism, and the index of refraction. L.U.

10. Prove the relation connecting the angle of a prism, the deviation of light which has passed symmetrically through it, and the index of refraction. L.U.

11. Prove that for a definite angle of incidence the deviation produced by a prism is a minimum. How would you determine the refractive index of the material of the prism? Allahabad University.

12. Discuss the conditions of achromatism of the combination of lenses used as the object glass of a telescope. Calcutta University.

13. Calculate the focal length of the lens constructed of flint glass of dispersive power 0·45 which will render achromatic a converging lens of crown glass of focal length 75 cm. and dispersive power 0·21. Also find the focal length of the combination.

14. An achromatic objective glass of focal length 150 cm. is to be constructed of crown glass ($\mu = 1{\cdot}51$, $\omega = 0{\cdot}21$) and of flint glass ($\mu = 1{\cdot}65$, $\omega = 0{\cdot}45$). Given that the component lenses are to have one face of each in complete contact, and that one face of the flint glass lens is to be plane, calculate the curvatures of the surfaces.

15. When using a single lens as a magnifying glass the image is frequently found to have coloured edges. Account for this and explain how this colour effect may be diminished by using suitable converging and diverging lenses in combination. L.U.

16. What is meant by *dispersion* of light ?
Explain how the various kinds of light obtained in a spectrum can be re-compounded to form white light. L.U.

17. Describe the essential parts of a spectroscope, and give some account of its uses. L.U.

CHAPTER XLIX

COLOUR

The spectrum.—For the study of colour a large and brilliant spectrum should be available. A single glass prism does not produce sufficient dispersion to form a long spectrum. Hence a train of prisms is sometimes used, as in Fig. 590, or a prism of highly dispersive material such as carbon bisulphide contained in a prismatic bottle. The extreme end of the visible spectrum is a dull red colour, gradually changing to brighter red, then orange, yellow, green, green-blue, blue and violet, the intensity then falling off gradually. The greatest visible intensity is in the yellow.

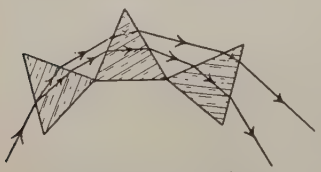

FIG. 590.—Train of prisms.

Since light and radiant heat have the same character, it is important to compare the heating effects of the different parts of the spectrum. This may be done roughly by passing the bulb of a delicate therometer along the spectrum and noting the effect, but it is much better to place a thermopile (p. 381) in different parts and note the galvanometer deflections.

These deflections are proportional to the rate at which radiant energy falls on the thermopile. It will be observed that the greatest heating effect is in the dark space just beyond the red end of the spectrum. This is called the **infra-red** portion of the spectrum. It should be remembered that with ordinary apparatus, the glass lenses, prism, etc., absorb a considerable portion of the radiant heat falling on them. Thus for research on the heating effect, rock-salt prisms and lenses must be used, as these are highly diathermanous (p. 386).

There is still another way of detecting light, namely, by its photographic effect. If the spectrum fall on a sensitive photographic plate, which is subsequently developed, the intensity of the photographic image is greatest at places beyond the violet end of the

spectrum. This is called the **ultra-violet** region, and the rays here are sometimes called **actinic rays**, on account of their photographic activity. They do not differ essentially in nature from the other rays, and the special name is hardly necessary.

Thus the whole spectrum is enormously greater than the visible part, the latter being merely that part to which the eye is sensitive. It will be seen in Chapter L. that light consists of undulations. The different parts of the spectrum consists of rays of different frequencies. Thus the waves constituting light have, in air, a wave-length of 0·0000589 cm. for the light of the sodium burner, 0·00004 cm. for the extreme end of the violet, and 0·00008 cm. for the extreme red. The longest infra-red rays known have a wave-length of about 0·01 cm. and the shortest known actinic rays 0·00001 cm.

EXPT. 146.—**Heating effect in the spectrum.** Place a thermopile in various parts of the spectrum produced by a carbon bisulphide prism, and tabulate the galvanometer deflections observed for different positions of the thermopile.

EXPT. 147.—**Photographic effect in the spectrum.** Place a piece of 'bromide' paper so that the spectrum falls upon it, leaving a length of the paper beyond the violet end. The room must, of course, be darkened. After an exposure of about a minute, when an electric arc is used as a source of light, remove the bromide paper, develop and fix it, and then replace it in its old position. Note that the blackening begins in the violet part of the spectrum and is continued far into the ultra-violet region.

Colours of bodies.—**Opaque bodies** owe their colour to the fact that they do not reflect all the rays composing white light to the same extent. Such a body appears red when it reflects red rays copiously, but does not reflect green and blue. Also, if the body reflects red and green, but absorbs blue light, it appears orange coloured. This may easily be demonstrated by placing a piece of red paper, or cloth, in different parts of the spectrum. In the red part, the cloth will probably appear its natural colour, but in the green and blue regions of the spectrum it will appear black, showing that its ordinary appearance is due to the fact that it reflects red rays copiously but does not reflect rays of any other colour. Similar tests may be made with green and blue bodies; it should be noted, however, that the blue dyes and pigments in general use are not pure colours, and the blue body will probably not appear black in any part of the spectrum. But in every case, if a body does not appear black, it will have the colour of the part of the spectrum in which it is placed.

Many **transparent bodies** are coloured. Thus a sheet of ruby glass only transmits red rays, and if a sheet of it be placed anywhere

in the path of the light forming the spectrum, only the red rays are transmitted, and the rest of the spectrum will be entirely cut off. Similarly, a good piece of green glass will cut off all but the green region of the spectrum. If now the red and green glasses be put together they will cut off all the light of the spectrum, for the green glass absorbs all but the green rays and the red glass all but the red rays.

Theory of colour vision.—From the foregoing experiments it will be seen that the rays of different colours affect the eye differently, and it is to the sensations produced, that the names of the colours properly belong. Nevertheless, it is convenient to speak of the rays which produce the sensation of redness as ' red ' rays, although as we saw (p. 637) that the only difference between these and others is one of frequency. If the two kinds of ray enter the eye at the

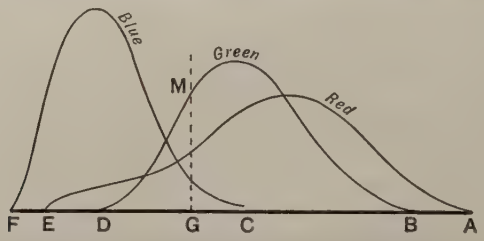

FIG. 591.—Intensities of primary colour sensations.

same time, the eye perceives a mixed sensation. When red and blue rays are mixed, the sensation is that of the colour we call crimson.

Many theories of colour vision have been propounded, but that which has had the widest acceptance and appears to account best for a variety of effects, is that of Young, afterwards further developed by Helmholtz, and known as the **Young-Helmholtz theory.** According to this theory there are three primary colour sensations, namely, a particular red, green, and blue, and all others are made up of mixtures of these. A curve of intensity of these three colour sensations has been constructed. This is given in Fig. 591.

It will be seen that for the spectrum of white light, the red curve extends from A to E, the green from B to D, and the blue from C to F. From A to B the sensation is pure red and this part of the spectrum has the primary red colour. From E to F the sensation is the pure blue, or rather violet. But from B to E all the colour sensations are mixed. The pure green is not seen anywhere in the spectrum, but the nearest approach to it is at G, where the mixture consists of red, green, and blue in the proportions in which they

occur in white light, with a considerable amount of green effect in excess.

Whether there are three sets of nerves in the retina, sensitive to the red, green and blue respectively, or whether the three sensations belong to three properties of the same nerve has not been satisfactorily settled. The phenomenon of **colour-blindness**, however, seems to point to the first conclusion. For many people are partially, or wholly, blind to one of these primary colour sensations, and in rare cases to two. Thus a red-blind person would have the 'red' curve from A to E (Fig. 591) missing, and his colour sensations would be due to mixtures of the remaining green and blue.

Complementary colours.—The mixture of all the coloured rays occurring in sunlight we call 'white.' If any colour, or colours,

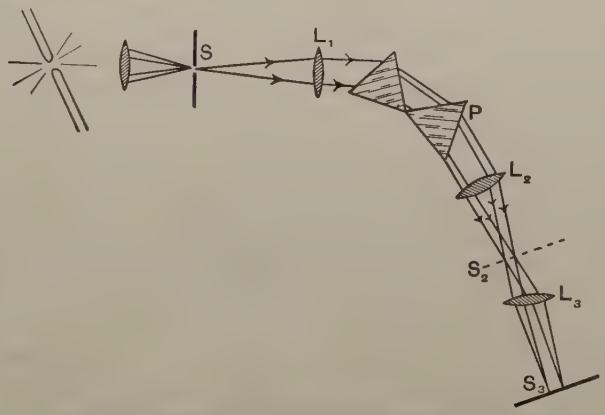

FIG. 592.—Apparatus for studying complementary colours.

be abstracted, the remaining mixture is no longer white. The two sets, however, if combined, will again form white light. Two colours are said to be **complementary** when on adding them together the result is white.

There are various methods of combining differently coloured beams, and the following, due to Sir Wm. Abney, is one of the best. Light from an electric arc is converged upon the slit S by the lens (Fig. 592). S is at the principal focus of the lens L and a parallel beam is therefore produced. The train of prisms produces dispersion, and the lens L_2 forms a spectrum at S_2 in the ordinary way. If a screen be placed at S_2 the spectrum may be seen. A lens placed at L_3 will now recombine the colours, forming a white patch upon the screen S_3. L_3 is so situated that it forms an image of the face P of the prism upon the screen S_3, the differently coloured images

of P coinciding on the screen and giving the white patch. By placing strips of metal or cardboard at S_2, parallel to the slit, any colour or colours may be abstracted from the spectrum, and the patch at S_3 will be a mixture of those remaining.

Obviously the colour mixture abstracted is complementary to that remaining, for the two together make up white light, and by suitably placing the strips to obstruct the light at S_2, colours that are complementary may be determined. The complementary colour to red is a bluish-green, and to yellow a particular blue.

A further use of this apparatus is to find the component colours of any given tint. For on placing the material of given colour at S_3 beside the patch of light, obstacles may be adjusted at S_2 until the patch matches the given colour, whence its component colours are known. It is not always possible to match the colour by mixing pure spectrum colours, without adding a certain amount of white light.

There is another method of finding approximately the complementary colour to a given colour in certain cases, by making use of the fatigue of the eye. Thus, if a bright red object be looked at steadily for about a minute, the retina becomes fatigued to this colour. On then looking at a white sheet of paper, the paper appears a bluish-green, owing to the fact that the eye is not fatigued for the green and blue rays, and therefore sees them more vividly than the red. If the object gazed at be yellow, the colour seen on looking at the white sheet is blue. It is very difficult to fatigue the retina to green, this being the most restful colour for the eye.

Pigments.—When differently coloured rays are received by the eye at the same time, we have seen above that a mixed sensation results. This must not be confused with the mixing of variously coloured pigments or paints. As an example, take the particular yellow and blue rays which are complementary, so that when mixed the resultant effect is white. If two similar yellow and blue pigments be mixed, the resulting mixture is green. The reason for this is that the yellow pigment absorbs the red and blue rays from white light, and the blue pigment absorbs red and yellow. Both, however, reflect the green, and this is the only colour reflected by the mixture, and is therefore its characteristic colour.

Persistence of vision.—The persistence of vision is sometimes employed for the mixing of colour effects. That the sensation produced by an image falling upon the retina does not disappear instantly when the light is cut off, is well exemplified in the cinematograph. Successive images thrown upon a screen, if they succeed each other sufficiently rapidly, produce a continuous impression.

In practice each picture remains stationary upon the screen about $\frac{1}{3\frac{1}{5}}$ second, and is then replaced by another, 16 pictures per second falling on the screen. It is well known that the eye receives a continuous impression.

Colour-top.—In the colour-top a number of coloured discs of paper or cardboard are placed on an axle. The discs are each slit along a radius so that they may overlap, and various amounts may be exposed. On rotating them rapidly the effect of each upon the eye persists, while the others are passing, so that the eye cannot perceive the separate sectors, and the sensation produced is that of a mixture of the separate colours. A match for any given colour can be produced by varying the angles of the sectors; further, a given mixture of red, green, and blue will give a grey colour when the discs are rotated. A good white can never be obtained. The difficulty is that the pigments are never pure colours, and each sector supplies only a fraction of the total illumination, whereas for a white disc each part of it supplies the full illumination.

Kinemacolor.—A further application of the persistence of colour impression has been made in the form of cinematograph known as the **kinemacolor**, for the projection of pictures in their natural colours. In taking the original photograph, a disc, having alternately red and green glasses, rotates in front of the lens of the camera, at such a rate that alternate negatives are taken with light that has passed through the red or the green glass. Thus, a red object would produce a dense part of the negative when taken through the red glass, and no effect when taken through the green. On producing a positive transparency, the dense parts of the negative give the transparent parts of the film for exhibition. A similar disc with red and green glasses is used when projecting the picture, and is so timed that the red glass is opposite the lens when the picture taken through the red glass is being shown. Hence the transparent parts will be bright red on the screen. A similar description applies to the pictures taken through the green glass, and the pictures alternate upon the screen so rapidly that the eye receives a mixed impression, and combines the colours in the pictures so that the natural colours appear. Reds, yellows, and greens are well reproduced, but since no blue pictures are used, the bluish-greens and blues are not so good.

Colour photographs.—Several modern processes of colour photography depend upon the use of coloured screens. A screen, having

a fine colour pattern upon it, is placed in contact with the photographic plate, and the exposure is made by light which has to pass through the colour pattern in order to reach the sensitive surface. To explain the action let us imagine the colour screen to be magnified and three of the coloured areas to be represented by R, G, and B (Fig. 593). R is red, G green, and B blue. The image of a green object falling on these three areas will be treated differently. The green light passes through G and reaches the plate, giving rise to a dark part of the negative, but is absorbed by R and B, so that the negative under these will be transparent. On producing a transparency from this negative by the ordinary process of contact printing, the part corresponding to G will now be transparent (owing to the density of the negative) while those corresponding to R and B will become opaque. Thus on covering the transparency with the original colour screen and holding it up to the light, G will appear bright green, but R and B will not be seen, and the original colour of the object is thus reproduced.

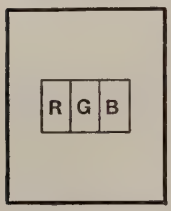

FIG. 593.—Colour screen.

In practice the whole colour screen is covered with a very fine pattern of these colour spaces, so that the eye cannot distinguish the separate spaces; but in the final transparency, with the colour screen correctly superimposed, each minute space transmits the amount of red, green, or blue light corresponding to the colour of the original object, and the eye sees the general colour effect. The difficulties are, to obtain the three correct colours in the original screen, and so to expose and develop the films that the proportionality of colour in the final transparency is correct.

Spectrum analysis.—On p. 630 it was seen that the light from the incandescent sodium vapour in a bunsen flame when common salt is placed in it, gave a spectrum consisting apparently of one yellow line only. Hence the characteristic yellow colour of the sodium flame. Actually there are two yellow lines very close together, but high resolving power is necessary for them to be seen as two separate lines. Every element in the gaseous state, when rendered luminous, has a characteristic spectrum, but it is not always easy to obtain the spectrum, and in very few cases is the spectrum so simple as that of sodium. Thus the spectrum of strontium vapour has many lines, but several in the red are particularly bright, and another in the blue, giving the characteristic crimson colour to the strontium flame. The hydrogen spectrum has

a number of lines, and the iron spectrum has such a vast number that the light emitted appears to the eye to be white. On the other hand the spectrum of thallium consists of a single green line. The lines produced by any element have characteristic places in the spectrum, and by observing the spectrum, the substance which gave rise to it may be identified. This process is known as **spectrum analysis.**

This analysis is carried out by means of the **spectroscope**, which is the same in principle as the spectrometer (p. 625), but the resolving power is usually greater. The telescope is usually fixed, and the eyepiece is often replaced by a photographic camera so that a photograph of the spectrum is obtained.

Continuous spectrum.—Bodies at ordinary temperatures do not emit radiations which affect the eye, but if the temperature of a solid body be gradually raised, at about 700° C. the body begins to be luminous, and appears a dull red colour. The spectrum of it will then be limited to a small piece at the extreme red end. As the temperature continues to rise, the spectrum will extend to the yellow, green, and eventually the blue, and at very high temperatures the whole of the visible spectrum will be present. The body then emits white light and is said to be ' white hot.' The spectrum of a white hot solid body has all the colours present and a **continuous spectrum** is presented. This is a contrast to the spectrum of a luminous gas or vapour, which consists of **lines, bands,** or **flutings.**

Absorption spectra.—It is a general rule that any vapour will absorb, at all temperatures, the particular kind of light which it itself emits at high temperatures. Thus if a beam of white light be allowed to pass through a bunsen flame impregnated with a sodium salt, the characteristic yellow light of sodium is absorbed from the beam of white light. The spectrum will then have the appearance of a continuous spectrum, with a dark line in the yellow corresponding to the sodium line.

If a spectrum of the light from an electric arc be examined by means of the spectrometer or spectroscope, and a sodium burner be placed between the arc and the slit, the dark line in the spectrum may be observed. For, since the flame is at a much lower temperature than the arc, it does not itself emit as much of the yellow light as it absorbs from the white light from the arc. On removing the arc and observing the spectrum of the sodium burner, a yellow line will be seen, and it will be observed that it has the position previously occupied by the dark absorption line.

A continuous spectrum, with certain characteristic lines absorbed from it by any substance, is called an **absorption spectrum.** Thus the absorption spectrum produced by any substance is complementary to its own bright line spectrum.

Solar spectrum.—The spectrum of sunlight has an immense number of absorption bands, called **Fraunhofer lines**, after their discoverer. These are produced in a manner somewhat similar to that just described. The visible part of the sun, called the **photosphere**, consists of luminous solid, or liquid, substance and emits white light. Surrounding this layer is the **chromosphere**, which is made up of gases and vapours at much lower temperature than the photosphere. Hence the light corresponding to the elements in the chromosphere is absorbed from the white light of the photosphere, giving rise to the Fraunhofer lines. Many elements, such as sodium, iron, hydrogen, etc., are by this means known to be present in the sun. Certain prominent lines are named A, B, C, D, E, F, G, and H. The D line is the yellow sodium line. Several of the sun's metals, such as caesium, rubidium, and thallium, have been discovered by their spectra, and helium was known in the sun's chromosphere long before it was found on the earth.

As evidence of the correctness of this explanation of the origin of the Fraunhofer lines, it may be mentioned that when the moon's disc just covers the photosphere in the event of a solar eclipse, the spectrum becomes reversed, the chromosphere then producing its own bright line spectrum.

Fluorescence.—Many substances possess the power of becoming luminous when light of certain wave-lengths falls upon them. This is not a case of reflection, for the light emitted is of a different wave-length from that which excites it. This phenomenon of absorbing one kind of light and in its place emitting another kind is called **fluorescence.** It was first noticed in fluorspar (calcium fluoride), but is strongly marked in many of the aniline dyes, such as eosin and fluorescine. Sulphate of quinine and paraffin oil also exhibit fluorescence.

If a parallel beam of white light be passed into a vessel containing a solution of fluorescine, the liquid will glow with a brilliant yellow-green light where the white beam enters, but the further path of the beam does not exhibit fluorescence. The reason for this is, that the particular rays in the white beam which excite fluorescence are absorbed by the solution, and this absorption takes place in the first parts of the solution traversed. Hence in the subsequent layers there is no exhibition of fluorescence.

On placing a tube of the solution in various parts of a bright spectrum, it will be found that the solution fluoresces when situated in the blue, violet, and ultra-violet, but not in the green and red. **The light emitted in fluorescence is nearly always nearer to the red end of the spectrum than the light which excites fluorescence.** If a flat cell containing the solution be placed in the path of the white beam which forms the spectrum, it will be found that the blue end of the spectrum is missing. **Hence the light which produces fluorescence is absorbed in doing so.** It is the energy of the light absorbed which supplies that necessary for the fluorescent radiation. Glass absorbs a considerable amount of the ultra-violet light from a beam of sunlight, or the light from the electric arc. Hence it is desirable in studying fluorescence to use quartz lenses and prisms, since quartz is much more transparent than glass to ultra-violet light.

Fluorspar fluoresces a pale blue, sulphate of quinine or paraffin oil also a pale blue, but eosin and fuchsin fluoresce a brilliant yellow-green, while sodium vapour in sunlight emits red, yellow and green rays.

Phosphorescence.—Strictly speaking, fluorescence ceases instantaneously with the cessation of the exciting light. Many substances, however, show a prolonged effect, which is then called **phosphorescence.** Thus diamond, calcium sulphide, etc., if illuminated in daylight for some time will glow when placed in the dark. The ultra-violet light is the most active in producing phosphorescence. Balmain's luminous paint is chiefly composed of calcium sulphide. This effect must not be confused with the glow sometimes arising from chemical action, as in the case of phosphorus, or decaying animal matter, which is sometimes called phosphorescence.

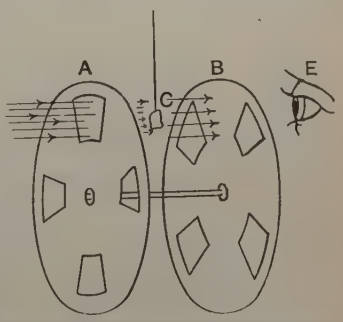

FIG. 594.—Becquerel's phosphoroscope.

Becquerel's phosphoroscope.—The substance to be examined is situated between two discs A and B (Fig. 594), which can be rotated rapidly by a mechanism not shown in the diagram. The discs have slots, those in A not being opposite to those in B. White light passing through a given slot in A falls on the body C under examination. As the discs rotate the light is cut off, and after a short interval a slot in B comes opposite to the body, so that it can be seen from E. If the body is phosphorescent it will

still be emitting light and will then be seen from E; but if it is fluorescent its luminosity ceases when A cuts off the illumination, and it will no longer be seen from E. By varying the speed of the discs the limiting times of phosphorescence may be determined. This time varies from a small fraction of a second up to several hours, according to the nature of the substance.

Photography.—It will have been noticed that the rays which are particularly active in producing fluorescence and phosphorescence are also those chiefly effective in producing photographic results. These rays of high frequency appear to be highly active in producing atomic and molecular changes, the atoms of matter readily absorbing the energy of the rays. Sometimes, as in fluorescence, the energy is again radiated as light, but in the case of some substances, notably certain silver salts, molecular changes are produced, resulting in new chemical arrangements, in some of which, silver is reduced to the opaque metallic form. The chemistry of photography cannot here be discussed, but we may note that, since the blue and ultra-violet rays alone are active, red and yellow objects, which appear bright to the eye, will be inactive photographically, so that a coloured object will not be reproduced in a photograph in its due proportion of light and shade. This defect would render impossible the colour photography described on p. 641. Several methods of rendering photographic plates sensitive to rays of all colours have been devised. Thus, if a plate, before use, be immersed in a solution of 1 part in 10,000 of erythrosin, it becomes active to light up to the yellow in the spectrum. Such plates are said to be **isochromatic** or **orthochromatic.** This treatment does not render the plates any less sensitive to the violet and ultra-violet rays, whose effect would still predominate, and mask the effect due to the green and yellow rays; the blue and ultra-violet rays are therefore reduced by the use of a lemon-yellow glass screen placed in front of the lens. This screen owes its colour to the fact that it is more transparent for the yellow and red parts of the spectrum than to the rays at the blue end.

By employing a solution of pinacyanol the plates may be rendered sensitive to the whole spectrum, including the red rays, and are then said to be **panchromatic.** A lemon-yellow screen must, of course, be used, and such plates must be developed in complete darkness. They are now largely employed in photographing spectra.

Exercises on Chapter XLIX.

1. Describe a simple form of spectroscope. How would you use it to observe the absorption spectrum of sodium vapour ?

2. Upon what does the colour of natural objects depend ? Explain why two coloured objects appear of different tints by daylight and by gaslight.

3. What is a pure spectrum ? Explain clearly how you would produce it. Account for the presence of **Fraunhofer's lines** in the solar spectrum.
Allahabad University.

4. Explain how you would set up the apparatus for showing the spectrum of an electric arc.
What would be the effect on the spectrum of placing a sheet of red glass between the lamp and the rest of the apparatus ? L.U.

5. How would you study the distribution of energy in the different positions of the spectrum of sunlight ?
State the results you would obtain. L.U.

6. Describe in detail the way in which you would obtain a pure spectrum of the light from the sun.
Describe briefly some method of examining the visible spectra of gases.
L.U.

7. Describe the arrangement of illuminated slit, lenses and prism to give a "pure spectrum."
How would you investigate the existence of radiation outside the limits of the visible spectrum ? L.U.

8. Give a short account of the Young-Helmholtz theory of colour vision.

9. Explain what you mean by complementary colours. How would you find the nature of the colour complementary to any given colour ?

10. Describe how you would find a set of spectrum colours which when mixed would match a given colour.

11. Give a short account of the principle of some process of colour photography.

12. What do you mean by the terms fluorescence and phosphorescence ? How could you decide whether a material was fluorescent or phosphorescent ?

CHAPTER L

POLARIMETRY AND SACCHARIMETRY

Polarisation.—The discussion of the wave theory of light and of polarisation is beyond the scope of this book, but a few words may serve to render intelligible their application to the subject of **saccharimetry**. It has already been pointed out (Chap. XLIX) that light and radiant heat are both wave phenomena, and differ only in frequency, or in wave-length. At the part of the spectrum of sunlight where the heating effect is great, and the sensation of light is not produced, the wave-length varies from about 10^{-2} cm. to 10^{-4} cm., but at the part of greatest luminosity the wave-length is about 6×10^{-5} cm.

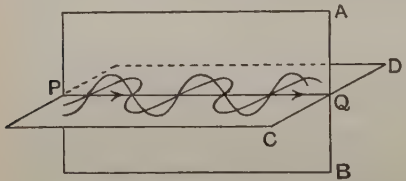

FIG. 595.—Plane polarised waves.

The waves constituting light are entirely transverse (Chap. LIV), and it will be proved later that the oscillations are at right angles to the direction in which the wave is travelling (p. 651). These oscillations occur in some plane or planes containing the ray. Thus they may occur in a plane AB (Fig. 595), or in two planes AB and CD, or may be distributed over an infinite number of planes containing the ray PQ, as in the case of ordinary light.

If the oscillations constituting the beam of light are confined to one plane, the light is said to be **plane polarised**. The properties of a plane polarised beam differ with respect to two planes, one of which contains the oscillations, while the other is at right angles to them.

Double refraction.—In transmitting a beam of light, many transparent crystalline materials have the property of splitting the

beam into two beams having different laws of refraction. For most directions of transmission, a single ray of ordinary light separates into two, one of which is called the **ordinary ray** because it obeys the ordinary laws of refraction, and the other is called the **extraordinary ray** because it does not obey the ordinary laws of refraction. The mineral calcite, or Iceland spar, exhibits this doubly refracting property most strongly.

EXPT. 148.—**Double refraction of calcite.** Place a slab of calcite, made by cleaving a natural crystal, on a piece of paper on which a dot has been marked at O (Fig. 596). On looking through the slab, two dots can be seen, one at O_O produced by the ordinary ray, and the other at O_E produced by the extraordinary ray. Keep the slab in contact with the paper and rotate it: note that O_E rotates about O_O.

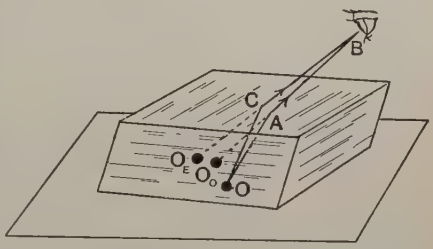

FIG. 596.—Slab of calcite producing double refraction.

The ordinary and extraordinary rays are both plane polarised, and the plane in which the oscillations of the ordinary ray occur, is at right angles to that in which the oscillations of the extraordinary ray occur.

Nicol's prism.—For experimental purposes it is desirable to separate the ordinary from the extraordinary ray. One of the best means for effecting this is provided by the **Nicol's prism**, in which the ordinary ray is suppressed, leaving the extraordinary ray remaining.

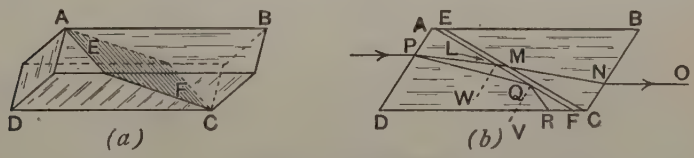

FIG. 597.—Nicol's prism.

A prism of calcite ABCD (Fig. 597 (*a*)) is obtained by cleaving a crystal along the natural cleavage planes. The crystal is now sawn across a plane EF, which makes an angle of about 22° with AB. The cut surfaces are then polished and cemented together with Canada balsam. Consider a ray of light which enters the face AD (Fig. 597 (*b*)). This separates into the ordinary ray PQ and the extraordinary ray PL. The critical angle for the passage of the ordinary

ray from calcite to Canada balsam is 69° 30′, and the angle PQV is greater than this; hence the ray PQ is totally reflected along QR, and is absorbed at R by the black covering of the Nicol's prism. Owing to the fact that the index of refraction of Canada balsam is greater than that of calcite, for the extraordinary ray, it follows that there is no critical angle in this case, so that total reflection does not occur, and this ray is therefore transmitted along LM, travels along MN, and finally emerges from the face BC as shown at NO. Thus the emergent ray is plane polarised.

Plane of polarisation.—The nature of the oscillations constituting light, and the actual plane in which they occur, will not be discussed here. It is necessary, however, to fix some plane relatively to the beam.

The usual convention is: call the plane containing the long diagonal of the face BC (Fig. 597) of the nicol and the emergent ray, the **plane of polarisation.** This is shown by an arrow in Fig. 598, which is an end view of the nicol. Thus, a ray of light polarised in this plane can be transmitted through a nicol, because it is the extraordinary ray, but a ray polarised in a plane at right angles to this (that is, parallel to the plane of BC), is the ordinary ray, and is suppressed. Thus a Nicol's prism serves to analyse light, that is, to detect polarisation; and also to polarise light, that is, to produce a polarised beam.

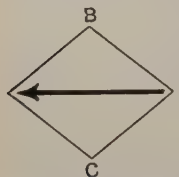

FIG. 598.—Plane of polarisation.

In reflecting light, most surfaces produce a certain amount of polarisation. This can be detected by viewing a sheet of glass, or the polished surface of a table, through a nicol. On rotating the nicol about its axis a variation in the intensity of illumination will be observed, because the reflected light generally consists of a mixture of ordinary light and plane polarised light. The plane of polarisation of the latter coincides with the plane of incidence and reflection.

Suppose that the beam of light emerging from a nicol A be allowed to fall on a second nicol B. The amount of light emerging from B, which has therefore traversed both nicols, will depend upon the relative positions of A and B. If the long diagonals of both prisms are parallel, the beam will traverse the nicol B with undiminished intensity. If B be rotated until the long diagonals are at right angles to each other, then the beam emerging from A will be completely

suppressed by B. The nicols are then said to be **crossed.** A may be called the **polarising** and B the **analysing** nicol.

The fact that the beam is completely stopped with crossed nicols proves that light consists entirely of transverse oscillations. Any longitudinal oscillation would be treated in the same manner by both nicols, as it would possess the same properties in all planes containing the ray, and would therefore be independent of the orientation of the nicols.

Rotation of the plane of polarisation.—On passing a beam of polarised light through certain substances, rotation of the plane of polarisation occurs. A crystal of quartz has this power when the beam travels along the direction of the axis of the crystal. Thus, if a plate of quartz, having faces perpendicular to the axis of the crystal, be interposed between crossed nicols, the beam is no longer completely cut off by the second nicol.

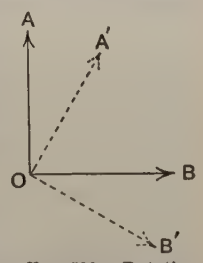

FIG. 599.—Rotation of the plane of polarisation.

Let OA (Fig. 599) be the plane of polarisation for the polarising nicol, and OB that for the analysing nicol. On interposing the quartz plate, the plane of polarisation is rotated by the plate into some such position as OA′. The analysing nicol must be rotated through an angle BOB′ = AOA′ in order that the beam shall be extinguished.

The angle of rotation depends upon the thickness of the quartz traversed by the beam, and also upon the colour of the light employed. We shall only deal here with the yellow light of the sodium flame (the D line of the spectrum). The angle of rotation for quartz is 21·72° per millimetre of path traversed. Some crystals of quartz produce rotation in one direction and others in the opposite direction. The convention adopted in this country is that, looking along the ray towards the source of light, the rotation of the plane of polarisation is said to be **right-handed** if clockwise, and **left-handed** if anticlockwise.

Many liquids and vapours produce a similar rotation of the plane of polarisation, but to a smaller extent than quartz. When two substances which alone would produce opposite rotations of equal magnitudes are present, the result would be zero rotation. And more generally, if two or more active substances are present, **the rotation actually produced is the algebraic sum of the rotations which would**

be produced by the substances separately, if each could be present to the extent of the same number of grams per cubic centimetre which it has in the mixture.

Specific rotation.—The angle of rotation of the plane of polarisation produced by any substance, in solution or otherwise, is proportional to (i) the length of path in the substance traversed by the plane polarised beam, and (ii) the actual density of the substance present. The amount of rotation produced by traversing a path of 10 cm. length in the substance when the density is unity is called the **specific rotation.** The actual rotation θ produced in any case is therefore given by

$$\theta = \frac{\mathsf{S}ld}{10}, \quad \text{or} \quad \mathsf{S} = \frac{10\theta}{ld},$$

where S is the specific rotation of the substance, l is the path traversed in centimetres, and d is the actual density of the active material in grams per c.c.

To find d it is necessary to determine d_1, the density of the solution, by means of the specific gravity bottle (p. 278). Then, if w grams of active substance are dissolved in 1 gram of non-active solvent, such as water,

$$d = d_1\left(\frac{w}{1+w}\right).$$

Substance.	Solvent.	Specific rotation.	Substance.	Solvent.	Specific rotation.
Cane sugar -	water	+66·67°	Quinine sulphate	water	−214°
Tartaric acid	water	+15·06°	Nicotine - -	pure	−162°
Turpentine -	pure	−37°	Camphor - -	alcohol	+54·4°

+ indicates right-handed, − indicates left-handed.

Polarimeters or Saccharimeters. In order to measure the amount of rotation of the plane of polarisation in any given case, a special piece of apparatus is generally used, called a **polarimeter or saccharimeter.** The form here described is due to Laurent.

The telescope T (Fig. 600) is focussed upon the plate E next to the nicol N_1, which produces the plane polarised beam from the light emitted from the sodium burner F. The analysing nicol N_2 is fixed in the tube A which carries the telescope; the tube is able to rotate on its axis, and its position is observed by means of the verniers V and the fixed circular scale S. G is a glass cell containing a solution of potassium chromate, which absorbs all but the yellow D light given out by the bunsen flame F. The position for extinction, in

which the nicols are crossed, can be found by looking through the telescope T, rotating the telescope and analysing nicol, and observing the position of the verniers when the field is dark. On interposing the tube of rotatory solution BC, having glass ends, a new position for extinction is found, and the travel of the verniers is a measure

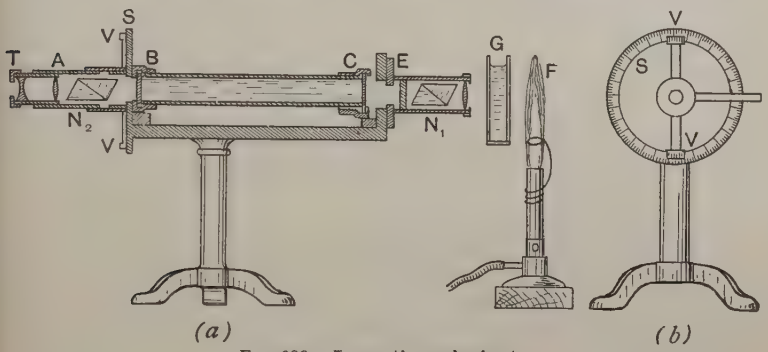

FIG. 600.—Laurent's saccharimeter.

of the rotation produced. Owing to ambiguity as to its value when the rotation approaches 90° or 180°, two lengths of tube BC are employed, one of which is 10 cm. long and the other 20 cm. One rotation must now be twice the other, which fact removes the ambiguity in the original reading.

It is not possible to judge accurately when complete extinction of the light occurs, and therefore the plate E is introduced in Laurent's saccharimeter. This plate is made in two parts. The half A (Fig. 601) being of glass, the plane of polarisation for this half is unchanged, and is shown by the arrow p_1. The half B consists of a crystalline plate of such thickness that the plane of polarisation of the light in passing through it is rotated through such an angle that it assumes the position indicated by the arrow p_2. If now the long diagonal of the analysing nicol is perpendicular to the bisector of the angle between p_1 and p_2, the two halves of the field are equally illuminated, since the beam is extinguished to the same extent in each half. This condition is one that the eye can judge very accurately. On introducing the tube of solution, both p_1 and p_2 are rotated to the same extent, and the amount of rotation of the analysing nicol required to restore equality in the illumination of the two halves of the field is a measure of the rotation of the plane of polarisation produced by the solution.

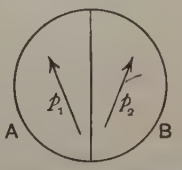

FIG. 601.—Half quartz plate of Laurent's saccharimeter.

EXPT. 149.—**Rotatory power of quartz.** Set up the Laurent's saccharimeter, and measure the rotation produced by a plate of quartz. Measure the thickness of the quartz by means of a micrometer or spherometer (pp. 16-18), and calculate the rotation produced by 1 mm. of quartz.

EXPT. 150.—**Specific rotation of cane sugar.** Make up four solutions of cane sugar of strengths approximately 5, 10, 15 and 20 per cent. by weight, and calculate their exact percentages from the weighings. Determine the density of each solution by means of the specific gravity bottle; calculate the actual density of the sugar in each solution. By means of the Laurent saccharimeter measure the rotation of the plane of polarisation produced by 10 cm. of each solution, and by the equation on p. 652 calculate four values of the specific rotatory power of the sugar and take the mean.

The polarimeter may be employed for analysis of solutions in which there is one substance present which produces rotation, as in the determination of the strength of a sugar solution. It also reveals differences in molecular structure between materials which to chemical analysis are identical. Thus the two forms of quartz rotate the plane of polarisation to the right or left respectively (p. 651), but they are otherwise indistinguishable. Similarly dextrose or grape sugar has the same composition as laevulose and differs from it chiefly in the fact that its solutions produce right-handed rotation, while those of laevulose produce left-handed rotation in the plane of polarisation of light.

EXERCISES ON CHAPTER L.

1. What do the terms "plane-polarised light" and "rotation of the plane of polarisation" mean?
How would you show that sugars differ in their optical properties as regards the transmission of polarised light through them? L.U.

2. Give an account of an experiment which proves that double refraction occurs in certain materials.

3. Describe a Nicol's prism. How would you examine a beam of light with a Nicol's prism in order to determine whether the beam is plane-polarised?

4. Give a description of some experiment which illustrates the rotation of the plane of polarisation of light by means of certain substances. How would you determine the amount of rotation produced by 1 millimetre of quartz?

5. Describe some form of polarimeter which can be used for measuring the specific rotatory power of cane sugar.

6. Define specific rotation. If 20 grams of a substance are dissolved in 70 grams of water and the density of the solution is found to be 1 21 grs. per c.c., and it is found that a length of 20 cm. of the solution produces

a rotation of 35° in the plane of polarisation, calculate the specific rotation of the substance.

7. Explain how you would find the strength of a sugar solution, if the specific rotation of the sugar were given.

8. Describe some method of producing a beam of plane-polarised light. How would you determine whether or not the beam were completely polarised ?

9. What do you understand by the term " polarisation of light " ? Describe some method of producing plane-polarised light, and explain what deductions can be made from the phenomenon of polarisation as to the nature of aether vibrations. L.U.

10. What is meant by *plane polarised* light, and how may it be produced ? Describe the property which a solution of sugar has with respect to such light. L.U.

CHAPTER LI

VELOCITY OF LIGHT

Römer (1675).—The velocity of light is so great that unless extreme precautions are taken, no interval can be observed between its starting from a source and its arrival at the observer situated at any terrestrial distance from the source. The astronomer Römer, while making observations upon the period of revolution of one of Jupiter's satellites, found certain discrepancies which he could not at first explain. With the earth and Jupiter situated as shown at E_1J_1 (Fig. 602), the period of revolution of the moon M_1 under observation is about $42\frac{1}{2}$ hours, and so long as the observations are not separated by great intervals of time it was found that the moon M_1 entered Jupiter's shadow at the calculated time. But if an interval of several weeks elapsed, it was found that the eclipses of M_1 did not appear to occur until after the calculated time, and this retardation increased for about six months. In the next six months the eclipses occurred after shorter and shorter intervals, until after a year they had again reached their calculated time.

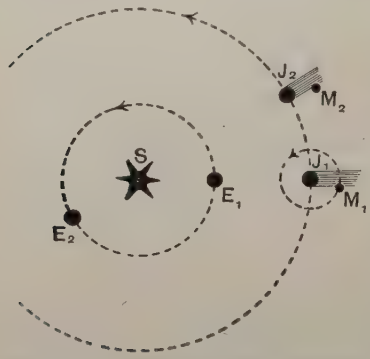

FIG. 602.—Determination of the velocity of light.

Römer explained this by assuming that the light took time to cross the earth's orbit. In approximately six months' time the earth is at E_2 and Jupiter at J_2, since Jupiter's period of revolution round the sun is nearly 12 of our years. The retardation in six months amounted to about $16\frac{1}{2}$ minutes or 990 seconds, and, taking

the sun's distance from the earth to be 93 million miles, this gives the velocity of light as $2 \times 93000000/990 = 190{,}000$ miles per second, or $3{\cdot}03 \times 10^{10}$ cm. per sec.

Bradley (1727).—In observing the position of the fixed stars at different times of year, Bradley found a small displacement in the apparent position, always in the direction in which the earth was travelling at the time of observation.

This is due to the relative motion of the earth and the light coming from the star. If the light is traversing the path SAB (Fig. 603) then, if the earth were at rest, the telescope would be directed along BA in order to observe the star. But if the earth moves through distance CB while the light is traversing the telescope, that is the distance AB, the telescope must be inclined at the angle CAB to the path of the light, and the star will appear to be in the direction CAS′.

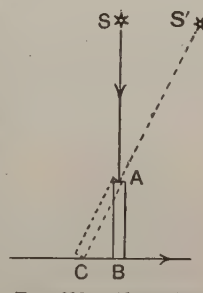

FIG. 603.—Aberration of light.

Now, $\tan \text{CAB} = \dfrac{\text{CB}}{\text{AB}} = \dfrac{\text{velocity of earth}}{\text{velocity of light}}$, when the earth is travelling at right angles to the path of the light. Hence a star near the pole of the earth's orbit will appear to describe an ellipse about its mean position, in the course of a year. The angle CAB or SAS′ is found to be 20·45 seconds of arc, and taking the earth's velocity in its orbit as 30557 metres per sec. and tan 20·45″ as 0·0001003

$$\tan 20{\cdot}45'' = 0{\cdot}0001003 = \frac{30557}{\text{vel. of light}},$$

which gives the velocity of light as $3{\cdot}047 \times 10^8$ metres, or $3{\cdot}047 \times 10^{10}$ cm. per sec.

Fizeau (1849).—The first successful attempt at measuring the velocity of light over terrestrial distances was made by Fizeau. Light from a source S falling on the lens L_3 is converged towards a focus (Fig. 604). A considerable part of it, however, is reflected by the plate glass M_1, and is thus brought to the focus F. This is at the principal focus of the lens L_1, so that the rays emerge from L_1 as a parallel beam. After traversing a distance of 8·633 kilometres the light arrives at the lens L_2, and is brought to a focus on the surface of a spherical mirror M_2 whose centre of curvature is on L_2. The light is there reflected and retraverses its path, some of it passing through the plain glass M_1 and the eyepiece E, thereafter entering the eye.

GF is a toothed wheel which can be rotated with great velocity about the axle A. GF is so situated that the light may be brought to a focus in the space between two teeth of the wheel, when, of course, the eye sees a bright image. When the light falls on a tooth at F, no bright image is seen. The wheel is then set in rotation,

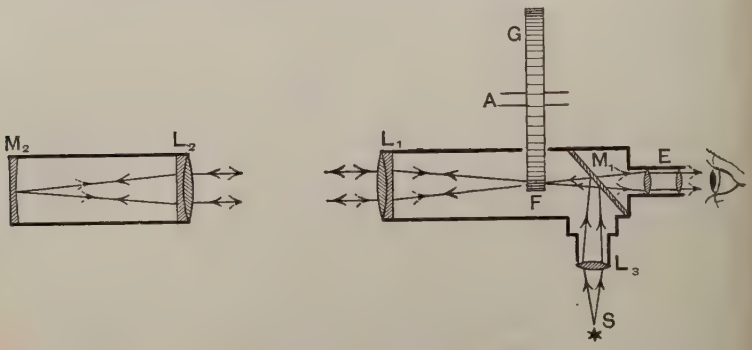

FIG. 604.—Velocity of light (Fizeau).

and if not more than about 10 spaces pass F per second, the eye sees a continuous bright illumination. But if the speed be increased until the light travels from F to M_2 and back again in the time that the wheel has advanced forwards through a distance equal to half that between the centres of two consecutive teeth, the light that passes through any space in going is stopped on its return, by the next tooth. In Fizeau's experiment the wheel had 720 teeth and made 12·6 revolutions per second when the eclipse of the image was produced. The time taken for the light to traverse the double path between F and M_2 is therefore $1/(2 \times 720 \times 12{\cdot}6)$, and the velocity of light, according to this experiment, is $2 \times 8{\cdot}633 \times 2 \times 720 \times 12{\cdot}6 = 3{\cdot}13 \times 10^5$ kilometres per second or $3{\cdot}13 \times 10^{10}$ centimetres per second.

If the wheel rotate at twice the above rate the light passing through any space will, on its return, pass through the next space and the image will be seen. At three times the speed the light will again be stopped, and so on.

Cornu (1874 and 1878).—By Fizeau's method, improving the apparatus and increasing the distance apart of the stations to 23 kilometres, Cornu obtained a result $3{\cdot}004 \times 10^{10}$ cm. per sec.

Foucault (1850).—The method of the rotating mirror was suggested by Fizeau and Foucault, but carried out by Foucault. Light from an illuminated slit S (Fig. 605), after reflection from a plane sheet of glass A, passes to the lens L, which would produce a real image at C′. The mirror M, however, intervenes, and the image is

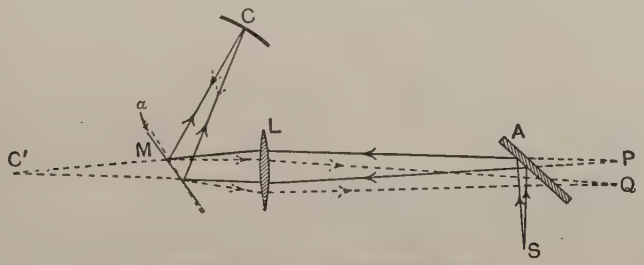

FIG. 605.—Velocity of light (Foucault).

produced on the concave mirror C, whose centre of curvature is at M. The light is there reflected normally and returns along the same path, retraversing the lens and forming a real image at P. If now the mirror M be rotated so rapidly that it has moved through an appreciable angle α while the light travels from M to C and back to M, the reflected light makes an angle 2α with its original path, and the real image is formed at Q. Knowing the distance MC ($=l$), and the displacement PQ ($=d$) the velocity of light can be found. Thus, the time taken to traverse the double path MC is $2l/v$, where v is the velocity of light. Also, if the mirror makes n revolutions per second its angular velocity (p. 53) is $2\pi n$ radians per second, and the time taken to rotate through the angle α is $\alpha/2\pi n$;

$$\therefore \frac{2l}{v}=\frac{\alpha}{2\pi n},$$

or

$$v=\frac{4\pi nl}{\alpha}.$$

Again, if the distance LP $=r$, then $2\alpha=\frac{d}{r}$;

$$\therefore v=\frac{8\pi nlr}{d}.$$

In Foucault's experiment the distance MC was about 20 metres and his value for the velocity of light is $2{\cdot}98\times10^{10}$ cm. per second.

By placing a tube of water in the path of the light between M and C the velocity of light in water was determined. It was found to be less than the velocity of light in air.

Michelson (1879 and 1882).—The method of the rotating mirror was improved by Michelson, amongst other alterations being that of increasing the distance between the rotating and the fixed mirrors to 600 metres. His final value for the velocity of light is $2{\cdot}998 \times 10^{10}$ cm. per second.

It is extremely likely that the actual velocity of light differs very little from the value $3{\cdot}000 \times 10^{10}$ cm. per second.

Exercises on Chapter LI.

1. Give an account of some method by which the velocity of light has been determined. L.U.

2. Explain how the speed of light may be found by observations of the eclipses of Jupiter's second satellite.

The greatest and least intervals between consecutive eclipses being 42 h. 28 m. 56 s. and 42 h. 28 m. 28 s., and the radius of the earth's orbit 92·7 million miles, calculate the speed of light in miles per second. L.U.

3. Describe some method in which the velocity of the earth is made use of in determining the velocity of light.

4. Describe Fizeau's method of finding the velocity of light.

5. How has it been shown that the velocity of light in water is less than the velocity in air ?

6. Consider that in apparatus arranged for Fizeau's experiment, the wheel has 600 teeth and that the two stations are situated 15 kilometres apart. What must be the number of revolutions per second of the toothed wheel in order that light passing through a space on the outward journey may be stopped by the next tooth on the return ? (Velocity of light $= 3{\cdot}0 \times 10^{10}$ cm. per sec.)

7. Calculate the angular velocity of a mirror in revolutions per second, if a beam of light reflected by it travels a distance of 750 metres, is then reflected back along its old path, and after reflection again by the rotating mirror, makes an angle of one minute of arc with its original path.

PART IV

SOUND

CHAPTER LII

SOUNDING BODIES—FREQUENCY

Sensation of sound.—Hearing, like sight and touch, is a primary sensation. A suitable stimulus, called a **sound**, applied to the ear produces the sensation of hearing. The source of this stimulus can always be traced to the movement of some body, although the movement may be so slight or so rapid that it is not visible. In the case of an explosion, the movement is conspicuous and the sound very great. But in the case of a tuning-fork, the sound may be appreciable although there is no apparent motion of the fork. This motion may be made obvious by bringing a light pith-ball suspended by a thread (Fig. 606) into contact with the fork, when the pith-ball will be kicked violently away.

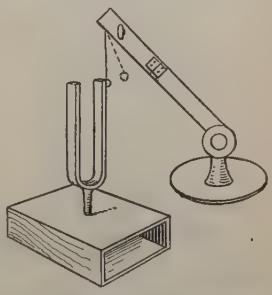

FIG. 606.—Motion of tuning-fork when sounding.

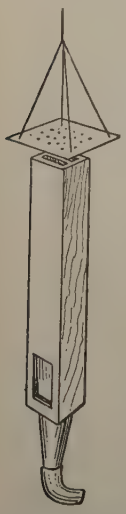

FIG. 607.—Motion of air in a sounding organ pipe.

Or the source of sound may be a column of air, as in the case of an organ pipe. On bringing a piece of paper with some fine dry sand over the mouth of the pipe when sounding, the sand will be seen to jump about upon the paper (Fig. 607). The reason is that the air is moving in and out of the pipe rapidly, and so sets the paper in vibration.

In either of these cases, or in the case of a vibrating stretched string, the motion may be felt by placing the finger upon the body. In the case of the fork, or the string, this will quickly bring the body to rest, with cessation of the sound.

Simple harmonic motion.—In the case of all continuous sounds, as apart from a mere explosion, or a blow, the source of sound is in a state of vibration. Most vibrating bodies, unless their motion be very great, execute a simple harmonic motion (p. 220), or their vibration consists of a combination of simple harmonic motions. Hence it is very important to understand the nature of simple harmonic motion, and the student is advised to read carefully pages 220 to 226.

There are several points in connection with simple harmonic motion that may with advantage be recapitulated here.

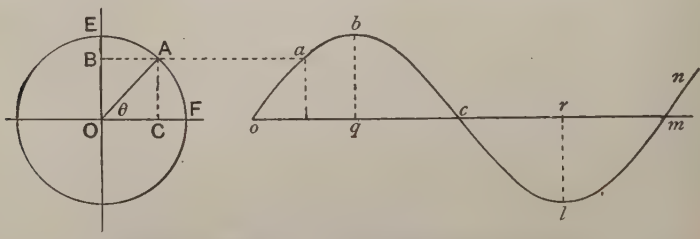

FIG. 608.—Diagram to represent simple harmonic motion.

Thus, if the vector OA rotates with uniform angular velocity about a point O (Fig. 608) its projection upon any fixed line, say OE, represents a simple harmonic motion. At the instant given, the vector is in the position OA, making angle θ, with OC. Then $BO = AC = OA \sin \theta$. If OA is in the position OF at time $t=0$, and the angular velocity of rotation of OA be ω, then $\theta = \omega t$.

Also, if OA be called the amplitude a, and $OB = y$,

$$y = a \sin \omega t.$$

A simple harmonic motion may also be represented by a curve *oabclmn*, where the abscissae are times, measured in seconds, and the ordinates are the displacements y.

Thus the point o corresponds to the instant at which A is at F; b to that at which A is at E, and so on. The curve *obclm*, corresponds to one complete rotation of OA. After this the curve repeats itself.

Amplitude and frequency.—A body capable of vibration, when disturbed from its equilibrium position, as in striking the prong of a tuning-fork, begins to vibrate about this equilibrium position. Except for the fact that the vibration slowly dies away, the extreme displacement on either side of the equilibrium position is called the **amplitude** of the vibration. This corresponds to OA, OE, qb, or rl in Fig. 608. The amplitude of vibration of a sounding body is generally very small, while that of a pendulum is comparatively large.

The number of **complete** vibrations made by the body in one second is called the **frequency** of the vibrating body.

EXPT. 151.—**To compare the frequencies of two tuning forks.** The two forks are securely clamped to a wooden block A (Fig. 609). A strip of glass B, which slides easily in an upright guide, can fall past the forks, and is released by burning the thread C, and so releasing a catch. Attached to a prong of each fork is an aluminium style whose point bears lightly upon the glass. The glass must have been previously

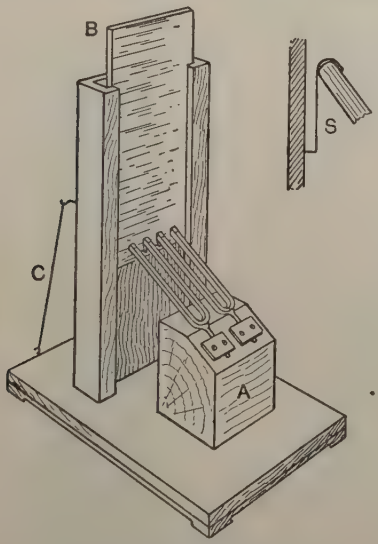

FIG. 609.—Dropping plate for comparing the frequencies of two forks.

FIG. 610.—Traces obtained on a dropping plate.

blackened in the smoky flame of a lamp. On drawing a violin bow across the forks they are set in vibration, and at the same instant the plate is released. Each style draws a trace upon the smoked glass, the number of vibrations made during the fall of the plate being easily discerned. In Fig. 610 such a trace for a pair of forks is shown. The starting points of the traces are A and B, and a line should be drawn through them. Two other lines CD and EF are then drawn, parallel to AB, so that the trace CE is made by one fork in the same time that the trace DF is made by the other. On counting the numbers of vibrations in CE and DF the ratio of the frequencies of the forks can be found. In the given case it is $\frac{26{\cdot}5}{20}=1{\cdot}325$.

The aluminium style should be bent to the shape shown at S (Fig. 609).

As the plate falls, the style can then bend slightly, owing to the plate pressing against it, but the level of the point is not altered by this bending.

Expt. 152.—**To find the absolute frequency of a fork by the dropping plate.** For the determination of the absolute frequency of a fork by the dropping plate method, the plate must fall freely. The guides are therefore dispensed with and the plate is suspended by the thread A (Fig. 611), and can be released by burning the thread at A. The style must be made as in the last experiment and the bowing of the fork performed as before.

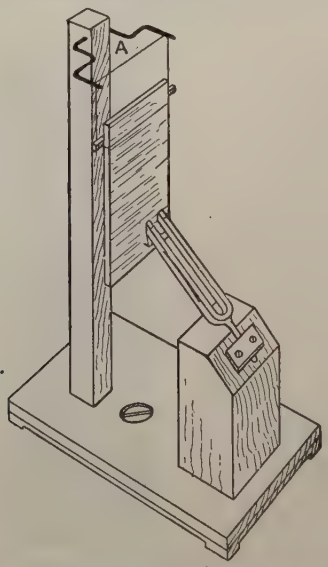

Fig. 611.—Dropping plate for measurement of absolute frequency.

Fig. 612.—Trace by single fork.

An example of the kind of trace produced is given in Fig. 612. The starting point A must be clearly marked. By means of a cathetometer or travelling microscope, the distances s_1 and s_2 to any clearly marked waves at B and C can be found.

Then, the time t_1 required for the plate to fall through the distance s_1 from the start is given by $s_1 = \frac{1}{2}gt_1^2$, where g is the acceleration of gravity (p. 33). Therefore $t_1 = \sqrt{\frac{2s_1}{g}}$.

Again, the time required by the plate to fall from A to C is $t_2 = \sqrt{\frac{2s_2}{g}}$. Hence the time occupied in falling from B to C is calculated from

$$t_1 = \sqrt{\frac{2s_2}{g}} - \sqrt{\frac{2s_1}{g}}.$$

In this time the number of vibrations n made by the fork can be counted, then

$$\text{Frequency of fork} = \frac{n}{t_2 - t_1}.$$

In the given case $n = 21$ and $s_1 = 1{\cdot}54$ cm., $s_2 = 9{\cdot}27$ cm.;

$\therefore$ taking g as 981 cm. per sec. per sec.

$$t_2 - t_1 = 0{\cdot}1375 - 0{\cdot}0561 = 0{\cdot}0814 \text{ sec.};$$

$$\therefore \quad n = \frac{21}{0{\cdot}0814} = 258{\cdot}0.$$

The nominal value of the frequency of the fork was 256.

Measurement of frequency by the chronograph.—A piece of smoked paper is wrapped round a drum D (Fig. 613) which can rotate about

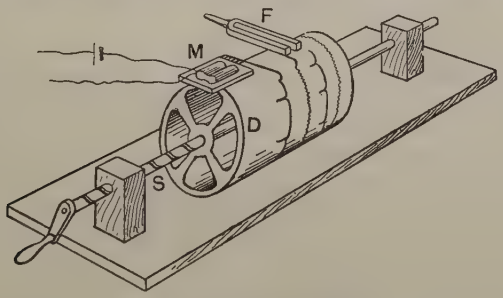

FIG. 613.—The chronograph.

an axle S, on which a screw thread of large pitch is cut. On rotating the drum by means of the handle, it advances as it rotates. The fork is provided with a style, so that it leaves a trace when the drum turns. In order to mark intervals of time, a small electromagnet M is provided with a style attached to its armature which also leaves a trace alongside of that due to the fork. The current for the electromagnet is supplied by a battery, and in series with it is a make and break actuated by a standard clock whose pendulum beats half seconds. Every half second, when the pendulum is at the bottom of its swing, contact is made, and the style then makes a little kick, which is recorded upon the blacked paper upon the drum. This arrangement is called the **chronograph**, and the number of vibrations of the fork between any half-second marks upon the paper can be counted, and the frequency of the fork determined.

The process may be inverted by employing the chronograph to measure small intervals of time. This necessitates a tuning-fork of known frequency. In this case the instants of time to be recorded are marked by the closing of the electrical circuit of the magnet M.

This causes a mark to be made on the drum D, and the interval between the occurrence of successive marks is known from the number of waves traced by the standard tuning-fork in the same interval.

Exercises on Chapter LII.

1. What is simple harmonic motion ? Show how it may be represented in a diagram of time and displacement.

2. Define the terms ' amplitude ' and ' frequency.'
Describe how the frequency of a tuning-fork may be measured.

3. How would you show that a source of sound is a vibrating body, in a case where the motion of the body is too small to be visible ?

4. Describe the dropping plate method for determining the absolute pitch of a tuning-fork. The plate drops from rest through a distance of 1·8 cm., and while dropping through the next 10·2 cm. the fork makes 35 vibrations. Calculate the frequency of the fork.

5. Describe how you would employ the chronograph to measure small intervals of time, if you were provided with a tuning-fork of known frequency.

6. Describe a method of comparing the frequencies of two tuning-forks.

CHAPTER LIII

PITCH, LOUDNESS, AND QUALITY

Pitch and frequency.—Our sense of hearing tells us whether a musical note is of high pitch or low pitch, but it does not tell us upon what the pitch of the note depends. If any sound or noise, such as the tapping of a hammer, recurs at regular intervals, it will be heard as a series of separate impulses. As the number of impulses per second increases, the separate sounds will be heard until the number reaches 25 to 30 per second, when they will cease to make separate impressions, and will be heard as a continuous humming note. As the frequency of the impulses increases, the pitch of the note will rise. Evidently the pitch of the note depends upon the frequency of the impulses.

Many examples may be found to illustrate this. Thus, if a long piece of clock spring be clamped at one end in a vice, and set vibrating, the vibrations, when very slow, do not produce any sensation of sound. On shortening the free part of the spring, the vibrations obviously become more rapid, and soon a low note will be heard. With further shortening, the frequency increases, and a note of higher and higher pitch will be heard, until, when the spring is something like 1 cm. in length, a very shrill note, that is, one of very high pitch, will be heard.

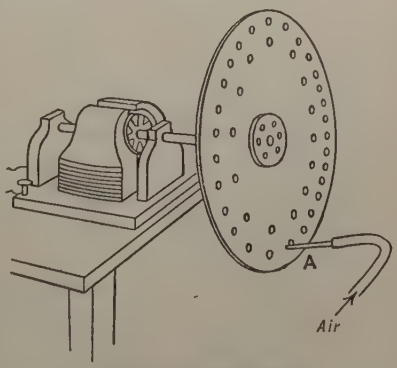

FIG. 614.—Disc siren.

That the relation between frequency and pitch is of a quantitative nature may be shown by means of the disc siren (Fig. 614).

A disc having two circles of holes in it is capable of being rapidly rotated, and a tube A, leading from a pair of blowpipe bellows, directs a draught of air against the holes. When the rotation is very slow, a puff of air passes through each hole as it comes opposite the tube, and the ear will hear the separate puffs. On increasing the speed the puffs blend into a note, whose pitch rises with increasing speed.

Further, if there are twice as many holes in the outer ring as in the inner ring, then, for any given speed, there will be an easily recognisable relation between the pitches of the notes produced by directing the jet of air against each set in turn. The note with twice the frequency will be the upper octave of the other note. This relation is true whatever the speed of the disc, so that whatever the absolute frequencies may be, **one note is the upper octave of another if it corresponds to twice the frequency.** This reasoning will be extended later to the case of intervals other than the octave, but it may be noted here that the interval between the pitches of any two notes depends upon the **ratio of their frequencies**, and not upon their absolute frequencies.

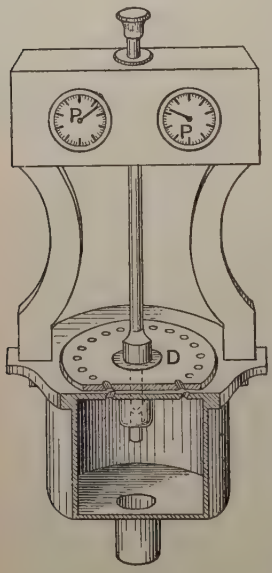

FIG. 615.

Thus, if one note corresponds to a frequency of 512 per sec. it is the upper octave of a note of 256 per sec., and is the lower octave of a note of frequency 1024 per sec.

The Siren.—The rotating disc of Fig. 614 has been developed into a useful piece of apparatus for determining frequency, called the **siren.** A disc D (Fig. 615) is mounted upon a vertical axis, the upper end of the axis being provided with a worm wheel which engages with a toothed wheel. This rotates a pair of pointers P, which record the single revolutions and tens of revolutions of the disc D. The disc is provided with a number of holes which, in one position, come over a similar set of holes in the top of the wind chest. Thus every time the upper and lower holes come together, puffs of air emerge, and when the speed of rotation is sufficient, these successive puffs give rise to a note. With n holes in the disc and N revolutions of the disc per second, the frequency is Nn.

The method of driving the disc should be noted. The holes are drilled obliquely through the plates, those in the disc being oppositely sloped to those in the top of the wind chest. As the air emerges it is driven against one side of each hole in the disc and so causes it to rotate. The speed of the disc can be regulated by varying the air pressure.

In order to measure the frequency of a source of sound such as a tuning-fork, the siren is regulated in speed until the pitch of the note is the same as that of the fork. The interval of time required to make a known number of revolutions is observed by watching the pointer and employing a stop-watch. There is often an arrangement supplied for engaging or disengaging the worm wheel from the toothed wheel at will, but it is better not to use this, as on engaging the wheel the speed of the siren is always caused to vary slightly. From the measured number of revolutions per second and the number of holes in the disc the frequency can be found.

Loudness.—Any note may vary in three ways, that is, in **pitch**, **loudness**, and **quality**. The pitch depends upon the frequency, the loudness depends upon the energy of the vibrating source, or of the wave transmitted to the ear. The quality will be discussed later, but it may be noted here that it is the quality which enables us to distinguish between notes of the same pitch sounded by different instruments.

The loudness of a note depends upon the **energy** of the vibration, and this in turn obviously depends upon the amplitude. The manner in which the energy depends upon the amplitude may be found as follows :

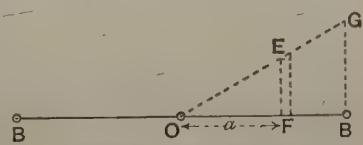

FIG. 616.—Energy diagram.

Let B, B, be the extreme positions of a body of mass m having simple harmonic vibrations. Let v be the velocity of the body when passing through O. Then its kinetic energy here is $\frac{1}{2}mv^2$ (p. 171). The body is under the action of a force which is continually directed towards the centre O, and is proportional to the displacement from O (p. 223). In passing from O to B, work is done against this force ; since the body comes to rest at B, the kinetic energy $\frac{1}{2}mv^2$ has been wholly expended in doing work against the force.

When the displacement is a, let the force be ka, where k is a constant. Let EF (Fig. 616) represent ka ; by erecting similar ordinates, the triangular diagram OBG is produced, and is the diagram of work done against the force (p. 169). If the amplitude OB = A, then

$BG = kA$. Evaluating the total work done, by taking the area of the diagram of work (p. 169), we have

$$\text{Work done} = OB \times \tfrac{1}{2}BG = A \times \tfrac{1}{2}kA$$
$$= \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2.$$

Since k is a constant, the work done, which also represents the energy of the vibration, is proportional to A^2, that is, **the energy of vibration is proportional to the square of the amplitude.** This is true for a vibrating body, and also for the sound waves (p. 683) by which the sound reaches the ear. The **loudness of the sound is therefore proportional to the square of the amplitude of the air vibration.**

The energy of a vibration also depends upon the frequency. Referring to Fig. 608, p. 664, we see that the periodic time of one oscillation $T = 2\pi/\omega = 1/n$, where n is the frequency, and ω the angular velocity of the rotating vector whose projection upon a straight line represents the simple harmonic motion. Further, $\omega = v/A$; hence,

$$\text{Kinetic energy at middle of oscillation} = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2}m\omega^2A^2$$
$$= 2\pi^2mn^2A^2.$$

Hence the energy of the oscillation is proportional to the square of the amplitude and also **proportional to the square of the frequency.**

By blowing a whistle by means of air at known pressure, Lord Rayleigh found the energy per second necessary to blow the whistle, also the distance at which it is just audible was found, and then allowing for the spreading out of the sound waves, it was found that the waves were just audible when the amplitude of vibration was $8{\cdot}1 \times 10^{-8}$ cm.

Limits of audibility.—There is no lower limit of audibility for a note, since, as the pitch becomes very low, the note merely becomes resolved into the separate impulses of which it is composed (p. 669). But if the frequency rises very much, the note becomes very shrill. At a frequency of 15000 the note is little more than a hiss, and at some frequency above this it ceases to be heard at all. Some people have a higher limit of audibility than others, being able to hear a note of frequency 20000 or even 25000. Young people have usually a higher limit of audibility than older people, and many people are unable to hear the squeak of a mouse owing to its high pitch. By means of the Galton's whistle (p. 710) a note of high pitch can be produced, and the pitch varied at will. In this way the limiting frequency of note which any person can hear may be found.

Quality.—Our ability to distinguish between notes of the same pitch produced by different sources is due to difference in **quality**

or **timbre**, which in its turn is due to the complexity of the vibration producing the note. It very rarely happens that the sounding body executes a simple harmonic vibration. The vibration of a tuning-fork is very nearly simple harmonic, and the note given by a fork mounted on a resonance box (Fig. 660) is the nearest approach to a simple tone which we have.

In all other cases the tone is not pure; that is, the vibration frequency which determines the pitch of the note is not the only one present, although the intensities of the others are usually less than that of **fundamental**, or note of lowest pitch. The other notes superimposed on this are called **overtones**, and are usually very simply related in frequency to the fundamental. They consist of notes having 2, 3, 4, etc., times that of the fundamental. In the case of a stretched string (Chap. LVIII.) the number and intensity of the overtones depend upon the place at which the string is struck or bowed, and in an organ pipe they depend upon whether the pipe is 'closed' or 'open' (Chap. LIX.).

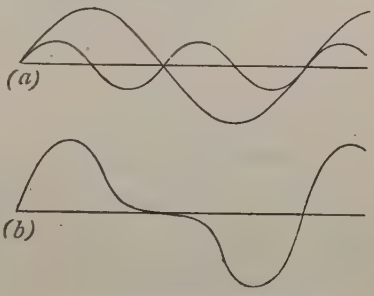

FIG. 617.—Combination of two waves.

The ear is capable of resolving unconsciously the complex wave due to such a source, and these overtones give rise to difference in quality of the notes which enables us from experience to recognise the source from which the note comes.

In Fig. 617 (*a*) two separate simple harmonic waves are represented by displacement-time curves, one of which has twice the frequency of the other. In Fig. 617 (*b*), the wave compounded of these two is given. It is obtained by adding the ordinates of the other two waves. The ear is capable of recognising the two vibrations, when a wave of type (*b*) falls upon the ear.

Compounding of vibrations.—In the preceding case, an example has been given of the compounding of two vibrations, but these have been supposed to take place in the same direction. The resulting vibration is therefore complex, but is still in the direction of its components. When a body has two simple harmonic motions at right angles to each other, they may be compounded and the resultant motion obtained by means of a simple construction.

Let one of the vibrations take place in the direction AB and the other in BC (Fig. 618), the frequencies being equal. If the two

vibrations are in the same phase (p. 225) they both leave their extreme positions simultaneously and also pass through their central positions simultaneously. Thus let the body start at the position A. Upon AB and BC construct semicircles and divide them into equal parts Aa_1, a_1b_1 ..., etc., Ba_2, a_2b_2, ..., etc. On account of the first vibration, the body's position is given by the projection of O_1a_1, and on account of the second vibration it would be given by the projection of O_2a_2. Its actual position will therefore be a. After the next interval it will be at b, then at c, d, e, etc. In fact its path will be along the diagonal AC. The second half of the vibration will be a return from C to A. The resultant motion will therefore be a simple harmonic motion along the diagonal AC.

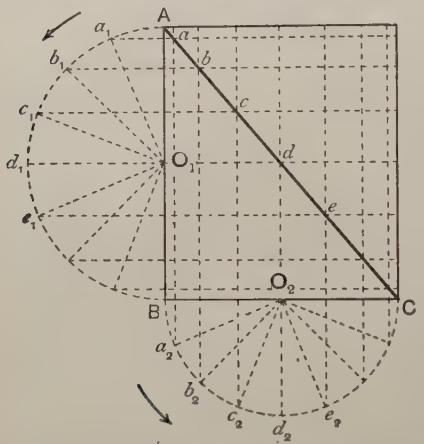

FIG. 618.—Two vibrations at right angles and in the same phase.

If the two component vibrations are not in the same phase, the path of the resultant motion will be an ellipse. Let the body start at the middle position for the vibration 2, and the end position for vibration 1. Its real position is then given by A and d_2; it is therefore at O (Fig. 619).

One-sixteenth of a vibration later it will be at 1; given by a_1 and e_2. Then at 2, 3, 4, etc. During the remaining part of its vibration it will complete the ellipse.

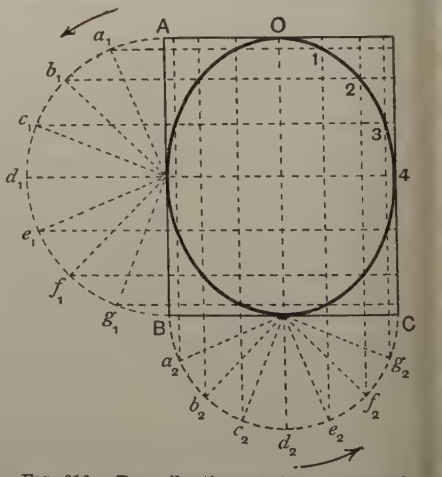

FIG. 619.—Two vibrations at right angles and differing in phase by $\pi/2$.

It will be seen that if the amplitudes of the two component vibrations are equal, AB = BC, and the motion will be a circle.

Thus, if the two component vibrations are in the same phase, the resultant motion is a straight line, but when they differ in phase by a quarter of a vibration the resultant is an elliptical, or circular motion.

These facts may be illustrated by means of a simple pendulum consisting of a weight hanging by a thread. When vibrating, if it receives an impulse at right angles to its plane of vibration, at the instant when it passes its central point, it will then execute a linear vibration as in Fig. 618. If, however, the impulse be given when at the end of a swing, the vibration will then become elliptical, or circular.

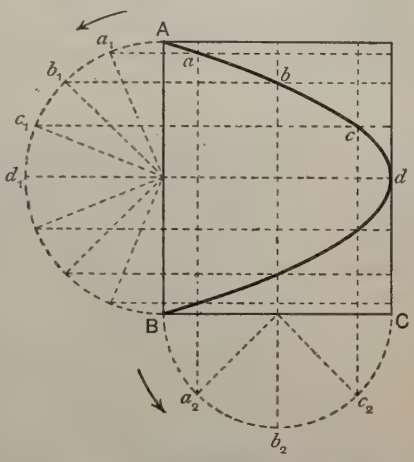

FIG. 620.—Two vibrations at right angles with frequencies as 2 : 1.

If one of the component vibrations has twice the frequency of the other, the resultant motion may be found as before. The method can be followed in Fig. 620, where the relative phases are such that the path is a curved one, A*abcd*B; and in Fig. 621 it is a figure 8. For different phases the figure changes, and if the ratio of the frequencies is not exactly the given fraction, then the form slowly changes from one shape to another. The three simplest cases of compound vibrations are given in Fig. 623, the frequencies being 1 : 1, 2 : 1 and 3 : 1.

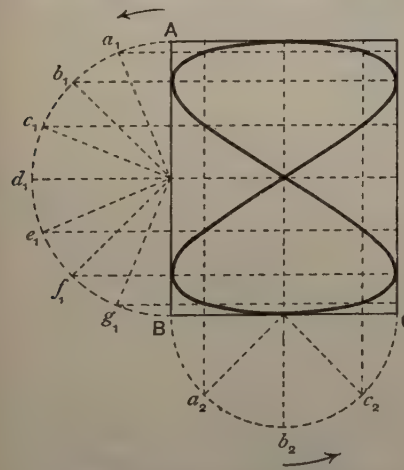

FIG. 621.—Two vibrations at right angles with frequencies as 2 : 1.

Lissajous figures. — The above curves may be produced by means of two tuning-forks, a beam of light being reflected from two mirrors in turn, one attached to each fork. If the mirrors are attached directly to the forks, the resulting motion of the beam of light is

so small that the figures can only be produced on a very small scale. But by attaching a small mirror, such as a galvanometer mirror, to a thin strip of mica fastened to the prong of the forks, as at M_1 and M_2 (Fig. 622), the amplitude of movement of the reflected beam of light may be considerable, and the figures are produced on a large scale upon a screen. After a very few vibrations, the motion of the mirror will be an accurate magnified copy of that of the forks. A powerful source of light is situated behind a screen having a small hole S in it. The lens L then is adjusted in position until it produces a sharp image of S at a distance equal to that of the screen. The mirror M_1 is now placed to receive the beam of light, which then passes to M_2 and thence to the screen. If the fork F_1 only is in vibration, the spot of light on the screen executes a vertical motion and

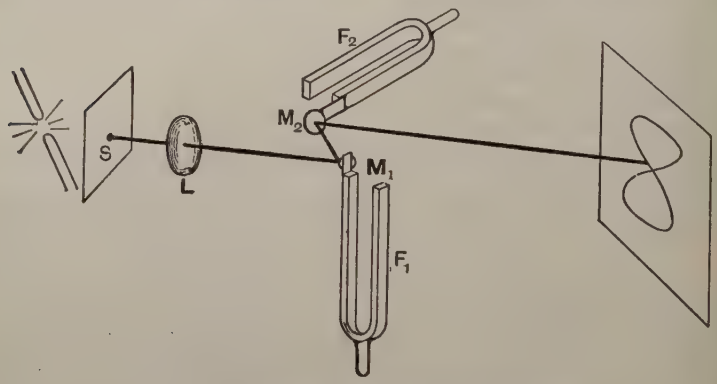

FIG. 622.—Apparatus for producing Lissajous figures.

appears drawn out into a vertical straight line. The vibration of F_2 alone would draw it out into a horizontal straight line. When both forks vibrate simultaneously the spot describes a path whose shape depends upon the frequencies and phases of the forks. Such figures are called Lissajous figures, and they enable us to determine with great accuracy the ratio of the frequencies of the two forks when the ratio consists of small whole numbers.

Thus, if the frequencies are 1 : 1 the figure will be of the form in Fig. 618 or Fig. 623. If the frequency is exactly 1 : 1 the Lissajous figure will remain constant, but if it differs from 1 : 1 by a very small amount, the figure will gradually change, going through the series and returning to its original shape when one fork has executed exactly one vibration more than the other.

Thus, if one fork has a frequency of 256 and the complete cycle of change of the Lissajous figure is made in 10 seconds, then

while one fork has made 2560 vibrations the other has made 2559 or 2561. Which of these numbers is correct can be determined, either by noting the direction of change of the figure, to see which

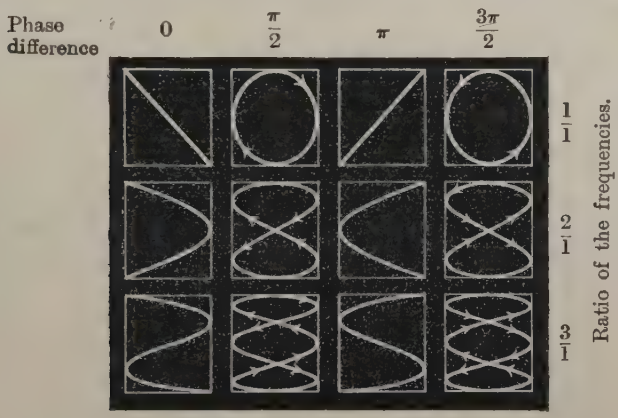

FIG. 623.—Lissajous figures.

fork gains on the other, or by slightly loading one fork to make it vibrate more slowly, and noting the change produced upon the figure. In Fig. 623, Lissajous figures for the frequency ratios 2 : 1 and 3 : 1 also are shown.

EXERCISES ON CHAPTER LIII.

1. How does a noise differ from a musical note ?

Describe a siren, and explain how you would use it to determine the frequency of a note.

2. Describe a method by which the frequency of a fork may be determined by tuning a siren to give the same note.

3. Show that the kinetic energy of a vibrating body is proportional to the square of the amplitude of vibration.

4. Describe how two simple harmonic motions taking place in the same direction may be compounded. Also, how two simple harmonic motions in directions at right angles to each other may be compounded.

5. What are Lissajous figures ? How may they be used to compare the frequencies of two tuning-forks ?

6. Explain fully how a siren may be used to determine the frequency of a tuning-fork ; and suggest some method for checking the result obtained.

A siren having a ring of 200 holes is making 132 revolutions a minute, and it is found to emit a note which is an octave lower than that of a given tuning-fork. Find the frequency of the latter. L.U.

7. Show that two simple harmonic motions at right angles to each other, simultaneously impressed upon a body, may give it an elliptical motion, which under certain circumstances becomes circular. L.U.

8. Two tuning-forks whose frequencies are approximately in the ratio 2 : 1 are employed to produce Lissajous figures, and it is observed that the figure goes through a cycle of change in 15 sec. On loading slightly the fork of upper pitch the figure goes through a cycle of change in 10 sec. If the fork of lower pitch has a frequency of 300, what is the frequency of the other fork before and after loading?

9. A body executes a simple harmonic motion of amplitude 1·5 cm., and frequency 88 per sec. If its mass be 0·8 gram, find the kinetic energy when at the middle of an oscillation.

10. Upon what does the quality of a musical note depend? How is it that the ear can detect a difference between the note produced by a tuning-fork and that produced by an organ pipe when both notes have the same pitch?

CHAPTER LIV

WAVE MOTION

Transmission of sound.—It is obvious that something passes from the sounding body to the ear, in order to produce the sensation of sound. From the motion of the sounding body we should conjecture that some sort of wave motion is passing outwards from it. That the air is the medium in which this wave motion takes place may be shown by hanging up an electric bell, inside an air-tight bell-jar, and gradually pumping the air out (Fig. 624). As the amount of air in the jar becomes smaller, the sound of the bell gets feebler. Complete silence is never attained, because the bell must be supported in some way, and waves pass to some extent through the supports, even though the bell be hung by india-rubber cords. Nevertheless, the great reduction in the sound shows that the air is the chief medium for the transmission of sound waves.

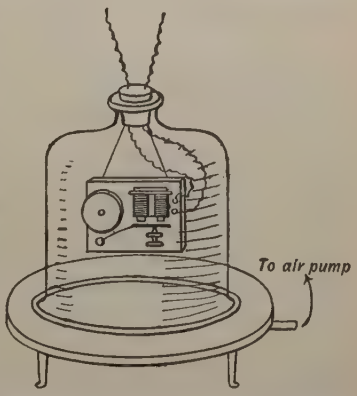

FIG. 624.—Experiment to show that sound is transmitted by air.

All material substances, however, can transmit sound waves; if a faint tapping or scratching be made at one end of a long table, the sound can be heard on putting the ear to the other end of the table, even when the noise is too faint to be heard when the ear is not placed close to the wood. This shows that the sound waves have been transmitted through the wood.

Transverse waves.—There are two types of wave motion. In one, the particles of the medium travel in paths at right angles to

the path of the wave, as in the case of waves upon the surface of water. In the other, the particles of the medium travel backwards and forwards in a path whose direction is the same as that in which the wave is travelling. The former kind is called a **transverse wave**, the latter a **longitudinal wave**. Sound waves are of the longitudinal type; these are not so easily represented, or studied, as transverse waves, which will therefore be described first.

The most familiar case of a transverse wave is perhaps that which travels along a rope or string if one end is suddenly jerked sideways. If the rope is not stretched too tight the wave can be seen travelling along it. If the end be given a backwards and forwards motion, a succession of waves will travel along it, and if the motion given to the end be simple harmonic, the wave can be conveniently represented by a sine curve. The displacement of each particle is lateral, and at any instant may be represented to scale by a curve of the form ABCDEF (Fig. 625). As the wave passes, each piece of the string executes a simple harmonic motion of amplitude B*b* or F*f*.

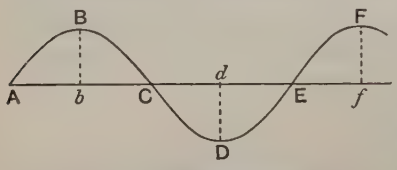

FIG. 625.—Graphical representation of a harmonic wave.

From analogy with waves on the surface of water, B and F are often called **crests**, and D a **hollow**.

The motion of any part of the string repeats itself after a length of wave equal to AE passes over it; AE is therefore called the **wave-length**. The wave-length is therefore the distance between any two consecutive parts of the medium whose vibrations are in the same **phase**. Thus, $BF = AE = \lambda$, the wave-length.

Relation between wave-length, frequency and velocity.—While the end of the string executes a complete vibration, the wave travels a distance λ along the string. Therefore, if n vibrations are made in one second, the disturbance emitted at the beginning of the second will have travelled a distance $n\lambda$ by the end of that second. This is the velocity V of the wave in the string;

$$\therefore V = n\lambda.$$

It is a general fact, quite independent of the form of the wave, that

Velocity of wave = frequency × wave-length.

Wave velocity and particle velocity.—It must be carefully borne in mind that the velocity of the wave is quite distinct from the velocity of a particle of the medium in which the wave is travelling.

Thus, if the wave ABC is travelling forward with velocity V, any particle such as a (Fig. 626) which vibrates transversely about some mean position, has a velocity v, which varies from instant to instant. **In a transverse wave v and V are at right angles to each other.**

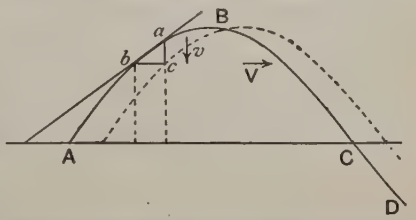

FIG. 626.—Diagram for finding particle velocity.

While the wave moves forwards through the small distance bc, the particle moves through the distance ac. Now ac/bc is known as the **slope** of the curve ABC;

$$\therefore \frac{\text{Particle velocity}}{\text{Wave velocity}} = \text{slope of wave curve.}$$

Thus, at B the particle velocity at the given instant is zero, while at A and C the particle velocity is greatest.

Equation of simple harmonic wave.—It has already been seen that a simple harmonic motion can be represented by the sine of a continually increasing angle (p. 664), and therefore has the form $y = a \sin \omega t$. The angular velocity ω is that of the rotating vector in Fig. 608. If T is the time for one rotation, $\omega = 2\pi/T$, and the equation for the simple harmonic motion may then be written

$$y = a \sin 2\pi \frac{t}{T}.$$

Therefore every particle of a medium through which a simple harmonic wave is passing, executes a vibration of the type given by this equation. But the vibrations are not all in the same phase, or there would be no progressive wave. Each particle is a little later than the one behind it, and this lateness in phase increases from point to point along the wave.

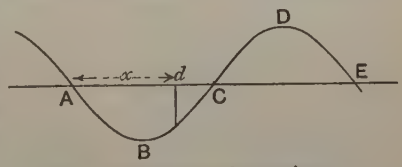

FIG. 627.—Curve for harmonic wave.

Thus the particle at B (Fig. 627) is always a quarter of a period later than one at A. That at C is half a period later than A and so on until at E the particle is a whole vibration (or 2π) later than A; this is equivalent to saying that it is in phase with A. Thus the equations to the motion of the particles B, C and D are respectively,

$$y = a \sin\left(2\pi \frac{t}{T} - \frac{\pi}{2}\right), \quad y = a \sin\left(2\pi \frac{t}{T} - \pi\right) \text{ and } y = a \sin\left(2\pi \frac{t}{T} - \frac{3\pi}{2}\right);$$

and for E, $$y = a \sin\left(2\pi \frac{t}{T} - 2\pi\right) = a \sin 2\pi \frac{t}{T}.$$

The distance from A to E is the whole wave-length λ. Measure distances from A as origin where the particle is just about to start a vibration, and let x be the distance from A of a particle at d. The distance Ad is x/λ of a wave-length, and the retardation in phase between A and d is $x/\lambda \cdot 2\pi$. Hence the equation of motion of the particle d is

$$y = a \sin\left(2\pi \frac{t}{\mathrm{T}} - 2\pi \frac{x}{\lambda}\right),$$

or

$$\mathbf{y = a \sin 2\pi\left(\frac{t}{\mathrm{T}} - \frac{x}{\lambda}\right)}.$$

This equation gives the complete form of the wave, for by putting any given constant value for x, we obtain the equation for the motion of the particle at this point. Again, if we put any constant value for t, it gives us the shape of the whole wave at this instant. Thus, when $t=0$, $y = -a \sin 2\pi x/\lambda$, and this is the equation to the curve at the instant shown in Fig. 627.

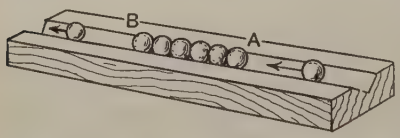

FIG. 628.—Demonstration of compression wave.

Longitudinal waves.—Sound waves in air are entirely longitudinal waves. Hence the form of the wave cannot be shown by a sine curve such as that of Fig. 627. In order to understand the propagation of a longitudinal wave, consider a row of marbles resting in a straight groove and touching each other (Fig. 628). If one marble be driven against the end of the row at A, one will fly off at the other end of the row at B. The reason is that when the marble at A is struck it is momentarily compressed. In recovering its shape it compresses the marble in front of it, and so on along the row. The last but one drives off the last marble in recovering. Thus a wave of compression travels down the row. If two marbles in contact be driven against A, two will fly off at B, since two compression waves, one following the other, pass down the row.

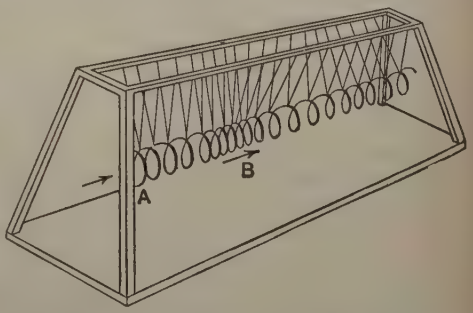

FIG. 629.—Demonstration of longitudinal waves.

A further illustration of a compression wave may be obtained

by suspending a spring from a frame-work by means of threads as in Fig. 629.

On pushing the end A of the spring suddenly forward, the end part becomes compressed. This part in recovering will compress that in front of it, and so on. If the spring is very light and strong the compression wave travels too quickly to be seen very well, but if the spring is of thin iron wire and is weighted by clamping little pieces of lead to it throughout its length, the compression wave can be followed as it travels along.

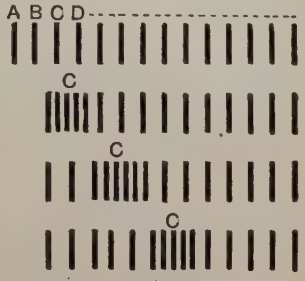

FIG. 630.—Compression wave.

We can conjecture from the above, what a compression wave in air is like. For, if ABCD, etc. (Fig. 630), represent equally spaced planes in air when the pressure is uniform throughout, then if A be suddenly driven to the right, the air in the neighbourhood of A and B is compressed. This in recovering compresses the air in front of it and so on. The successive rows represent stages in the progression of the compression wave C from left to right.

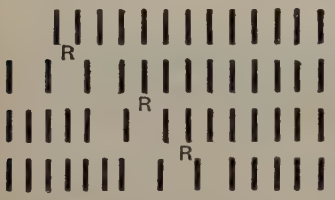

FIG. 631.—Rarefaction wave.

If, on the other hand, A had been pulled to the left, a lowering of pressure would have been produced, and in a similar way a rarefaction R would have travelled along, as represented in Fig. 631.

Now, if A is the prong of a tuning-fork, when it travels to the right a compression is started. When it moves to the left a rarefaction is started. These follow one another, and as the fork vibrates, a succession of compressions and rarefactions will travel forward.

In all these cases, the movement of the air particles is in the same direction as that in which the wave travels, and the waves are therefore of the longitudinal type.

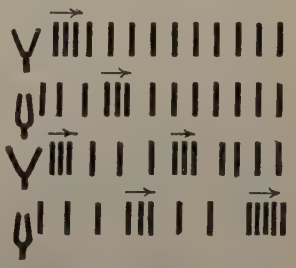
FIG. 632.—Longitudinal wave.

If the prong of the fork moves very slowly backwards and forwards, no wave is produced, because the air merely flows round the fork as it moves. The motion of the

fork must be sufficiently rapid to produce compression in the air, or there will be no wave. The action is similar to that of gunpowder. If this burns slowly in the open, the gases produced are able to pass slowly outwards and there is no explosion. If, however, the powder can be exploded instantaneously there is no time for the gases produced to escape, and they suddenly compress the air surrounding them, giving rise to a compression wave which produces the characteristic sound of an explosion.

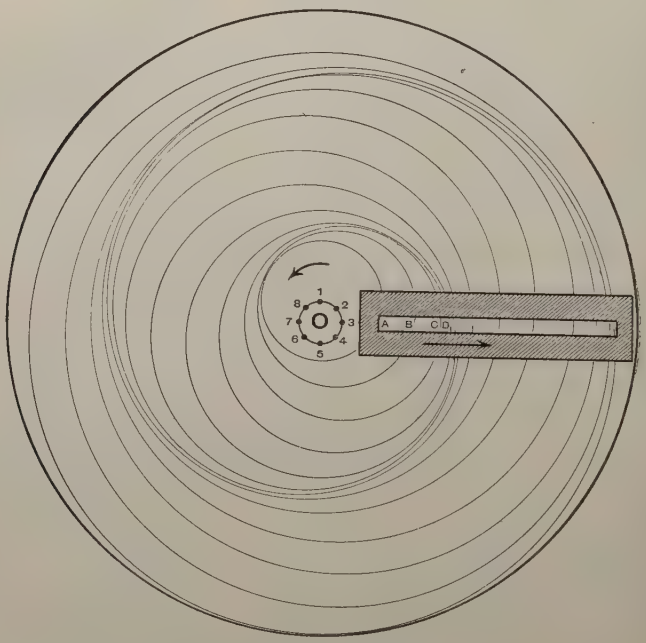

FIG. 633.—Crova's disc.

The motion of the air through which a harmonic wave is passing may be illustrated very well by means of **Crova's disc.** A number of circles increasing in radius by small steps are drawn with centres equally spaced round a small circle 123, etc. (Fig. 633). A strip of cardboard or metal has a rectangular slot cut in it and is placed with the slot over the circles so that small parts ABC, etc., of the circles can be seen. On rotating the large circles about the centre O, each small part ABC, etc., moves backwards and forwards along the slot over a path equal to the diameter of the circle 123, etc.

Compression and rarefaction waves will then be seen to travel successively along the slot.

Representation of a longitudinal wave by a sine curve.—Since the particles of the medium move in the direction of propagation of the wave, it follows that in the case of a longitudinal wave, particles originally in a straight line in the direction of the wave remain in that straight line. This row of particles will never take the shape of a sine curve, as do the particles in the case of a transverse wave (Fig. 625). The correct representation of the wave must therefore

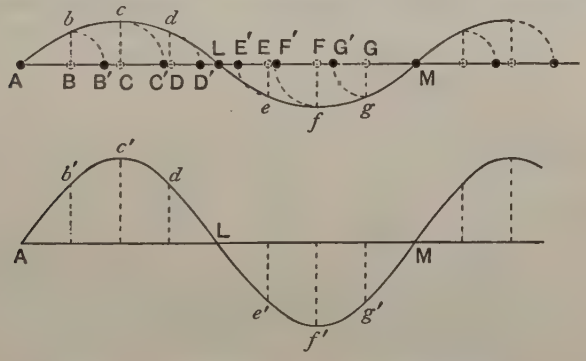

FIG. 634.—Displacement curve for longitudinal wave.

be by means of a row of strokes or dots as in Fig. 632. Nevertheless the wave may be represented **diagrammatically** to scale by means of a sine curve in the following manner :

If ABC ... EFG (Fig. 634), are the undisturbed positions of certain particles, then at a given instant during the passage of the wave, their positions may be B′, C′, ... etc. ... E′F′. If we make Bb=BB′, Cc=CC′, etc., and Ff=FF′, measuring a displacement to the right hand upwards, and to the left hand downwards, the curve A*bcd* ... etc. ... *efg* is obtained, in which the ordinates represent the **displacements** of the particles. Such a curve is called a **displacement curve** for the wave, and the change in displacement of the particles may be found by moving such a curve with the velocity of propagation of the wave, as on p. 681.

Owing to the usual smallness of the displacements in the case of an actual air wave the curve is very flat, but there is no reason why the displacements should not be represented by ordinates

drawn to scale. Thus, in the curve A$b'c'd'$ ... etc. ... $e'f'g'$, every ordinate is twice the corresponding ordinate of the curve Abcd ... efg.

Compression curve.—It will be seen in Fig. 634 that L is a point of compression, because the air both in front and behind is displaced towards L, so that the density of the air is above the average. Similarly A and M are points of rarefaction, because the greatest separation of the air particles occurs there.

It is now possible to draw a new curve which represents the amount of compression, or rarefaction, at each point of the wave. Let ABCDE (Fig. 635 (a)) be the displacement curve for a given wave. For two neighbouring points, e and f, the displacements are eg

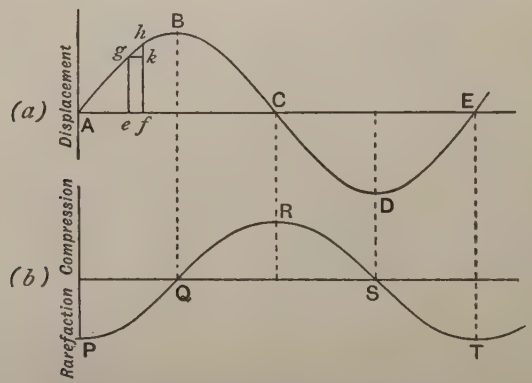

FIG. 635.—Compression curve for longitudinal wave.

and fh. Consider ACE to be the axis of a cylinder of unit cross-section; when there is no displacement, the air within this cylinder is in a similar condition to the air outside, and the volume of air between two planes, at right angles to the axis and passing through e and f, is ef. Now when the wave is in the position shown (Fig. 635 (a)), the plane through e is displaced forwards a distance eg (to scale) and the plane through f is displaced through distance fh. If these displacements were equal, the volume of the air would still be ef, but as they are not equal there is a change in volume equal to $fh - eg = hk$ (to scale). Hence the amount of **compression or rarefaction**, that is, the **volumetric strain** (p. 154) in this air is equal to hk/ef. When e and f are very close together hk/ef is the slope of the curve, and hence **the slope of the displacement curve measures the amount of compression or rarefaction** at any point. Comparison with Fig. 634 shows that when the slope is in one direction it indicates a compression, and in the other direction a rarefaction,

and where the displacement curve is horizontal there is neither compression nor rarefaction, the pressure being normal. In Fig. 635 (*b*) the slope of the curve ABCDE has been plotted to scale. Thus there is maximum compression at R, and maximum rarefaction at P and T

Exercises on Chapter LIV.

1. Explain the difference between a transverse wave and a longitudinal wave. Of which type is a sound wave?

2. Explain the terms 'wave-length' and 'frequency.'
A given tuning-fork produces air waves of which the wave-length is $2\frac{1}{2}$ feet. If the velocity of the waves is 1100 feet per second, what is the frequency of the fork?

3. Show how a transverse wave may be represented by means of a curve, and also how the velocity of any particle may be found from this curve.

4. Describe how a longitudinal wave may be represented by a sine curve. What is the distinction between a displacement curve and a compression curve?

The disc of a siren having a circle of 40 holes rotates uniformly 500 times in 1 minute 24 seconds. Find the frequency of the note emitted, and its wave-length in air if the velocity of sound in air is 34,000 cm. per second. L.U.

6. Explain the terms 'frequency' and 'wave-length.'
Draw diagrams to illustrate the difference in character of a compression wave and a transverse wave having the same frequency and wave-length. L.U.

7. Define and explain the terms *frequency*, *amplitude*, and *wave-length* as applied to sound waves in air. What are the differences in the sensations perceived which correspond to differences in these quantities?
The shortest wave-length that is audible is about 1·8 cm., and the longest about 900 cm. What is the frequency in each case? What is approximately the number of octaves interval between them?
Velocity of sound in air, 33,000 cm. per sec. L.U.

8. Discuss the nature of the vibrations in air, when sound is transmitted through it.
In the case of a musical note, what characteristics of the vibration determine the pitch, intensity, and quality, respectively, of the note? L.U.

CHAPTER LV

SOUND WAVES, VELOCITY, Etc.

Velocity of sound waves.—Air has only one kind of elasticity, that is, bulk elasticity, or **elasticity for compression** (p. 154). Hence the only kind of wave that can be transmitted by air is the compression wave. It is of importance to determine the velocity of such waves in air, and two methods for doing this are open to us. The velocity can be calculated from the known elasticity and density of the air (p. 412), or it may be determined by direct measurement.

That the velocity of sound waves is not exceedingly great is obvious, for on observing anyone at a distance hammering, the sound of the blow will be heard after the hammer is seen to fall, and the interval is greater, the greater the distance between the observer and the hammer; also the interval that elapses between lightning and thunder is familiar to everyone. The further one is from the actual flash of lightning, the more the sound of thunder will be delayed. About five seconds per mile gives a rough estimate of the distance.

Velocity of sound by gun method.—Many determinations of the velocity of sound have been made by measuring the interval between seeing the flash and hearing the report of a distant gun. The method, although simple in principle, is liable to a number of errors.

(i) **Effect of wind.**—When the whole mass of air is moving, that is, when there is a wind, the velocity of the sound waves relatively to the ground is not the same as the rate of propagation through the air. Thus if the wind is blowing from the gun to the observer, the velocity of sound is increased by the velocity of the wind; when the wind blows in the opposite direction it is diminished by an equal amount. The effect of the wind may be eliminated by taking observations in both directions. Thus one observer fires a gun and another at a distance measures the time interval. Then the second observer fires a gun and the first observes the interval. By means

of these reciprocal observations, the true velocity is found, being the mean of the two observed velocities. When the direction of the wind is oblique to the line joining the observers, the calculation is more complicated; but the simple mean value is very near the truth, except when the velocity of the wind is very great.

(ii) **Personal equation.**—The time observations are made by means of a stop-watch, chronometer, or better still, by the chronograph driven by clockwork (p. 667). As, however, the sight impression of the flash of the gun and the sound impression made by the report take different times to react upon the observer and to be recorded by him, another error is introduced. This is known as the **personal equation** of the observer, and it differs for different observers. It is necessary to measure the personal equations of the observers by separate experiments and to apply the corrections to the velocity observations.

(iii) **Temperature and hygrometric state of the air.**—The velocity of sound depends upon the temperature and density of the air, in a manner to be explained later (p. 693). It is therefore necessary to correct the observations in order to find the velocity in dry air at 0° C.

The mean of the best results of observations of the velocity of sound waves in air by direct experiment gives the value 332 metres per second at 0° C.

Velocity in tubes.—Several experimenters have measured the velocity of sound in tubes, and the experiments of Regnault are the best known. Regnault endeavoured to avoid the error due to personal equation by using mechanical means to record the times. A wire, stretched over the muzzle of the gun is broken when the gun is fired. This breaks an electric current circuit and causes a signal to be recorded upon the revolving drum of a chronograph (p. 667). At the receiving end, the wave enters a cone, ending in a cylinder, across the base of which is stretched an india-rubber membrane. The compression wave drives the membrane forward, and in doing so either makes or breaks another electric circuit, producing another mark upon the same drum of the chronograph used for recording the gun firing. The personal equation is not really eliminated, as this, even in the case of the human ear, is partly due to physical causes. In fact the arrangement of india-rubber drum, electric circuit, and chronograph has a "personal" equation, although this is likely to be more nearly constant than in the case of an animate observer.

Regnault found that the velocity of sound becomes greater with increasing intensity of wave. As the intensity gets less the velocity decreases for a time, but reaches a limiting constant velocity. This limiting velocity for feeble sounds in open air at 0° C. he found to be 330·6 m. per sec.

In tubes, the diameter affects the velocity up to a diameter of about 1 metre. For wider tubes the velocity is the same as in open air. In a tube of 10·8 cm. diameter the velocity was 324·25 m. per sec., and in smaller tubes the velocity was found to be still less.

* **Velocity of sound by calculation.**—The velocity of a compression wave can be deduced from a knowledge of the elasticity and density of the medium through which the wave is passing. For any small part of the medium, the motion is determined by the fundamental relation,

$$\text{Force} = \text{mass} \times \text{acceleration} \quad \text{(p. 67)}.$$

Let AB (Fig. 636) be the displacement curve for any compression wave in general. Then for a thin layer of gas, *ef*, it was seen on p. 686 that the volumetric strain is hk/kg.

Now, Bulk modulus of elasticity $= \dfrac{\text{stress}}{\text{strain}} = \mathrm{K}$ (p. 156).

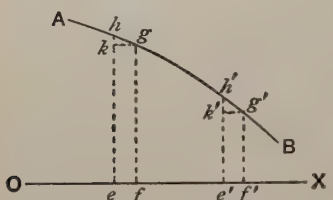

FIG. 636.—Displacement curve.

But, $\text{strain} = \dfrac{hk}{kg}$;

$$\therefore \text{stress} = \mathrm{K} \cdot \frac{hk}{kg}.$$

Again, at $e'f'$, $\text{stress} = \mathrm{K} \cdot \dfrac{h'k'}{k'g'}$.

But the stress is the excess of pressure at any point above the normal pressure throughout the medium. Hence the excess of pressure at *ef* is

$$p = \mathrm{K} \cdot \frac{hk}{kg},$$

and at $e'f'$,

$$p' = \mathrm{K} \cdot \frac{h'k'}{k'g'};$$

$\therefore$ Difference of pressure at *ef* and $e'f'$

$$= p - p' = \mathrm{K}\left(\frac{hk}{hg} - \frac{h'k'}{h'g'}\right).$$

Now, consider a tube of the gas of unit cross-section, the tube being parallel to OX, the direction of propagation. The layer of gas between *e* and *e'* has a force p exerted upon the face *e*, and an opposite force p' upon *e'*. Hence the resultant force acting on this layer is $p - p'$. Again, the mass of gas in this layer is

$$\text{vol.} \times \text{density} = ee' \times d.$$

* May be omitted on first reading.

But Force = mass × acceleration ;

$$\therefore\ p - p' = ee' \times d \times \text{acceleration} ;$$

$$\therefore\ \text{acceleration} = \frac{p-p'}{ee' \times d} = \frac{\mathsf{K}\left(\frac{hk}{kg} - \frac{h'k'}{k'g'}\right)}{ee' \times d}$$

$$= \frac{\mathsf{K}}{d} \cdot \frac{\frac{hk}{kg} - \frac{h'k'}{k'g'}}{ee'} . \qquad \text{.....(1)}$$

Again, let V be the velocity of propagation of the wave in the direction OX; then the time required for the wave to travel over the distance ef is ef/V, and in this time the displacement at e or f changes by the amount $(eh - fg) = hk$;

$$\therefore\ \text{velocity of particle at } ef = \frac{hk}{\frac{ef}{\mathsf{V}}} = v ;$$

$$\therefore\ v = \mathsf{V}\frac{hk}{ef},$$

and velocity of particle at $e'f' = \mathsf{V}\dfrac{h'k'}{e'f'} = v'$;

$\therefore$ while the wave travels from ef to $e'f'$ the particle velocity changes from v to v' and the time taken is ee'/V ;

$$\therefore\ \text{acceleration of particle} = \frac{v-v'}{\frac{ee'}{\mathsf{V}}} = \mathsf{V}\frac{v-v'}{ee'}$$

$$= \frac{\mathsf{V}}{ee'} \cdot \mathsf{V}\left(\frac{hk}{ef} - \frac{h'k'}{e'f'}\right)$$

$$= \mathsf{V}^2 \frac{\left(\frac{hk}{ef} - \frac{h'k'}{e'f'}\right)}{ee'} . \qquad \text{.....(2)}$$

Equating the expressions for the acceleration given by (1) and (2), we obtain

$$\mathsf{V}^2 = \frac{\mathsf{K}}{d},$$

or

$$\mathsf{V} = \sqrt{\frac{\mathsf{K}}{d}} .$$

Velocity of sound in terms of pressure and density.—The fact that the velocity of sound is equal to $\sqrt{\dfrac{\text{modulus of elasticity}}{\text{density}}}$ was first established by Newton. He took the elasticity to be the pressure, in absolute measure, of the atmosphere. Thus, consider a given

quantity of air to have pressure p_1 and volume v_1, and let these change by a very small amount **at constant temperature**, becoming p_2 and v_2.

Then, from Boyle's law (p. 396),

$$p_1v_1 = p_2v_2.$$

Also the stress $p_2 - p_1$, produces the strain $\frac{v_1 - v_2}{v_1}$, and from the definition of the bulk modulus of elasticity,

$$K = \frac{\text{stress}}{\text{strain}} = \left(\frac{p_2 - p_1}{v_1 - v_2}\right) v_1 = \frac{p_2v_1 - p_1v_1}{v_1 - v_2}.$$

Now, replacing p_1v_1 by p_2v_2, we have

$$K = \frac{p_2v_1 - p_2v_2}{v_1 - v_2} = p_2.$$

Hence the elasticity is represented by the absolute pressure, provided that the temperature is constant.

For air at 0° C., $p = 13{\cdot}6 \times 76 \times 981$ dynes per sq. cm.,

and $d = 0{\cdot}00129$ grams per c.c. ;

$$\therefore \text{ Velocity of sound} = \sqrt{\frac{13{\cdot}6 \times 76 \times 981}{0{\cdot}00129}} = 28100 \text{ cm. per sec.}$$

This value is too low, and the reason for the discrepancy was pointed out by Laplace. The compressions and rarefactions in a sound wave occur so rapidly that the rise of temperature during compression, and the lowering of temperature during rarefaction have not time to become obliterated by conduction. When a gas expands, or is compressed so rapidly that no heat leaks in or out, the change is said to be **adiabatic**, and Boyle's law no longer applies. In its place, the relation,

$$p_1v_1^{\gamma} = p_2v_2^{\gamma},$$

holds good (p. 430), where γ is the ratio of the specific heat of air at constant pressure to that at constant volume, which in the case of air is 1·41.

Now the relation between pressure and volume may be written

$$p_1v_1^{\gamma} = p_2(v_1 - v_1 + v_2)^{\gamma}$$

$$= p_2v_1^{\gamma}\left(1 - \frac{v_1 - v_2}{v_1}\right)^{\gamma}.$$

Expanding the last expression by the binomial theorem, we have

$$1 - \gamma\frac{v_1 - v_2}{v_1} + \frac{\gamma(\gamma - 1)}{1 \cdot 2}\left(\frac{v_1 - v_2}{v_1}\right)^2 - \ldots .$$

But if the change in volume, that is, $v_1 - v_2$ be extremely small, the terms containing $\left(\frac{v_1 - v_2}{v_1}\right)^2$ and higher powers may be neglected;

$$\therefore\ p_1 = p_2\left(1 - \gamma\frac{v_1 - v_2}{v_1}\right),$$

or
$$\frac{p_2 - p_1}{v_1 - v_2}v_1 = \gamma p_2,$$

that is, modulus of elasticity $\mathsf{K} = \gamma p_2$,

and velocity of sound $= \sqrt{\frac{\gamma p}{d}}$.

Using the same data as before for the atmosphere,

$$\mathsf{V} = \sqrt{\frac{1{\cdot}41 \times 13{\cdot}6 \times 76 \times 981}{0{\cdot}00129}} = 33160 \text{ cm. per sec.}$$

This is in close agreement with the value obtained by experiment, and therefore establishes the truth of Laplace's correction.

Effect of pressure upon the velocity of sound.—If the pressure of the atmosphere could be changed without altering the density, the velocity of sound would be changed. But, provided that the temperature remains constant, the density of the atmosphere is proportional to the pressure, from Boyle's law. Hence the numerator and denominator in the expression $\sqrt{\gamma p/d}$ both vary in the same ratio. It follows that the velocity of sound in any gas is independent of the pressure, so long as Boyle's law holds good. The variation in height of the barometer does not, therefore, affect the velocity of sound. It has been established by direct experiment that the velocity of sound at a considerable height up a mountain is the same as at the sea-level.

Effect of temperature upon the velocity of sound.—The pressure of the atmosphere does not depend upon the temperature of the atmosphere; consequently the numerator in the expression $\sqrt{\gamma p/d}$ does not alter with change of temperature. But the density of the atmosphere does depend upon the temperature. For a given mass of gas, let the density at T_0 be d_0, where T_0 is the absolute temperature. Then if the temperature becomes T_1 the density changes to d_1, and

$$\frac{d_0}{d_1} = \frac{\mathsf{T}_1}{\mathsf{T}_0} \text{ (p. 411).}$$

Now the velocity of sound at 0° C., or T_0, is given by the equation

$$V_0 = \sqrt{\frac{\gamma p}{d_0}},$$

and

$$V_1 = \sqrt{\frac{\gamma p}{d_1}} = \sqrt{\frac{\gamma p \cdot T_1}{d_0 T_0}};$$

$$\therefore V_1 = V_0 \sqrt{\frac{T_1}{T_0}}, \quad \text{or} \quad \frac{V_1}{V_0} = \sqrt{\frac{T_1}{T_0}}.$$

That is, **the velocity of sound in air is proportional to the square root of the absolute temperature.** If the temperatures are expressed upon the Centigrade scale, the relation becomes

$$V_t = V_0 \sqrt{(1 + \alpha t)},$$

where V_t is the velocity at $t°$ C., and α is the coefficient of expansion or $\frac{1}{273}$.

By means of this relation the velocity of sound, measured at any temperature, may be corrected to 0° C. For ordinary atmospheric temperatures this correction amounts to about 61 cm. per sec. per degree.

Velocity of sound in gases other than air.—Since other gases have densities differing from that of air, the velocity of sound in them will differ from that in air. The relation between the velocities may be seen from the following :

$$V_{air} = \sqrt{\frac{\gamma p}{d_{air}}}, \quad V_{gas} = \sqrt{\frac{\gamma p}{d_{gas}}};$$

$$\therefore \frac{V_{air}}{V_{gas}} = \sqrt{\frac{d_{gas}}{d_{air}}}.$$

That is, **the velocity of sound in a gas varies inversely as the square root of the density of the gas,** provided that γ has constant value.

Thus, since oxygen has 16 times the density of hydrogen,

$$\frac{V_{hydrogen}}{V_{oxygen}} = \sqrt{\frac{16}{1}} = 4,$$

and again,

$$\frac{V_{hydrogen}}{V_{air}} = \sqrt{\frac{1 \cdot 29}{0 \cdot 0899}} = 3 \cdot 79,$$

$$V_{hydrogen} = 332 \times 3 \cdot 79 = 1260 \text{ m. per sec. at } 0° \text{ C.}$$

The **correction for moisture in the atmosphere** follows from the above considerations. Thus, if the relative humidity of the air be known, its density compared with dry air at the temperature of observation can be found, and, from the ratio of the densities, the

velocity of sound in dry air can be calculated, knowing the observed velocity in the moist air.

It should be noted that γ, the ratio of the specific heats (p. 692), has been assumed to have the same value for all the gases considered. This is justified in the case of ordinary gases, but for some, such as mercury vapour, helium, or argon, this quantity has a value near to 1·66. In fact, an experimental determination of the velocity of sound, together with a measurement of the pressure and density, affords a method of determining γ, the ratio of the specific heats in cases where these cannot be measured directly.

Velocity of sound waves in water.—The expression

$$\sqrt{\frac{\text{modulus of elasticity}}{\text{density}}}$$

correctly represents the velocity of all waves which owe their propagation to the elasticity of the medium in which they exist. It is not, however, always clear at first sight which modulus of elasticity must be taken. It was seen on p. 693 that in the case of a gas γp represents the correct modulus of elasticity for the calculation of the velocity of sound waves. In the case of a liquid such as water, the bulk modulus must be used, but this is not so simply obtained as in the case of a gas. The expansibility of a liquid being very much less than that of a gas, there is much less error in using the isothermal elasticity instead of the adiabatic elasticity. Taking the bulk modulus of water to be $2{\cdot}04 \times 10^{10}$ and the density to be unity, the velocity of sound should be $\sqrt{2{\cdot}04 \times 10^{10}} = 143000$ cm. per sec.

This is not far from the experimental determination of 139900 cm. per sec. at 4° C. made by Martini in 1888. Colladon and Sturm arranged for a bell to be struck under water in Lake Geneva, the sound being detected at a distance by a large trumpet-shaped receiver, the larger lower end of which was immersed, while the small upper end was placed to the ear. The striking of the bell was accompanied by the ignition of some gunpowder above the surface of the water. The observer timed the interval between the flash and the sound. The value obtained was 143500 cm. per sec. at 8·1° C.

Velocity of sound in rods.—Solids are capable of experiencing various kinds of strain, and therefore there are possibly many forms of wave transmitted. The only kind that will be considered here is that of a compression wave travelling along a narrow rod or wire. In this case the required modulus of elasticity is obviously Young's modulus (p. 156), and the expression

$$\sqrt{\frac{\text{Young's modulus of elasticity}}{\text{density}}}$$

gives correctly the velocity of sound in such bodies.

The velocity may be determined by experiment, as described in Chap. LIX.

The following table gives the velocity of sound in rods for different substances.

VELOCITY OF SOUND.

Material.	Cm. per sec.
Aluminium - -	$5{\cdot}10 \times 10^5$
Glass (soda) - -	$5{\cdot}0 - 5{\cdot}3 \times 10^5$
„ (flint) - -	$4{\cdot}0 \times 10^5$
Brass - - -	$3{\cdot}65 \times 10^5$
Deal (along the grain)	$5{\cdot}0 \times 10^5$
Fir „ „ -	$4{\cdot}5 - 5{\cdot}3 \times 10^5$
Oak „ „ -	$4{\cdot}0 - 4{\cdot}4 \times 10^5$
Pine „ „ -	$3{\cdot}3 \times 10^5$

Inverse square law.—The falling off in the intensity of sound due to a sounding body as the distance from it increases, may easily be found, provided that the sound waves spread out uniformly from the body in all directions. If, however, this uniform spreading out is interfered with by the process of reflection or refraction, the following simple law no longer holds.

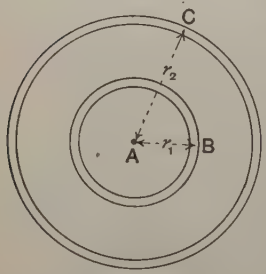

FIG. 637.—Illustration of the law of inverse squares.

Let A (Fig. 637) be the sounding body, and let the energy radiated in the form of sound waves in one second be a constant quantity. Then the energy in a spherical shell of radius r_1 and thickness unity is, at any moment, equal to

$$\text{Vol. of shell} \times \text{energy per c.c.} = 4\pi r_1^2 \times I_1,$$

where I_1 is the energy per c.c., or the intensity of the sound at the point B.

Similarly, the energy in a shell of radius r_2 at any instant is given by

$$\text{Vol. of shell} \times \text{energy per c.c.} = 4\pi r_2^2 \times I_2.$$

Now, the energy emitted in the form of a wave motion, in a given time, is constant and therefore the amount in each of these shells of equal thickness is constant;

$$\therefore\ 4\pi r_1^2 \times I_1 = 4\pi r_2^2 \times I_2$$

or $$\frac{I_1}{I_2} = \frac{{r_2}^2}{{r_1}^2},$$

or $$\frac{\text{Intensity of sound at distance } r_1}{\text{Intensity of sound at distance } r_2} = \frac{{r_2}^2}{{r_1}^2}.$$

That is, **the intensity of sound due to a given source varies inversely as the square of the distance from the source.**

Speaking tube and sounding board.—The inverse square law only applies when the sound wave spreads uniformly in all directions. A sounding body near the surface of the earth produces sound waves which can spread outwards in one direction only. Thus, a man at the top of a ladder can hear another on the ground speaking, better than he can be heard by him. This is sometimes explained by the statement that "sound rises," but this is obviously untrue. The explanation is that the sound waves from the man at the top of the ladder spread out vertically both upwards and downwards, and

FIG. 638.—Speaking tube.

therefore have less intensity at a given distance, than the waves from the man on the ground, since these only spread upwards over a hemisphere. If the waves are prevented from spreading altogether, it follows that they will fall off very little in intensity as they travel.

Thus, the waves produced at one end of a tube will travel down the tube without spreading, and can be heard clearly by an ear placed at the other end of the tube. Except for the energy wasted through friction between the air and the tube there is no reduction in the energy of such waves. This arrangement is known as the **speaking tube**, and is used in place of the telephone for transmission of speech for small distances.

The sounding board sometimes placed over the speaker's head in very large buildings prevents the spreading of the sound waves in an upward direction, so that the whole energy is given out in an approximately horizontal direction. The falling off in intensity is then more nearly inversely as the distance than inversely as the square of the distance.

Reflection of sound waves.—When a compression meets a rigid wall it can, in recovering, compress only the air behind it. The direction of a compression wave is therefore reversed on meeting a rigid obstacle. This process is called **reflection.** Reflection

may occur under a variety of conditions, but wherever there is a discontinuity of the medium, reflection occurs. We shall see later (Chap. LIX.) a case where reflection occurs at the open end of a tube.

If A (Fig. 639) be a sounding body, the waves of compression and rarefaction on meeting a rigid wall at C are reversed in direction, and as each part of the spherical wave arrives, the component of its velocity perpendicular to the wall is reversed. The reflected wave is still spherical, but is travelling outwards from A′ as centre instead of A. A′ is said to be the **sound, or acoustical, image** of A.

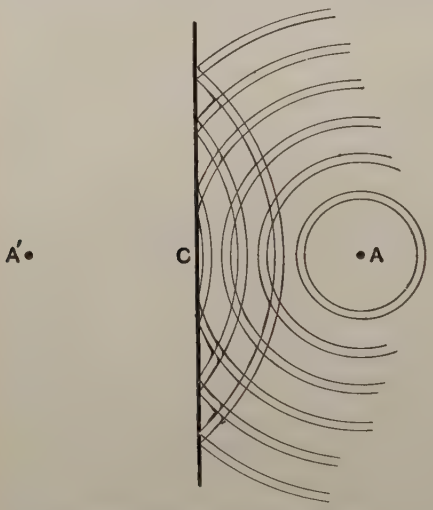

FIG. 639.—Reflection of sound waves.

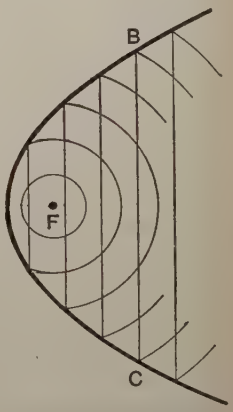

FIG. 640.—Parabolic reflector.

Reflection by curved surface.—In Fig. 639 the reflected waves have the same curvature as the incident waves; but when the reflecting surface is itself curved, this is no longer the case. Thus, if the sounding body be placed at F (Fig. 640) which is suitably placed on the axis of a parabolic reflector BAC, the reflected waves are plane. Conversely, if plane waves fall upon the reflector they pass after reflection through the point F. F is called the **principal focus** of the mirror. By means of two such mirrors the process of reflection may easily be demonstrated.

A source of feeble sound waves, such as a watch, should be placed at the focus F_1 of a parabolic reflector (Fig. 641); the plane waves emitted are allowed to fall on the second parabolic reflector whose axis is coincident with that of the first. The waves are then

reflected, become spherical and travel towards the focus F_2. On placing a funnel at F_2, an india-rubber tube from it being brought to the ear at E, the ticking of the watch may be heard.

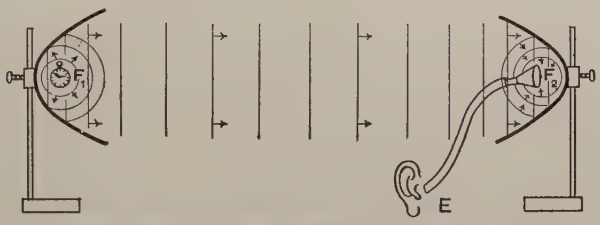

FIG. 641.—Pair of parabolic reflectors.

Echoes.—The reflection from a plane wall gives rise to a sound image. Thus, an observer situated at E (Fig. 642) receives sound waves directly from a source A, and also waves which have been once reflected and are then travelling from A′. In general, the two sets of waves arrive at E with such a small interval of time between them that they cannot be distinguished, one from the other. In an ordinary room, with plane walls, floor and ceiling, the only effect of the reflection is to increase the intensity of sound observed. Anyone who compares the difficulty of speaking or singing in the open air to that in a small room will appreciate this fact.

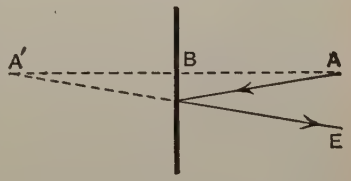

FIG. 642.—Production of echoes.

If, however, the distance AB is such that the interval between the arrival of the direct and reflected waves is about $\frac{1}{30}$ sec. the sound, if of a sudden nature, like a clap of the hands, will appear to be drawn out. With greater distance, so that the interval is, say, $\frac{1}{20}$ sec., two separate sounds are distinguishable, and at still greater distances the interval is such that two distinct sounds can easily be heard. The name **echo** is properly reserved for this case in which the two sounds are distinctly separated.

Remembering that for the interval to be $\frac{1}{20}$ sec., the time for the wave to travel from A to B (Fig. 642) is $\frac{1}{40}$ sec., we see that

$$\frac{AB}{\frac{1}{40}} = \text{Vel. of sound} = 332 \text{ m. per sec.};$$

$$\therefore AB = \frac{332}{40} = 8{\cdot}3 \text{ metres.}$$

Echoes occur in nature, and are produced by cliffs, hillsides, woods, or even by large buildings. When the distance is great enough, quite a large number of consecutive sounds may be repeated by the echo.

In large rooms, such as public halls, the echo may become highly objectionable. Even though the sounds may not be quite separated, the prolongation produced by the echo may lead to great indistinctness. It is very much less when the hall is filled with people than when it is empty.

Refraction of sound waves. Whenever sound waves pass from one medium to another of different density, the waves are bent

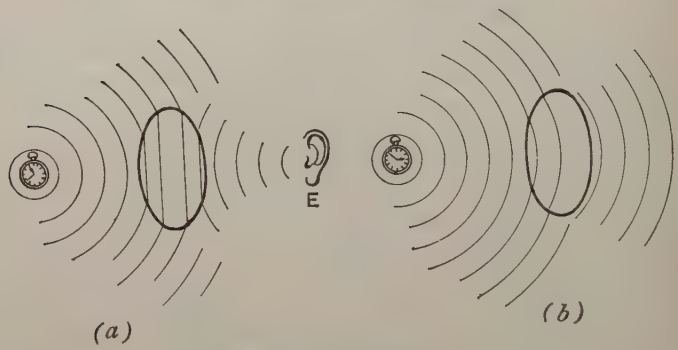

FIG. 643.—Acoustic lens.

or refracted in a manner somewhat similar to that for light (p. 574). Thus, by constructing a lens-shaped vessel, such as an india-rubber balloon, with a gas which is denser than air (such as carbon-dioxide) the counterpart to a converging lens may be produced (p. 587). If the gas be less dense than air, say hydrogen, or coal gas, the resulting lens is of the divergent kind.

In the first case (Fig. 643 (*a*)) the sound waves converge to a focus E, and the ear placed there will hear the ticking of the watch from which the sound waves arise. In the second case (Fig. 643 (*b*)) there is no focus, as the waves diverge more after passing through the lens than before falling upon it.

Effect of wind upon sound waves.—When the wind is blowing from the sounding body towards the observer, the sound is more clearly heard than if there were no wind, and *vice versa*. If the wind consisted of the whole mass of air all moving with the same velocity, there is no reason why the sound should be heard more clearly in

one direction than another; the velocity of sound would, of course, be changed, but not its intensity. However, the velocity of the wind is not everywhere the same; it is least near the ground and increases with increasing altitude. Consider plane compression waves *abc*, etc. (Fig. 644 (*a*)). The wind being considered to travel in the same direction as the waves, the parts at higher altitude will have greater velocity than the lower parts. Hence the waves, originally vertical, become more and more tilted as they travel. But they always travel at right angles to their own planes, hence their direction will be inclined downwards, and an observer at A will hear the sound more clearly than if there were no wind. On the other hand, when the wind is in the opposite direction, the tilt of the waves is opposite, and their direction of motion again becomes inclined, but in such a manner that they travel away from the ground. An observer at B (Fig. 644 (*b*)) will, therefore, not hear the sound so plainly as he would if there were no wind.

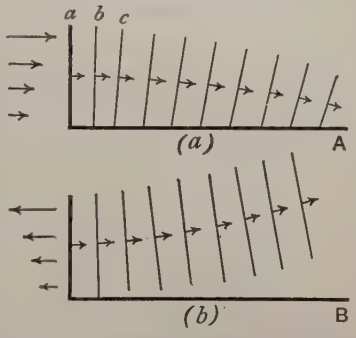

FIG. 644.—Effect of wind upon sound waves.

Atmospheric refraction.—The velocity of sound waves being greater the higher the temperature, it follows that any variation of temperature from one place to another causes refraction of sound waves. During the daytime, the lower layers of the atmosphere are at a higher temperature than the upper layers. It follows, therefore, that a sound wave will be refracted upwards as it travels, exactly as in Fig. 644 (*b*), although from a different cause. Hence there is a tendency for the wave to travel away from the ground, and the intensity at a distance is decreased due to this effect. On the other hand, if the lower layers are at lower temperature than the upper layers, as frequently happens on a still evening and particularly over the surface of water, the refraction of the sound waves is then downwards, as in Fig. 644 (*a*). In this case sounds from distant sources can be heard much more clearly than during the daytime.

Upward streams of hot air, frequently produced on warm days, produce a dispersive effect upon sound waves, since they cause refraction. When the atmosphere is homogeneous, as often happens in a fog, sound waves are propagated regularly, and sounds are much more audible at a distance from the sounding body than when irregularities of temperature occur.

Doppler effect.—It has probably been noticed by everyone, that a sounding body changes its apparent pitch when it passes the observer with considerable velocity. Thus the whistle of a railway engine appears to become considerably lower in pitch as it passes the observer. As the engine approaches the observer, each compression or rarefaction leaving the sounding body is a little nearer to the compression in front than would have been the case if the body were at rest. Hence the frequency with which these compressions arrive at the observer is greater than it would have been if the sounding body had been at rest. As the engine recedes from the observer the frequency appears to be lowered correspondingly.

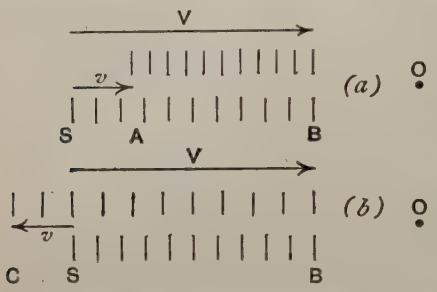

FIG. 645.—The Doppler effect.

Suppose the sounding body S (Fig. 645 (*a*)) to have a frequency n, so that it gives out n waves per second. If V be the velocity of sound, the waves given out in one second will, at the end of the second, occupy the distance SB equal to V. If now S has velocity v, it will at the end of the second be at A, where $SA = v$. All waves except the first are now a little nearer the observer at O than they would have been if S remained at rest. Now, $AB = V - v$, so that n waves now occupy a distance $V - v$ instead of V. The wavelength is therefore changed from V/n to $(V - v)/n$ by the motion of the body. The apparent frequency n', to an observer, is therefore

$$n' = \frac{V}{\frac{V - v}{n}} = n \cdot \frac{V}{V - v}.$$

Of course, if the velocity of S had been in the opposite direction, as in Fig. 645 (*b*)), the apparent frequency to an observer at O would have been

$$n' = n \frac{V}{V + v}.$$

It follows therefore that when the sounding body approaches the observer the pitch is apparently raised, and when it recedes from him the pitch appears to be lowered.

EXAMPLE.—A train approaches an observer at the rate of 72 km. per hour, the whistle of the engine sounding continuously. Find the change in pitch, stated as the ratio of the apparent frequencies, as the train passes the observer, taking the velocity of sound to be 333 m. per sec.

Let the actual frequency of the note $=n$.

Then, apparent frequency during approach $= n \cdot \frac{V}{V-v}$,

and " " receding $= n \cdot \frac{V}{V+v}$;

$$\therefore \text{ Change in pitch} = n \cdot \frac{V}{V-v} \cdot \frac{1}{n} \cdot \frac{V+v}{V}$$

$$= \frac{V+v}{V-v}.$$

Now, $V = 333$ m. per sec.,

and $v = \frac{72000}{60 \times 60} = 20$ m. per sec.;

$$\therefore \text{ Change in pitch} = \frac{333+20}{333-20}$$

$$= \frac{353}{313} = 1{\cdot}128.$$

General expression for the Doppler effect.—It is clear that the motion of the observer must affect the apparent pitch of a note, and the wind may also produce a change. To find the effect of the motion of the source when there is wind of velocity w in the same direction as v, proceed as before. The waves emitted in one second now occupy the distance $AB = V + w - v$ (Fig. 646).

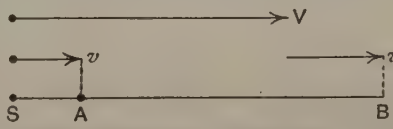

FIG. 646.—Diagram for Doppler effect, moving source.

Thus, the wave-length is changed in the ratio $(V+w-v)/V$, and the apparent frequency to a fixed observer changes from n to $nV/(V+w-v)$.

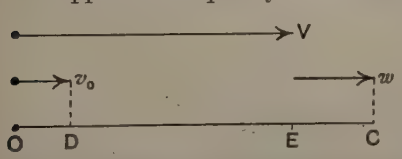

FIG. 647.—Diagram for Doppler effect, moving observer.

Now consider the motion of the observer. If there were no wind, and if the observer were at rest, the n waves occupying a distance $OE = V$ (Fig. 647) would pass him in one second. But actually, the waves occupying a distance $DC = V + w - v_0$, where v_0 is the velocity of the observer, will pass him in one second. Hence on account

of the wind and his own velocity, the frequency changes in the ratio $(V+w-v_0)/V$.

Thus, the total change in pitch due to **motion of observer and source** measured by the ratio of the apparent frequencies, is given by

$$\frac{V+w-v_0}{V} \times \frac{V}{V+w-v} = \frac{V+w-v_0}{V+w-v}.$$

Thus, if n is the actual frequency of the source, the apparent frequency is

$$n\frac{V+w-v_0}{V+w-v}.$$

If v and v_0 are zero, that is, if both source and observer are at rest, or even if they have equal velocities in the same direction, it will be noticed that the fraction reduces to unity. Therefore, unless the bodies have some motion relative to each other, the wind does not affect the apparent pitch. Also, when the wind velocity is small relatively to the velocity of sound, as is usually the case, the apparent pitch is $n\frac{V-v_0}{V-v}$.

Exercises on Chapter LV.

1. How may the velocity of sound in air be measured? Is the velocity of sound in air affected by (*a*) a change of temperature, (*b*) a change of pressure, and, if so, in what manner? Sen. Cam. Loc.

2. How is the velocity of sound in air found by experiments between two stations?

A steamer approaching a cliff whistles, and the echo is heard after an interval of 10 seconds. Five minutes later the interval is found to be 8 seconds. How far is the steamer now from the cliff and how fast is she going?

(Take the velocity of sound in air as 1120 ft. per second.)

3. Explain why sound is heard at greater distances (*a*) over the surface of water than over land (*b*) in the direction of the wind than in the opposite direction. L.U.

4. Explain why, when soldiers are marching in a column in time with the music of a band in front, as they hear it, they are seen not to be all in step together.

Do their steps sound together to a man (*a*) in the front, (*b*) in the rear, of the column? Give reasons for your answer. L.U.

5. Explain how the velocity of sound in air may be determined. A man standing between two parallel cliffs fires a rifle. He hears one echo after $1\frac{1}{2}$ seconds, one after $2\frac{1}{2}$ seconds, and one after 4 seconds. Explain how these echoes reach him, and calculate the distance apart of the two cliffs. The velocity of sound under the given conditions is 1120 feet per second. L.U.

6. A person standing in a railway cutting hears the whistle of an approaching train. Two notes of different pitch are, however, heard, one being due to reflection of sound from a bridge beyond the train. Explain why the notes heard are not of the same pitch, and show how to calculate what the alteration in pitch amounts to. L.U.

7. Describe one method by which the velocity of sound in air has been measured.

Explain how the error due to the wind may be eliminated.

8. Find an expression for the velocity of propagation of sound through a gas. Find approximately the ratio of the velocity of sound in hydrogen at 100° C. to that in air at 0° C. Madras University.

9. How has the speed of sound in free air been determined?

A body of troops marching behind a band always appears to an onlooker to be slightly out of step. Account for this and find what length of column marching 130 steps per minute would have the rear ranks apparently out of step but keeping time with the front ranks.

(Velocity of sound in air = 1120 feet per second.) L.U.

10. Describe a method of measuring the speed of sound in air.

An engine is approaching a tunnel surmounted by a cliff, and emits a short whistle when half a mile away. The echo reaches the engine after $4\frac{1}{2}$ seconds. Calculate the speed of the engine, assuming the velocity of sound to be 1100 feet per second. L.U.

11. What is the "Doppler effect"?

A source of sound moves with a velocity v towards a stationary observer. Taking the velocity of sound in the intervening medium as V, determine the ratio of the observed frequency to the real frequency of the note emitted.

Show that this ratio is not identical with that corresponding to the case in which the observer moves towards a stationary source. L.U.

CHAPTER LVI

INTERFERENCE. RESONANCE

Principle of interference.—When two sounding bodies of the same frequency are situated near each other, or whenever two sets of waves of the same frequency overlap, the medium will at every point experience the resultant effect of the two wave systems. This may be effectively illustrated by considering the waves on the surface of water, or the ripples on the surface of mercury. Let A and B (Fig. 648) be two points at which the surface is disturbed synchronously, that is, the two disturbances have always the same phase. Then from each of these points circular waves spread outwards. These waves consist of alternating crests and hollows. For convenience the crests are supposed to be represented by thick and the hollows by thin lines. At points such as C, D, E, etc., both sources are tending to produce hollows at the given instant. Half a period later both sources are producing crests at these same points, which are therefore points of great disturbance. At points such as P, Q, R, and S a crest from one source reaches the point at the same instant as a hollow from the other source. The two together will, if the waves are of equal intensity, cancel each other out, so that the resultant displacement at the point is zero. At these points the effects of the two waves will continually be zero.

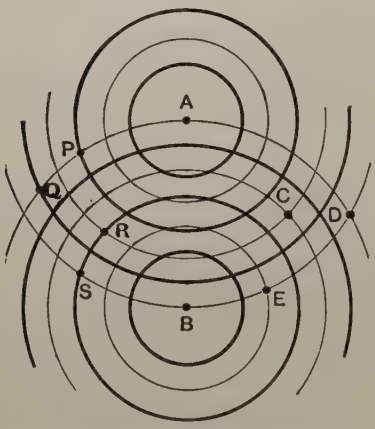

FIG. 648.—Diagram for interference between two sets of ripples.

The result is that at some points the waves from the two sources reinforce each other and at others destroy each other, so that at some points the disturbance is greater than that due to one set alone, while at other points it is zero. The two sets of waves are said to **interfere**, and the phenomenon is called **interference**.

FIG. 649.—Interference of mercury ripples.

It may be very clearly shown by means of the ripples on mercury. In Fig. 649 the ripples produced by two wires attached to the prong of a tuning-fork are shown.

Tuning-fork.—Compression waves may also exhibit interference, but, except in certain cases, this is not easily observed. In the case of a tuning-fork it may be noticed that when the prongs move apart there is a compression produced outside at A and B (Fig. 650), and a rarefaction in the space C between them. When the prongs move inwards there is compression at C and rarefaction at A and B. Hence the waves starting from AB and from C are always in opposite phases, and may be represented by the circles in Fig. 650. It will then be seen that along the four dotted lines radiating from the fork, the two sets of waves interfere, and there will be silence.

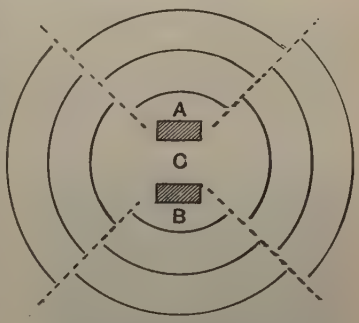

FIG. 650.—Interference in sound waves from a tuning-fork.

At any point on the dotted lines, a compression from A or B arrives at the same time as a rarefaction from C, so that the resultant effect is zero. This may be detected by striking a fork and turning it round on its stem between the fingers, when alternations of loudness and softness will be heard, the softness occurring when the ear is in the position of one of the dotted lines of Fig. 650.

Manometric flame.—For the detection of compression waves

in air, the manometric capsule or flame is of great convenience. Gas entering a small chamber by the pipe A (Fig. 651), leaves by the jet B, where it burns, forming a tall thin flame. One boundary

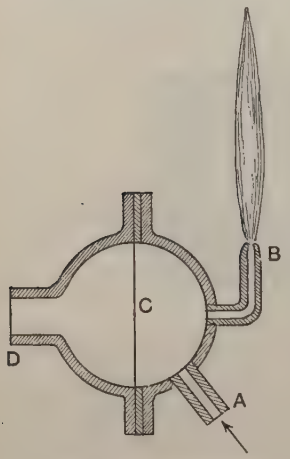

FIG. 651.—Manometric flame.

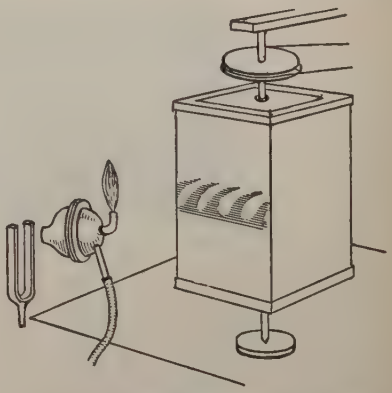

FIG. 652.—Revolving mirror for viewing manometric flame.

of the chamber is the thin india-rubber membrane C. The sound waves enter at D, and the variation in pressure drives the membrane C in and out. This communicates a varying pressure to the gas and the jet jumps up and down correspondingly. On viewing the image of the jet in a revolving mirror (Fig. 652) the image will appear serrated when sound waves are entering the capsule, and the nature of the serrations gives some clue to the character of the sound waves.

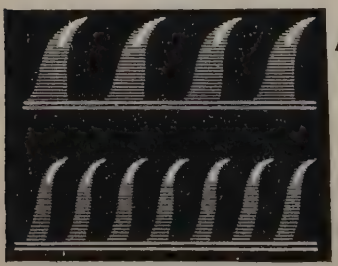

FIG. 653.—Manometric flame seen in revolving mirror.

Some of the appearances of the flame frequently met with are given in Fig. 653. A is that due to an organ pipe blown gently, and B that due to the same pipe blown hard, so that the frequency is doubled.

Interference produced by a divided tube.—A tuning fork F (Fig. 654) of fairly high frequency is sounded opposite the opening A of a tube AB. The compression waves travelling down AB, enter the tube CBD

at B. They divide at B, part passing along BC and the remainder along BD, and they finally unite at E, pass down EG and affect the manometric flame at G. The D part of the tube is made to slide into the other part so that the length of path BDE may be varied. If D is pushed in so that the two paths BCE and BDE are the same length, the waves take the same time to travel by either path from B to E, and therefore the two sets always arrive in the same phase. They will therefore reinforce each other and produce considerable disturbance of the manometric flame. If, however, D is drawn out until the path BDE is half a wave-length longer than BCE, a compression by one path arrives at the same time as a rarefaction by

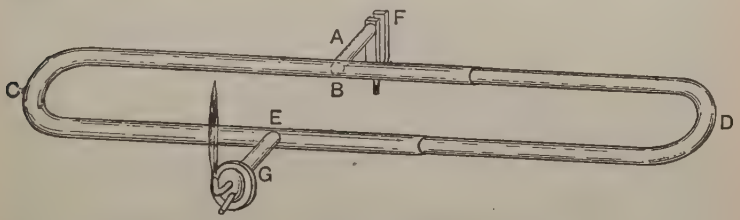

FIG. 654.—Divided tube for producing interference.

the other, and the two neutralising each other, silence results, and the manometric flame ceases to be disturbed. On further withdrawing D, a point is reached for which the path BDE is one wave-length greater than BCE. The two waves are then in phase at E and disturbance of the flame results.

Thus a series of positions of D can be found, for which the flame is at rest, and another series for which it is most violently disturbed. The first positions are such that the path BDE is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc., greater than BCE, where λ is the wave-length. The second positions are such that BDE is 0, λ, 2λ, 3λ, etc., greater than BCE. By measuring these positions it is clear that λ may be found.

Determination of the pitch of a high note by interference.—If the note is a shrill one, it affects a sensitive flame. This affords a means of detecting the presence of interference. The source of sound may be a Galton's whistle, A (Fig. 655), which can be blown steadily by means of air which has been collected in an ordinary gas bag. A is situated at a distance of a metre or so from a vertical plane B, which may be part of the wall of the room, or a large drawing-board placed vertically. The sound waves from A are reflected at B, and travel backwards as though they came from A′, the image of A (p. 698).

If the distances AC and A′C differ by a whole number of wave-lengths, the direct and reflected waves assist each other, and if

placed at this point, the sensitive flame will roar. If the flame be now moved to a point D, such that A′D and AD differ by an odd number of half wave-lengths, the direct and reflected waves destroy

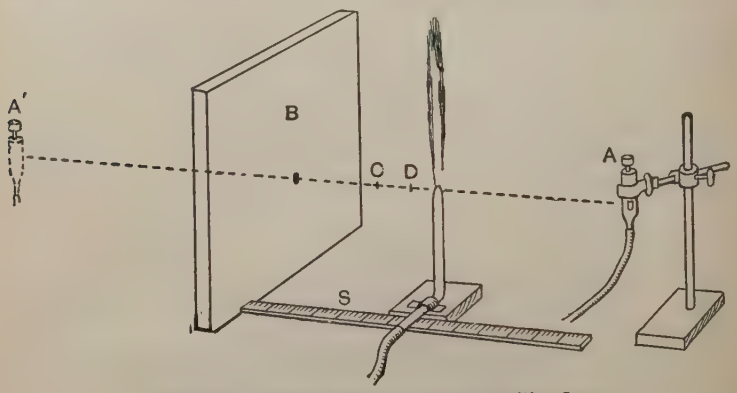

FIG. 655.—Determination of pitch by sensitive flame

each other at D, and the sensitive flame will burn up brightly and silently. A series of points of alternating quiet and disturbance can be found in this manner.

Since, $$A'C - AC = n\lambda,$$

and $$A'D - AD = n\lambda + \frac{\lambda}{2},$$

$$(A'D - A'C) + (AC - AD) = \frac{\lambda}{2},$$

$$2CD = \frac{\lambda}{2},$$

or, $$CD = \frac{\lambda}{4}.$$

Thus, the distance between a point of disturbance and the nearest point of silence is a quarter of a wave-length, and therefore the distance between consecutive points of silence is a half wave-length. Knowing the velocity of sound V, and the wave-length λ as determined from the experiment, we can find the frequency from the equation $V = n\lambda$ (p. 680).

EXPT. 153.—**Pitch by interference.** Arrange the Galton's whistle and sensitive flame near a wall, as shown in Fig. 655. If the flame is fed by gas from an ordinary gas bag, regulate the pressure by placing weights on the bag, until the flame roars when a whistle or hiss is made in its neighbourhood, or a bunch of keys is rattled. Let the stand for the sensitive

flame be placed alongside of a centimetre scale S. Move the flame to and fro parallel to this scale until the flame shoots up and becomes quiet, showing that a point of silence has been found. Note the position upon the scale S. Move the flame away from the wall and find the next point of silence. In this way find the position upon the scale for as many points of silence as possible, and calculate the mean distance between two consecutive points. Since this distance is $\lambda/2$, λ can be found. Taking the velocity of sound at the existing temperature (p. 694) to be V, calculate the frequency n from the relation

$$V = n\lambda.$$

Beats.—When two sounding bodies are of exactly the same frequency, there may be interference between the waves emitted by them, but at any given place the state of disturbance remains constant. Thus, at D, C, or E (Fig. 648) there is always great disturbance, while at P, Q, or R there is always a minimum of disturbance. If, however, the two bodies have not exactly the same frequency, the condition at any fixed point in their neighbourhood is continually changing. At one moment the compressions from both sounds arrive simultaneously, as do the rarefactions. The result is that there is great disturbance at the given point. A short time later, the more rapidly vibrating body is half a vibration ahead of the other, and the compressions from one will arrive at the same time as the rarefactions from the other, producing a minimum of sound. Thus, in addition to the note due to the vibrating bodies, there will be a pulsating effect produced. The alternate loud and soft pulses are called **beats**.

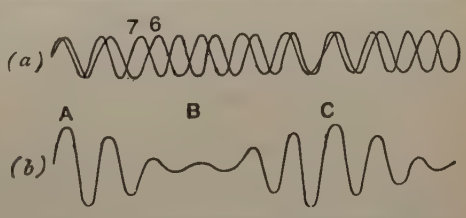

FIG. 656.—Curves illustrating the production of beats.

In Fig. 656 (*a*) the displacement curves for two waves whose frequencies are in the ratio 6 : 7 are drawn. It will be seen that in the neighbourhood of A they are in the same phase and assist each other. At B they are exactly out of step with each other, that is to say, they are in opposite phases, and neutralise each other. At C they are again in the same phase. The curve (*b*) is the resultant of the other two, and shows clearly the alternations in loudness and softness which are called **beats**.

In Fig. 656 it will be seen that if the distance AC includes the waves emitted in one second, there is one beat per second. Or, if the conditions represented by AC be repeated any number of times per second, just so many beats will there be per second, and this number will also represent the difference in the frequencies of the two sounding bodies. In fact, the **difference in the frequencies of two sounding bodies is equal to the number of beats produced per second.**

For the clear production of beats, the two sounding bodies should be of the same kind. If two tuning-forks mounted upon sounding boxes and having frequencies differing by, say, three, be sounded, three beats per second may easily be heard. If it is desired to determine which of the two forks has the higher frequency, one of them may be loaded by a piece of soft wax. This lowers the frequency of the fork, and if the rate of beating is now less than before, it follows that this fork has the higher of the two frequencies, since on lowering it, it is brought more nearly into unison with the other fork.

In the case of two stretched strings, if it is desired to tune one to unison with the other, the beats are of great service, for as the two are brought more and more nearly into unison, the beats become slower, and eventually disappear. If the hand be placed upon the board upon which the two strings are stretched, the beats can be *felt* as well as *heard*.

Beats are used for the production of certain effects in organs. In the case of the *vox-humana* and *vox-angelica* stops, two pipes having nearly the same frequency are used. The beating between the two gives the tremulous effect which is intended to imitate the human voice.

Combination tones.—When the beats produced by two sounding bodies become sufficiently rapid, they produce a tone known as a **beat tone**, whose frequency is the difference of the frequency of the two separate notes. Of the presence of this tone there is no doubt, but its cause is not so clear. A tone is due to a succession of impulses, whereas the succession of loud and soft intervals in beats does not consist of impulses in the ordinary sense of the word. However, whenever two pure tones are sounded together a series of other tones will be produced, although in general these are too feeble to be distinguished. One such tone is called a **summation tone** and has a frequency equal to the sum of the frequencies of the separate tones, while another, the **difference tone**, has a frequency equal to their difference. These two are the **first combination tones.**

There are also other combination tones produced between the first combination tones and the primary tones, and also **self-**

combination tones, having twice the frequencies of the primary tones. There are therefore, with two notes of frequencies m and n, the following tones :

$m,\ n$	primaries.
$m+n$	first summation tone.
$m-n$	first difference tone.
$2m,\ 2n$	self-combination tones.
$2m+n$	
$2m-n$	
.....	

The origin of these tones is somewhat obscure, but they have been explained on account of the want of symmetry in the vibrating apparatus, either of the ear, or in some cases of the sounding body.

The first summation tones may be detected when two organ pipes, or notes on the harmonium, are sounded strongly, the ear being placed close to the instrument. In order to detect them it is advantageous to determine by calculation what the pitch of such note should be and to sound it first, to prepare the ear for the detection of the combination tone.

Resonance.—It is frequently possible to set a body of great inertia in vibration by means of a very feeble force, provided that this force is applied at regular suitable instants. Thus, if a heavy body be suspended by a wire, it can be set in vibration by pulling a fine silk cord attached to it, provided that the cord is pulled intermittently, at such times that the pulls help to increase the velocity of the body. In this way a vibration of large amplitude may be set up, although the cord may be so thin that it would break rather than pull the body steadily through any considerable fraction of the resulting amplitude. The setting up of the vibration depends upon the timing of the impulses.

Whenever two similar bodies have exactly the same frequency, one can set the other in vibration when they are suitably connected. Thus, if two forks have exactly the same frequency, and one be vigorously bowed, and then held near the other, the other will be found to be set in vibration. The air waves from the first fork are always suitably timed to increase the vibration of the second ; they act like the pulls upon the silk fibre in the above case. Again, if two strings stretched upon the same board have the same frequency, then, on bowing one of them, the other will begin to vibrate.

This process is called **resonance**; one body is said to **resound** to the other. One body will only resound to another **when both have the same frequency.**

We shall come across many cases of resonance in studying sound; the following illustration is of interest. A company of soldiers in crossing a plank, or a suspension bridge, is always ordered to break step. For should the frequency of their step coincide with the frequency of vibration of the bridge, dangerously large vibrations will be set up in the bridge, and may threaten it with breaking. One man, by jumping repeatedly at the proper time, can set a long plank in vigorous vibration.

Forced vibration.—In the case of resonance, the applied force has a frequency equal to the natural frequency of vibration of the body to which it is applied, and the resulting vibration is then very great. But whenever a simple harmonic force is applied to a body, it produces a simple harmonic motion in it, although the amplitude of this may in many cases be extremely small. This resulting motion is called a **forced vibration.** As an example consider a small pendulum AB (Fig. 657 (*a*)) hanging from a larger pendulum AC. The point A executes a simple harmonic motion, and this will give the pendulum B a harmonic motion of the same period as A. Its amplitude will, however, depend upon the relation of the natural period of vibration of B to the period of vibration of A. The motion of B due to that of A is a forced vibration. There are three cases of forced vibration to consider.

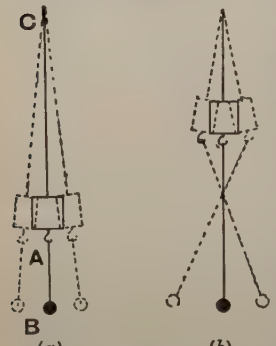

FIG. 657.—Forced vibration.

In case (i) the frequency of the applied force is less than the natural frequency of the body, and the amplitude is a copy of that applied, and on a slightly larger scale. Thus, in Fig. 657 (*a*) the amplitude of B is slightly greater than that of A, and the two are always in the same phase.

In case (ii) the frequencies are equal: this is the case of resonance, and the amplitude of the forced vibration becomes very great. It can never be infinite, because friction will always prevent the amplitude exceeding a certain limit.

In case (iii) the frequency of the applied force is greater than the natural frequency of the body. The motion in this case is opposite in phase to that of the applied force, as may be seen in Fig. 657 (*b*).

In all these cases, when the harmonic force is first applied, there

may be a certain amount of natural vibration set up. This, however, will soon die away, leaving the forced vibration only. The amplitude of the forced vibration may be calculated in terms of that of the applied force and of the two frequencies concerned. The calculation is beyond the scope of this book, but the result may be exhibited in the form of a curve. Thus, in Fig. 658 Ox is the ratio of the frequency of the applied force to that of the natural frequency of the body to which it is applied, while the ordinates represent the ratio of the amplitude of the forced vibration to that of the applied force. It will be seen that if Ox = unity, the ordinate is infinite. This is the case of resonance. An infinite amplitude is never attained in practice, owing to the damping effect of friction. For less frequencies of the applied force, the forced amplitude rapidly falls, until, for small values $OA = 1$, that is, the two amplitudes are equal. For greater frequencies of the applied force, the forced amplitude again falls, and in this case falls to zero for very great frequency, the two phases always being opposite, as is shown by the curve lying below Ox, the ratio of the amplitudes, being negative.

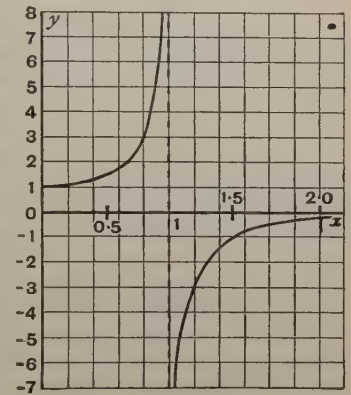

FIG. 658.—Graph of amplitude of forced vibration.

Sounding-board.—If a stretched string, or a tuning-fork, be set in vibration, the sound from it is very feeble, unless it be attached to some sounding-board. Thus, if one end of the string be clamped

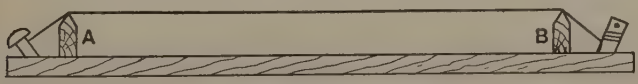

FIG. 659.—Sounding-board.

in a vice and a weight hang from the other end, very little sound will be heard when the string is plucked or bowed. If, however, the string be stretched upon a board and passed over two bridges, A and B (Fig. 659), then, on bowing or plucking it, quite a loud sound will be heard. The string alone is in contact with only a small quantity of air, so that when it vibrates it sets a small quantity of air only in vibration. Also, since compression occurs on one side of the string at the same time as rarefaction on the other, interference further reduces the effective sound (p. 707). When,

however, the string is stretched upon a board, the string exerts a periodic force upon the bridges A and B, which in turn give the sounding-board upon which they rest a periodic motion. As the board is in contact with a comparatively large amount of air, owing to its large surface, the energy of the air set in motion in a given time is much greater than if there were no board. Of course, the vibrations of the string die away more rapidly with the sounding-board, as the energy is then radiated away in the form of sound waves much more rapidly.

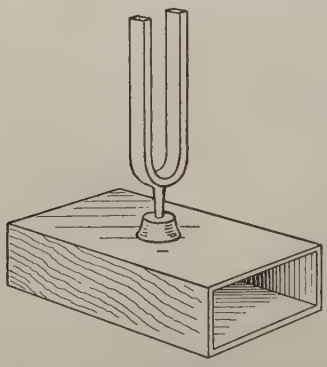

FIG. 660.—Tuning-fork on sounding-box.

The case of the tuning-fork is similar to that of the string. If held in the hand the sound from the fork is feeble, but if the stem be pressed upon a table or board, this is set in forced vibration. The vibration is thus communicated to a much greater quantity of air than before.

Resonators. — Sometimes tuning-forks are mounted upon hollow boxes, the box being of such a size and shape that the air inside the box has a natural period of vibration equal to that of the fork. When the fork is set in vibration, the wood of the box executes a forced vibration, and the air in the box will resound. In this way the sound due to the fork may be very great, but it must be noted that for resonance, the box must be properly tuned to the fork.

Another form of resonator due to Helmholtz is shown in Fig. 661. It consists of a brass shell of approximately spherical form, with a large opening at A and a small one at B. The air within it can resound to one particular frequency, and when waves of this frequency arrive, the air resounds, and the sound appears to be much magnified on applying the ear to B. Even if the wave is only a small component of the actual wave arriving, the resonator will pick out and magnify this component. Such resonators are made in sets, the pitch of each one being known, and they can then be used to detect the components in a complex note, although these components might be too feeble to be detected by the ear alone.

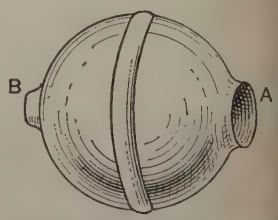

FIG. 661.—Helmholtz resonator.

Large sea shells produce a vague roaring noise, popularly said to be 'roaring of the sea,' when placed to the ear. The reason is

that they are resonators, but not of such a simple type as that of Fig. 661. They magnify certain tones, and these tones are always present to a certain extent, and when magnified they blend into the familiar roar of the shell.

Exercises on Chapter LVI.

1. Give an account of what you mean by 'beats,' and describe an experiment by which they can be illustrated.

2. A tuning-fork is struck and then slowly rotated about its long axis. Describe and explain the changes in the sound heard during one complete rotation of the fork.

3. Describe what is meant by 'interference.' What will be the form of the interference pattern produced by two synchronous disturbances upon the surface of a pond ?

4. Describe how the difference in pitch of two tuning-forks may be determined by the method of beats, and how the fork of higher frequency can be identified.

5. What is meant by the term 'resonance' ? How can the principle of resonance be utilised in tuning a stretched wire to a tuning-fork ? L.U.

6. Two tuning-forks, whose pitches differ by four vibrations per second, are placed near to each other and set in vibration simultaneously. Describe and explain the character of the resulting sound. L.U.

7. What are beats, and how are they produced ? Two strings vibrating transversely emit fundamental notes of frequencies 300 and 302 per second respectively. How many beats per second are produced (1) by the fundamental notes, and (2) by their first overtones ? L.U.

8. If the handle of a vibrating tuning-fork is held against a wooden board, the amount of sound produced is considerably increased. Explain carefully why this happens. Is the time during which the fork goes on vibrating affected, and, if so, why ? L.U.

9. Describe the manometric flame, and the method of using it for the detection of air waves.

10. Sound waves from a tuning-fork A reach a point B by two separate paths ACB and ADB. When ADB is greater then ACB by 16 cm. there is silence at B. When the difference is 32 cm. there is sound at B and when 48 cm. silence, and so on. Explain this effect and calculate the frequency of the fork if the velocity of sound be taken to be 332 metres per sec.

11. Give a short account of the combination tones that would be produced when notes of frequencies 512 and 768 are sounded simultaneously.

12. State what you mean by the term forced vibration, and describe how the amplitude of a forced vibration depends upon the frequency of the vibrating body.

13. Explain how 'beats' are produced.

Two tuning forks make 4 beats per second when sounded simultaneously. One fork makes 256 vibrations per second, and the beats cease when the other fork is loaded with a small piece of wax. What is the frequency of the second fork? L.U.

14. Explain the cause of 'beats.'

The points of the prongs of a tuning-fork A originally in unison with a fork B of frequency 512 are filed, and the forks produce 5 beats per second when sounded together. What is the pitch of A after filing? Allabahad University.

CHAPTER LVII

INTERVALS AND SCALES

Musical intervals.—It was seen earlier (p. 670) that when the frequency of one note is twice that of another, the former is the upper octave of the latter, and that this relation holds good whatever the absolute frequencies may be. Thus a note of frequency 400 is the upper octave of one of frequency 200; one of frequency 600 is the upper octave of one of frequency 300, and so on. Besides the octave, there are other easily recognisable musical intervals, and it is a universal rule that **the interval between two musical notes is determined by the ratio of their frequencies.**

Thus, a **fifth** corresponds to a ratio of $\frac{3}{2}$, and a **fourth** to a ratio of $\frac{4}{3}$. The following is a list of the intervals most commonly employed:

Unison	1 : 1	Fourth	4 : 3
Semitone	16 : 15	Fifth	3 : 2
Minor tone	10 : 9	Minor sixth	8 : 5
Major tone	9 : 8	Major sixth	5 : 3
Minor third	6 : 5	Seventh	15 : 8
Major third	5 : 4	Octave	2 : 1

In adding two intervals together, the frequency ratios must be multiplied.

Thus, minor third + major third $= \frac{6}{5} \times \frac{5}{4} = \frac{3}{2} =$ fifth,

again fifth + fourth $= \frac{3}{2} \times \frac{4}{3} = \frac{2}{1} =$ octave.

Diatonic scale.—The diatonic scale is the one universally employed in modern music, and it is from the positions of the notes in this scale that the above names of the intervals are derived. The octave, as its name implies, is an interval of an eighth, and the other notes of a scale are arranged in the following manner within the octave, the ratios of the frequencies for consecutive notes being as follows:

$$\underset{\frac{9}{8}}{c}\ \underset{\frac{10}{9}}{d}\ \underset{\frac{16}{15}}{e}\ \underset{\frac{9}{8}}{f}\ \underset{\frac{10}{9}}{g}\ \underset{\frac{9}{8}}{a}\ \underset{\frac{16}{15}}{b}\ c'$$

It will be seen that the interval c–d is a major tone, d–e a minor tone; c-e is a major third, c-g a fifth, and so on. Whatever the pitch of the lower c may be, these intervals must be maintained throughout the octave, and are repeated in the octaves above and below. The simplest whole numbers that will exactly represent these intervals are easy to remember, they are :

c	d	e	f	g	a	b	c'
24	27	30	32	36	40	45	48

For the representation of higher octaves c'', c''', etc., are used, and for the lower octaves C, C_1, etc. The positions of these notes on the usual musical cleffs are shown in Fig. 662.

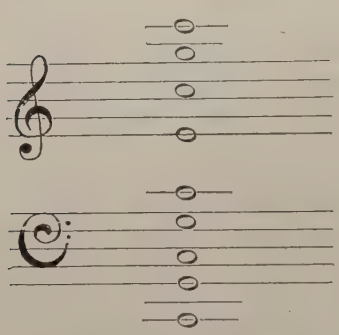

FIG. 662.—Musical cleffs.

Standards of pitch.—For scientific purposes the standard of pitch is a frequency of 512 for c'' and 256 for c'. This is a convenient standard, as it allows most of the notes to be expressed by whole numbers. Sometimes standard forks of frequency 50, 100, or 200 are constructed.

For musical purposes, the Philharmonic Society has fixed upon 439 for a' at a temperature of 68° F. It is necessary to define the temperature, as most instruments if tuned accurately at one temperature will alter in pitch if the temperature changes. The temperature 68° F. is about an average temperature for a concert room. This standard is known as the Low Pitch, as previously to 1896 the standard had been 452·4 for a' at 60° F., which is known as the High Pitch. The frequency of 439 for a' is equivalent to 526·8 for c''. The philharmonic scale is, therefore, slighter higher than the scale used for scientific purposes.

Concord and discord.—There are certain characteristic sensations produced by certain intervals, by means of which these intervals can be recognised. Thus, an octave, a fifth, or a third, can easily be distinguished. The octave has the most smooth sound of all the intervals, while the roughest is probably the semitone. The reason for the difference in the quality of various intervals was stated by Helmholtz to be due to the beats produced between the notes and their overtones. We shall see later that in the case of most notes produced by strings, or by wind instruments, the fundamental note is accompanied by overtones whose frequencies form

a series, 1 : 2 : 3 : 4, etc., where 1 represents the fundamental. Now beats, when few in number, do not produce a disagreeable sensation, but if they increase, a disagreeable harshness is produced which is called **discord**. The actual number of beats per second which produces maximum roughness or discord varies, but with a frequency of 512, 32 beats per second, or a note of

$$512 + 32 = 544$$

corresponds to maximum discord. This corresponds to an interval of $\frac{17}{16}$, or slightly less than a semitone.

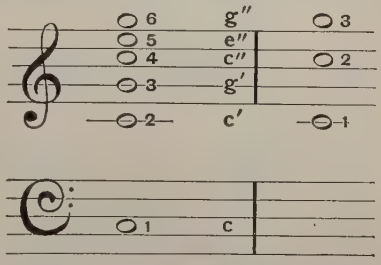

FIG. 663.—Overtones present with the octave.

The reason for the difference in smoothness of the various intervals can now be seen. In the case of the octave, the first five overtones of the lower note *c* are *c′g′c″e″g″* (Fig. 663), while the first three of the upper note *c′* are *c″g″c‴*, all of which coincide with some of the overtones of the lower note, so that there is no beating at all. If, however, the octave is not perfectly in tune, all the overtones of the higher note will beat with some overtone of the lower note, and great roughness ensues. Thus the octave is easy to tune.

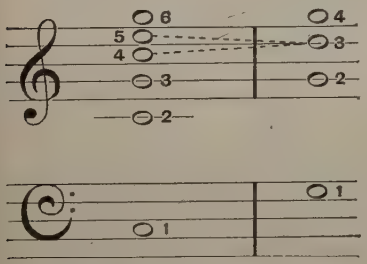

FIG. 664.—Overtones present with the fifth.

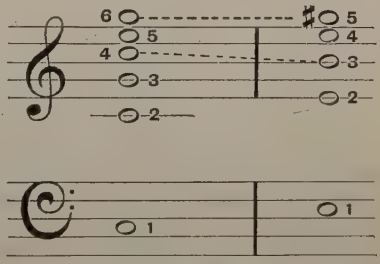

FIG. 665.—Overtones present with the major third.

In the case of the **fifth** (Fig. 664), it will be seen that overtone 3 of the lower note is the same as 2 of the upper. But 3 of the upper note is within beating distance of 4 and 5 of the lower. The **concord** is therefore not perfect in the case of the fifth, but the amount of discord is not great.

In the **major third** (Fig. 665) there is beating between the overtone 4 of the lower and 3 of the upper, and 6 of the lower and 5 of the upper. Overtone 2 of the upper is too far from 2 and 3 of the lower to produce beating.

In a similar manner the amount of discord for the other intervals may be found.

Helmholtz constructed a curve showing the relative amount of discord for various intervals throughout the octave (Fig. 666). It will be observed that the discord is greatest just below the semitone

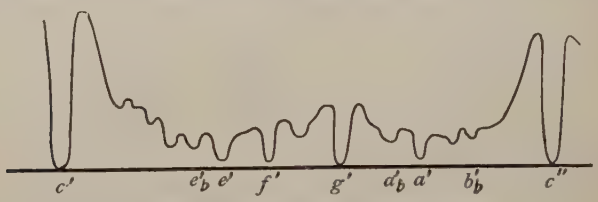

FIG. 666.—Helmholtz's curve of dissonance.

and above the seventh, and least at the octave and fifth. The octave and fifth are bounded by very steep parts to the curve, showing that a slight amount out in the tuning produces considerable dissonance and is readily detected. These are consequently the easiest intervals to tune accurately.

Temperament.—It is clear that an instrument, on which the diatonic scale beginning with C is to be played, must have eight notes to the octave. On taking the scale beginning with $c' = 512$, the first two octaves would be as follows :

c'	d'	e'	f'	g'	a'	b'	
512	576	640	682·7	768	853·3	960	
c''	d''	e''	f''	g''	a''	b''	c'''
1024	1152	1280	1365	1536	1706·7	1920	2048

But since the tone is rather a large interval, the scale is with advantage completed by adding semitones between c' and d', d' and e', f' and g', g' and a', a' and b', and the full scale would then be as shown in Fig. 667 (1).

There would now be twelve separate notes to the octave; the five sharps are the black notes on the piano.

Suppose that it is now desired to begin the scale with the note g', the intervals being as before; the requisite frequencies are given in Fig. 667 (3). It will be seen that the notes, with the exception of the second and seventh, are present in the scale of c'. Thus, in order to play the scale of g' major, two new notes would have to be added. The other notes may be treated in the same way. Each new note which is taken as the **tonic**, or **keynote**, of a new scale requires new notes to be added to the instrument. Thus, if any scale at will is to be played upon one instrument of fixed keys, the number of keys to the octave would have to be very great—

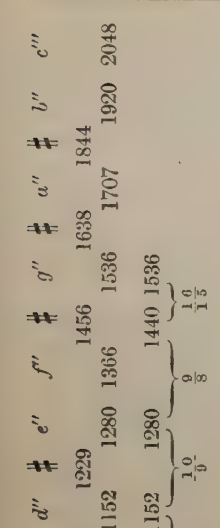

FIG. 667.—Scales,—diatonic and equal temperament.

prohibitively great. For this reason the scale is usually **tempered**; that is, a compromise is effected, which is sufficiently near the truth for all scales, but is not absolutely correct for any one of them. Thus $f''\sharp$ will do for the seventh in the scale of g major, and so on.

In instruments such as the violin, in which the notes are not fixed, but are judged by the player, it is, of course, possible to play the scale, using any note as tonic, with perfect accuracy. This is also the case with the human voice.

Scale of equal temperament.—There are various ways of tempering the scale of an instrument of fixed notes, such as the piano or organ. The method now universally adopted is to tune the notes to a scale which is called the scale of **equal temperament**, in which the octave is divided into twelve exactly equal semitones. It is obvious that this is not a truly diatonic scale for any key, but it is equally good, or bad, for all keys.

In order to find the value of this equally tempered semitone, remember that, repeated 12 times, it doubles the frequency of the note, that is, it raises it to the upper octave. Let x be the ratio of the frequencies for this semitone, then,

$$x \times x \times x \times \ldots \text{ 12 times} = 2,$$

or,
$$x^{12} = 2;$$
$$\therefore\ x = 2^{\frac{1}{12}} = \sqrt[12]{2}$$
$$= 1{\cdot}0595.$$

The semitone on the diatonic scale is $\frac{16}{15} = 1{\cdot}067$, which is, therefore, slightly greater than the semitone on the equally tempered scale. Employing the interval 1·0595 and applying it 12 times, the notes of the scale of c' are obtained and are given in Fig. 667 (4). It will be seen that no interval is perfect, but none

is very bad. The worst is $a'\sharp$ with a frequency of 912·3 instead of 922. This error would only be detected by a trained ear, and in the other cases the agreement is sufficiently perfect for ordinary purposes. It is clear that, whatever is taken as the tonic or key-note, the scale built upon it is of the same quality as the scale of *c*.

Exercises on Chapter LVII.

1. Explain what is meant by a musical interval. Show that the sum of two intervals is obtained by multiplying the ratios of the frequencies of the constituent notes.

2. Write down the relative frequencies of the notes composing the diatonic scale, pointing out the names of the intervals between consecutive notes.

3. Give some reason for the occurrence of discord with several intervals, and explain why it is much more easy to tune to strings one octave apart than a fourth apart.

4 Describe how a scale of equal temperament is constructed for an instrument having fixed strings.

CHAPTER LVIII

STRINGS

Wave in a stretched string.—A stretched string has the properties necessary for the propagation of transverse waves. Thus, if part of it be drawn aside, the tension in the string tends to bring it back to its position of static equilibrium. Also, the string having inertia, the force which causes the displacement requires time to produce its full effect. Thus a wave can travel along the string with definite velocity. The passage of such a wave is easily seen if a rope fixed at A (Fig. 668) be held in the hand at B, and lightly stretched into a nearly horizontal position. Then, on giving the end B a lateral jerk a wave 1 will be started which will travel along as shown at 2 and 3. If the rope is not too tightly stretched, the velocity of this wave will not be too great for it to be followed by the eye. The tighter the rope is stretched the more rapidly will the displacement travel along it. Also, if a series of to-and-fro movements be given to the rope, a wave something like a harmonic wave will travel along it.

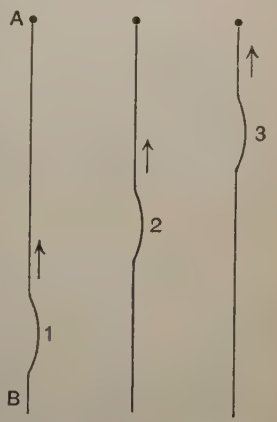

FIG. 668.—Wave in stretched string.

On arrival at the end A, the wave is reflected. The laws of this reflection will be studied later (p. 729).

* **Velocity of wave in stretched string.**—The ordinary dynamical equation

$$\text{force} = \text{mass} \times \text{acceleration}$$

may be applied to finding the velocity of a transverse wave in a

* May be omitted on first reading.

string, in a manner somewhat similar to that used in the case of a compression wave (p. 690). It is first necessary to find the restoring force acting upon any part of the string when this is bent out of the straight line.

Consider a small piece of the string AB (Fig. 669) in which the tension is t dynes. A force t then acts at either end of this portion AB, and when this is curved the two forces t are not in a straight line, owing to the curvature of the string; therefore they have a resultant, tending to bring the string to its straight position of static equilibrium.

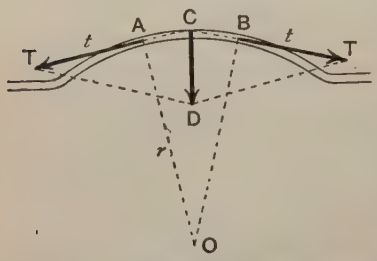

FIG. 669.—Forces acting on a small portion of a string.

The resultant of the two forces t is the diagonal CD of the parallelogram having the forces represented by CT, CT as sides. AB being very short, the two perpendiculars at its extremities A and B meet at O, where $OA = OB = r$, the radius of curvature of the string at this place. Again, ABO is for all practical purposes a triangle, similar to the triangle CDT;

$$\therefore \frac{CD}{CT} = \frac{AB}{OA};$$

$$\therefore CD = CT \cdot \frac{AB}{OA},$$

that is, $$\text{Restoring force} = t \cdot \frac{AB}{r}.$$

If the mass per unit length of the string is m, the mass of AB is $AB \times m$.

Again, $$\text{force} = \text{mass} \times \text{acceleration};$$

$$\therefore \text{acceleration of a point on the string} = t \cdot \frac{AB}{r} \times \frac{1}{AB \times m},$$

or, $$\text{acceleration} = \frac{t}{mr}. \quad \text{.....(1)}$$

It is next necessary to determine the acceleration of any point of the string in terms of the velocity V with which the wave is propagated along it. Consider a small piece PQ of the string (Fig. 670) where the velocity is changing.

Now, $$\frac{\text{velocity of P}}{\text{velocity of wave}} = \frac{PR}{RL}, \quad \text{(p. 681)}$$

that is, $$\text{velocity of P} = V \cdot \frac{PR}{RL}$$

$$= V\alpha_2,$$

since the angle α_2 is very small and is therefore equal to PR/RL. It must be remembered that the displacements are represented to an exaggerated scale in Fig. 670. The angle α_2 is in all practical cases very small.

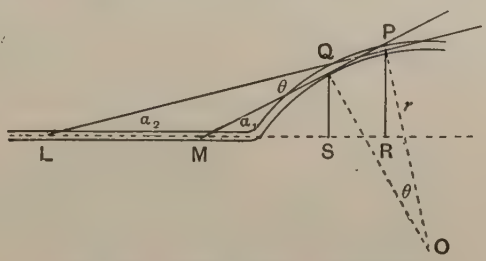

FIG. 670.—Velocity diagram for the string.

Similarly, velocity of Q $= V\alpha_1$;

$\therefore$ while the wave travels from Q to P, the velocity of the point changes from $V\alpha_2$ to $V\alpha_1$;

$$\therefore \text{ acceleration of point} = \frac{V\alpha_1 - V\alpha_2}{\text{time for wave to travel distance PQ}}.$$

The radii OQ and OP to the curve make an angle θ between them, equal to the angle betweem LP and MQ ;

$$\therefore \text{ PQ} = r\theta,$$

and the time for the wave to travel over PQ is equal to

$$\frac{\text{PQ}}{V} = \frac{r\theta}{V};$$

$$\therefore \text{ acceleration} = \frac{V(\alpha_1 - \alpha_2)}{\frac{r\theta}{V}} = \frac{V^2}{r}, \quad \ldots\ldots\ldots\ldots(2)$$

since $(\alpha_1 - \alpha_2) = \theta$.

From the expressions (1) and (2) for the acceleration, we now have

$$\frac{V^2}{r} = \frac{t}{mr};$$

$$\therefore V = \sqrt{\frac{t}{m}}. \quad \ldots\ldots\ldots\ldots(3)$$

Thus a transverse wave will travel along the string with velocity $\sqrt{t/m}$, provided that the lateral displacement of the string is always small, and that the stiffness of the string is negligible. If the latter condition is not fulfilled, then the bending of the string would produce restoring forces, etc. (p. 160), and the problem would be very complex. In all the cases with which we shall deal, the string is considered to be so thin that its stiffness can be left out of account.

Harmonic waves in a string.—In dealing with transverse waves in general, it was seen that a harmonic wave could be represented by the equation

$$y = a \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right), \quad \text{(p. 682)}$$

and that the motion of a point on the string is represented by the equation

$$y = a \sin 2\pi \frac{t}{\mathsf{T}},$$

obtained by putting $x = 0$.

Now, if a point in the middle of a string be given a simple harmonic motion, it is clear that waves will start in both directions from this point. Thus, if the point O of the string AB (Fig. 671) be moved in

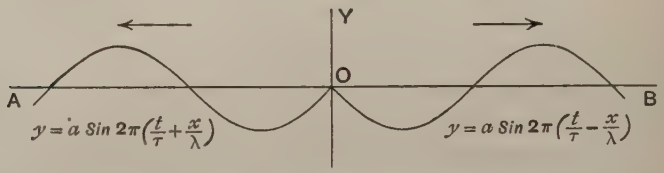

FIG. 671.—Two waves arising at a vibrating point.

a simple harmonic manner along the axis OY, one wave will travel towards the right, that is, in the positive direction of x, and the other towards the left, that is, in the negative direction of x.

The former wave has the equation

$$y = a \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right),$$

since, for points further to the right, the phase of the vibration is later the greater the value of x. On the other hand, the wave which travels towards the left will have the equation

$$y = a \sin 2\pi \left(\frac{t}{\mathsf{T}} + \frac{x}{\lambda}\right),$$

because points having a greater value of $(-x)$ occur in later phase.

Thus, the direction of the wave is indicated by the sign of x; it is negative for a wave travelling in the positive direction of x, and positive for a wave travelling in the negative direction of x.

Both equations take the form $y = a \sin 2\pi \frac{t}{\mathsf{T}}$, when $x = 0$.

Reflection of waves in strings.—When the waves in a stretched string reach a point at which the string is clamped, reflection occurs. In order to understand what is happening when reflection occurs, consider a point of the string to be clamped in a vice at P (Fig. 672).

The clamp prevents the motion of this part of the string. If the clamp were absent, the point P of the string would execute a simple harmonic motion. Since this is prevented, it follows that the string exerts a simple harmonic force on the clamp, and therefore the clamp exerts a simple harmonic force upon the string.

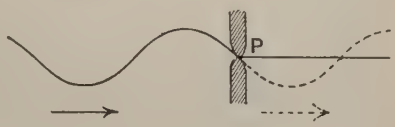

FIG. 672.—Point of the string maintained at rest.

Now a simple harmonic force applied to a string starts two harmonic waves, travelling in opposite senses from it (p. 728). Since the resultant motion of all points to the right of P (Fig. 672) is zero, it follows that the harmonic wave started in this sense by P is exactly equal and opposite to the continuation of the incident wave beyond P. The other wave, which travels in the opposite sense (p. 728), **is the reflected wave.**

There is no disturbance of the string to the right of P, so that it is immaterial whether this part of the string is present or not. P may be a clamped end of the string. The only real parts of the waves are the incident wave and the reflected wave, and the phase of the reflected wave may always be found from the above condition.

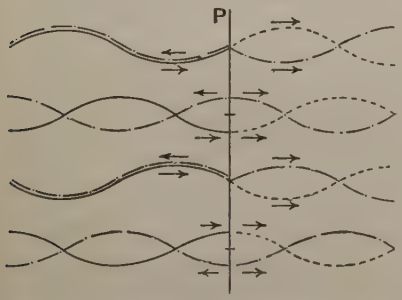

FIG. 673.—Reflection at the clamped end of a string.

In Fig. 673 four stages in the reflection of the wave are depicted. The incident wave is shown in full line; its continuation past the fixed point is given by a dotted line, and the two waves started by the fixed point P are given by chain lines. It will be noticed that the phase of the reflected wave at P is always such that the resultant displacement of P due to the incident and reflected waves is zero.

These waves may be conveniently represented by equations, thus,

Incident wave, $y = a \sin 2\pi \left(\frac{t}{\mathrm{T}} - \frac{x}{\lambda}\right)$.

In order to cancel this out, the wave

$$y = -a \sin 2\pi \left(\frac{t}{\mathrm{T}} - \frac{x}{\lambda}\right)$$

must be considered to originate at P and travel towards the right.

The companion wave to this, started simultaneously by P, and travelling towards the left (p. 728) is,

$$y = -a \sin 2\pi \left(\frac{t}{\mathsf{T}} + \frac{x}{\lambda}\right).$$

This is obtained by changing the sign of x, and is the **equation of the reflected wave.**

Stationary vibrations and interference.—It has already been seen (p. 706) that two sets of waves of the same frequency, travelling over the same medium, produce interference, points of maximum disturbance alternating with points of no disturbance. This is exactly the state of affairs in a stretched string with one end fixed. The incident waves are reflected, and the two sets of waves give rise to a state of steady vibration in the string.

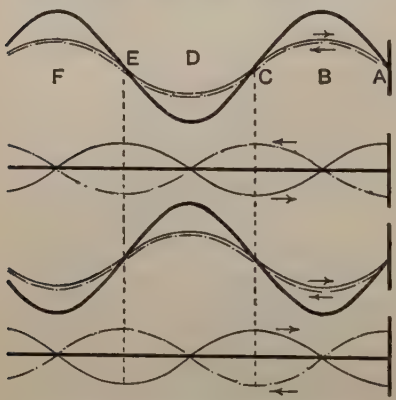

FIG. 674.—Resultant vibration of string.

Referring to Fig. 673, we may deduce what the relative phases of the incident and reflected waves will be during one whole period of vibration. In Fig. 674 these two waves are compounded, and the thick line curve is the resultant; this is, of course, the actual shape of the string, and is shown at quarter period intervals during one complete vibration. The points A, C, E, etc., are at rest, that is, they are points of no disturbance, and are called **nodes.** B, D, F, etc., are points of maximum disturbance, and are called **antinodes.** The wave-length is thus equal to the distance between **alternate nodes** or **alternate antinodes.**

The segment of the string AC, or CE, vibrates from side to side, each point of the string executing a simple harmonic motion, but **consecutive segments are in opposite phases.** Also **all points of any one segment are always in the same phase.**

The equation of motion of the string may easily be found from the equations of the incident and reflected waves.

Thus,
$$y = a \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right) \ldots \text{(incident wave)},$$

$$y = -a \sin 2\pi \left(\frac{t}{\mathsf{T}} + \frac{x}{\lambda}\right) \ldots \text{(reflected wave)}.$$

$\therefore$ the resultant of the two waves is given by the equation,

$$y = a \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right) - a \sin 2\pi \left(\frac{t}{\mathsf{T}} + \frac{x}{\lambda}\right)$$

$$= 2a \sin 2\pi \frac{x}{\lambda} \cos 2\pi \frac{t}{\mathsf{T}},$$

or,
$$y = \mathsf{A} \cos 2\pi \frac{t}{\mathsf{T}},$$

where
$$\mathsf{A} = 2a \sin 2\pi \frac{x}{\lambda}.$$

Thus each point of the string executes a simple harmonic motion of amplitude A, this amplitude varying from point to point along the string. At time $t=0$,

$$\cos 2\pi \frac{t}{\mathsf{T}} = 1,$$

and $\mathsf{A} = 2a \sin 2\pi \frac{x}{\lambda}$ is the equation giving the shape of the string at this instant. At points $x=0$, $x=\lambda/2$, $x=\lambda$, $x=3\lambda/2$, etc., the amplitude of vibration is zero ; these are the nodes.

At points $x=\lambda/4$, $x=3\lambda/4$, $x=5\lambda/4$, etc., the amplitude of vibration is $2a$ (or $-2a$), which points are therefore antinodes.

String fixed at both ends.—If a simple harmonic motion be given to a point in a string which is fixed at both ends, the waves travel to both ends, are reflected to the opposite ends and are again reflected and return to the original point, after having traversed twice the length of the string. If these waves, on arriving at the original point of disturbance are exactly in phase with the disturbance then being produced at the point, the waves will be reinforced and the process continues with increased amplitude. Thus the wave has travelled twice the length of the string, and for the simplest type of vibration of the string, the interval of time is that required to make one complete vibration, and this distance must, therefore, equal the wave-length λ;

$$\therefore\ 2l = \lambda,$$

where l is the length of the string.

But,
$$\mathsf{V} = n\lambda, \quad \text{(p. 680)}$$

and
$$\mathsf{V} = \sqrt{\frac{t}{m}}; \quad \text{(p. 727)}$$

$$\therefore\ n = \frac{1}{2l}\sqrt{\frac{t}{m}}.$$

This is the frequency of the simplest type of vibration of which the string is capable.

If two vibrations are completed while the wave travels twice the length of the string, $2l = 2\lambda$,

and $$n = \frac{1}{l}\sqrt{\frac{t}{m}}$$

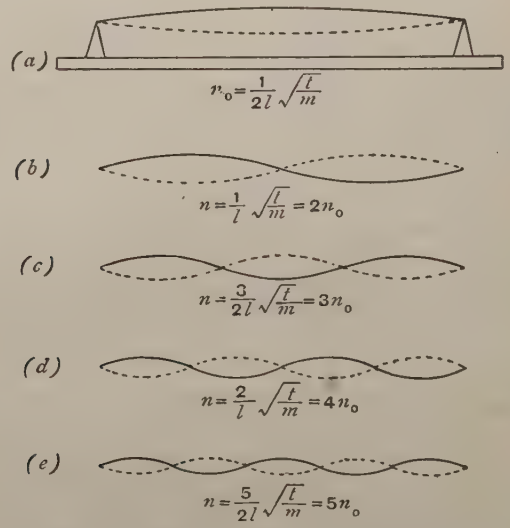

FIG. 675.—Modes of vibration of a stretched string.

In a similar manner, we may have

$$n = \frac{3}{2l}\sqrt{\frac{t}{m}}, \text{ etc.}$$

Thus there are many possible frequencies of vibration for a string, the vibration frequencies being proportional to the numbers

$$1 : 2 : 3 : 4 : \text{etc.}$$

The same result may be obtained in a somewhat simpler manner by remembering that the fixed ends of a string must be nodes. Hence for the simplest mode of vibration of a string there is a node at each end and an antinode in the middle, as at Fig. 675 (*a*). The length of the string is then one-half wave-length, or

$$l = \frac{\lambda}{2}, \text{ and } n_0 = \frac{1}{2l}\sqrt{\frac{t}{m}}.$$

This is the lowest frequency of vibration of which the string is capable, and n_0 is the fundamental note of the string.

Again, the next mode of vibration in order of complexity is that of Fig. 675 (*b*) with a node at each end and another in the middle. Here

$$l=\lambda, \quad \text{and} \quad n=\frac{1}{l}\sqrt{\frac{t}{m}}=2n_0.$$

The note then emitted is the **first overtone** or **harmonic**. Similarly the modes (*c*), (*d*), etc., give frequencies $3n_0$, $4n_0$, etc. Overtones with frequencies as high as $10n_0$ may often be detected.

Monochord or sonometer.—For studying the vibration of strings an instrument called the **monochord**, or **sonometer** is frequently employed. It consists of a sounding-board upon which is a stretched

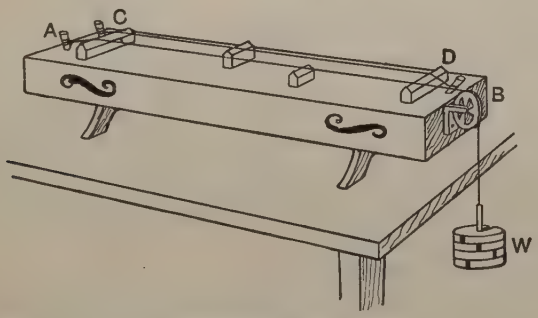

FIG. 676.—The monochord.

string. The end of the string A (Fig. 676) is fixed to a key and the end B passes over a pulley and is stretched by a known weight W. At C and D are two fixed bridges and other movable bridges are also provided. A second fixed string also is sometimes used. If the string be bowed or plucked at any position, that point cannot be a node; hence, if the string be bowed in the middle, the frequencies $2n_0$, $4n_0$, etc., are certainly absent and the fundamental predominates. Also, by touching the string lightly at any point with a brush or piece of paper, a node will be produced at this point. By carefully choosing the positions of bowing and stopping, any of the forms of vibration of Fig. 675 may be produced, and the corresponding pitches of the notes may be recognised. Also, the vibration of the string may be detected by placing light paper riders upon it. These are thrown off at the antinodes, but remain on the nodes.

In Fig. 677 the string is stopped at C, one-third of its length from A, and is bowed at B, midway between A and C. The mode of vibration is therefore (*c*) of Fig. 675. The riders are thrown off at D and F and that at E remains.

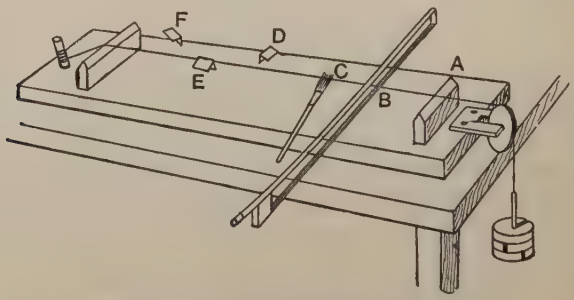

FIG. 677.—Detection of vibration of a string.

EXPT. 154.—**Proof that the frequency of a stretched string varies inversely as the length.** Obtain several tuning-forks of known frequency. By means of the movable bridge of the monochord, tune the string until a length l of it is in unison with the fork of frequency n. Measure carefully the length l of the string. Repeat with the other forks, keeping the weight which stretches the string constant.

Tabulate the results as follows:

Frequency of fork.	Length of string.	$n \times l$.

In the last column record the product **frequency × length.** This should be constant, showing that the frequency varies inversely as the length of the string.

EXPT. 155.—**Proof that the frequency of a string varies directly as the square root of the tension.** Place a bridge under the fixed string of the monochord so that one segment gives, when plucked, a note of suitable pitch. With a known weight stretching the other string, adjust the movable bridge under it until a length is found, in unison with the first string, and measure the length l. Vary the stretching force t, and

tune and measure again, repeating several times. Record the results as

Stretching weight.	Length of string.	$\frac{l}{\sqrt{t}}$.

The quotient $l/\sqrt{t}$ in the last column should be constant. This result shows that the length for constant frequency varies directly as $\sqrt{t}$. But the frequency for constant t varies inversely as l, therefore, if the original length were maintained, the frequency would vary directly as $\sqrt{t}$.

EXPT. 156.—**Proof that frequency varies inversely as the square root of mass per unit length.** Repeat the last experiment, with the sole difference that instead of changing the weight, change the string for another of different diameter or material. Repeat with several strings, in each case cutting off a piece and weighing it to determine the mass m per unit length. Record the results as follows:

Mass per unit length.	Length of string.	$l\sqrt{m}$.

The product $l\sqrt{m}$ should be constant, which would show, as in the last case, that the frequency of the string varies inversely as $\sqrt{m}$.

EXPT. 157.—**Absolute pitch of a fork.** Tune a piece of the string to unison with the fork, and measure the length l. Note the tension t in dynes. Cut off a piece of the string and weigh it, to determine m. Calculate the frequency from the equation

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}.$$

Melde's experiment.—A stretched string may be set in vibration by means of a tuning-fork, provided that the natural frequency of the string is equal to that of the fork.

Let the string AB, which may be a piece of stout thread or thin cord, be attached to one prong of the fork B (Fig. 678), the other end passing over a pulley and having a weight W attached. Then by moving the fork nearer to or further from the pulley, a length of the string may be found which will resound to the fork. Also by doubling the length of the string, or suitably changing W, the

string may be set in vibration in the mode (*b*) of Fig. 675. By suitable alterations, the string may be set into vibration in any

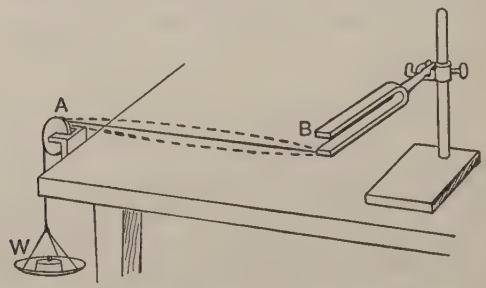

FIG. 678.—Melde's experiment (first arrangement).

desired number of segments, but in each case the frequency of the string is equal to that of the fork.

The string may also be set in vibration with the fork vibrating longitudinally to it as in Fig. 679, but in this case the vibration is produced by the alternating pulls upon the end of the string by the prong of the fork. Each movement of the prong to the right pulls the string tight, and this occurs in the middle of its swing, that is, twice in every vibration. Thus, in this case, the frequency of the string is **half that of the fork.**

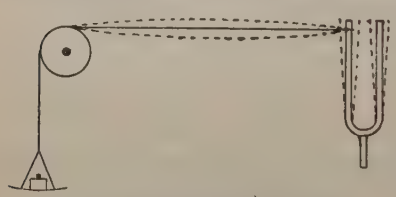

FIG. 679.—Melde's experiment (second arrangement).

Transverse vibrations of rods.—When a rod is bent, couples are brought into play tending to restore it to its original shape. Hence a rod is capable of transverse vibrations. The calculation of the form and period of vibration is difficult. The rod may vibrate in various ways according to whether it is unclamped, clamped at both ends, or clamped at one end. The last case is the most important, as rods clamped at one end are used as reeds in organ pipes and in musical boxes, and a tuning-fork may be looked upon as a pair of such rods united at their bases.

A thin rod such as a piece of clock spring with one end clamped is capable of vibrating in one segment as in Fig. 680 (*a*), or in two segments as in (*b*) with a node at A. In this case the frequency is about $6\frac{1}{2}$ times that in (*a*), and A is situated about one-fifth of the length of the rod from the free end. It can also vibrate as in (*c*), the frequency being about $17\frac{1}{2}$ times that of case (*a*). Thus the overtones are not simply related to the fundamental. In the

case of the tuning-fork, the first overtone is so feeble and of such high pitch that the fork gives a tone which is very nearly pure, being the most nearly simple harmonic vibration that produces sound with which we are acquainted.

Since the prongs of the fork vibrate in arcs, the centre of mass of each prong moves up and down slightly during the vibration. This gives rise to a harmonic force upon the stem, and if the stem is in contact with a table or sounding-board, the latter is set in vibration by this harmonic force.

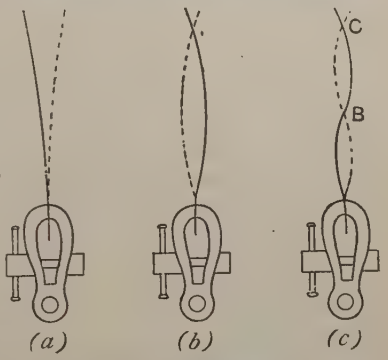

FIG. 680.—Transverse vibration of a spring.

In order to adjust the frequency of a tuning-fork, the metal is filed away slightly. To raise the pitch, metal is removed from the ends of the prongs, the effect of which is to decrease the moment of inertia of the prongs without altering the stiffness. To lower the pitch, metal is removed from the base where the prongs unite. This diminishes the elastic stiffness without sensibly altering the moment of inertia.

The effect of rise of temperature upon a tuning-fork is to increase its size and to lower the elasticity of the metal. The latter effect is the more important in the class of steel used for tuning-forks. A rise of 1° C. produces a lowering in the pitch of a tuning-fork by about 0·01 per cent.

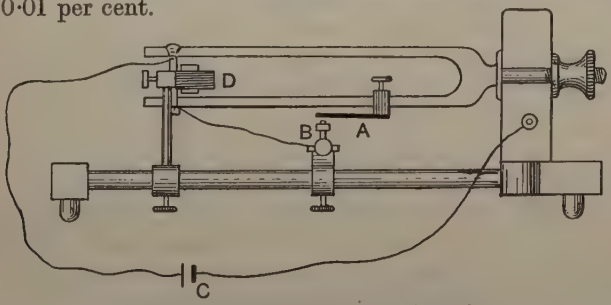

FIG. 681.—Electrically-driven tuning-fork.

Electrically driven tuning-fork.—For many purposes it is desirable to maintain a tuning-fork in continuous vibration without having to bow, or strike it. The fork is clamped to a stand and is provided with a short metal tongue A (Fig. 681) having a platinum

contact piece which touches a similar one at B. A secondary cell C is connected to the stand and then through the body of the fork to A. B is connected to the coil of a short electro-magnet D, situated between the prongs of the fork, the return wire then passing to the cell C. When contact is made at B, the current flows, and the electro-magnet D pulls the prongs of the fork together slightly, thus breaking the contact at B, and causing the pull of D upon the prongs of the fork to cease. The return motion of the prong makes contact again at B, and the process is repeated. The prongs of the fork thus receive impulses driving them together at regular intervals, determined by the fork's own period, and the fork is maintained in continuous vibration. Care has to be taken in adjusting the position of B so that the contact is just made for a short part of the vibration of the fork.

Stroboscopic method of determining frequency.—A very accurate method of measuring the frequency of a standard fork, known as

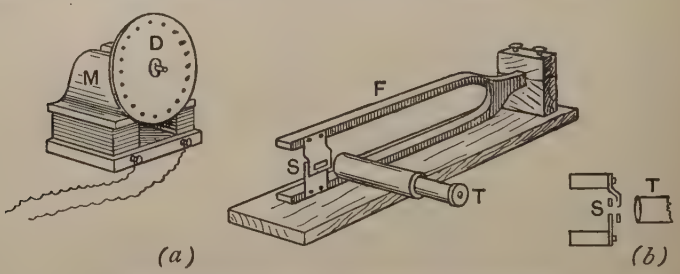

FIG. 682.—Stroboscopic method of obtaining frequency.

the **stroboscopic method**, is sometimes used. It depends upon the fact that a regular body in rotation, such as a wheel, appears to be stationary if intermittently illuminated, the instants of illumination being such that the wheel turns through the angle between the spokes in the time between successive illuminations. The same effect is produced if the wheel be viewed intermittently. In Fig. 682 (*a*) the disc D, driven by the electro-motor M, is viewed through the telescope T. The disc has a circle of dots or marks, equally spaced, and between the disc and the telescope is situated the tuning-fork F, whose frequency it is required to measure. Attached to the prongs of the fork are two light screens S, shown in section in Fig. 682 (*b*). Each screen has a slot in it, in such a position that the telescope and slots are in line with the dots on the disc when the fork is at rest. The fork may be of the electrically driven type shown in Fig. 681, but the driving mechanism is not shown in Fig. 682.

On causing the prongs of the fork to vibrate, the slots S will pass each other, and allow the circle of dots to be seen by an observer

looking through the telescope, twice in every complete vibration of the prongs. The speed of the motor can be regulated so that each time the circle of dots is seen, the dots appear to be in the same position, each dot having taken the position occupied by the dot in front of it when last seen. The dots will then appear to be stationary. If the disc is driven too slowly, each dot is a little behind the position occupied by the one in front when last seen, and the dots will appear to an observer to move slowly backwards. For a similar reason, when the disc is driven slightly too rapidly, the dots will appear to move forwards in the direction of rotation of the disc. By careful adjustment, the dots can be kept apparently stationary for a considerable time. The student will notice that if the disc be driven at twice or three times the speed, the dots will again appear stationary.

In order to determine the frequency of the fork, the motor must be provided with a revolution counter, and the number of revolutions in a known time observed. Multiplying the number of revolutions per second by the number of dots in the circle, we obtain a number which is twice the frequency of the fork, the dots being seen twice in each vibration of the fork. If the frequency of the fork be known, the method can be applied to the measurement of the speed of rotation of the disc, and serves as a very delicate method of observing whether the speed remains constant.

Vibration of plates.—Chladni's figures.—A rectangular plate may be looked upon as a very broad rod. Thus the square plate ABCD (Fig. 683) may be considered to be a rod of length AD and width AB, or as a rod of length AB and width AD. In the former case it could vibrate about EG as a nodal line, and in the latter about FH as a nodal line. If the plate be clamped at its middle point O, and bowed at one corner, EG and FH are nodal lines, and the corners are points of greatest vibration. A very great number of modes of vibration may be produced in plates, the theoretical determination of which presents great difficulties. These forms may be produced, however, by a method due to Chladni. The plate is clamped at one point, usually the middle, and bowed at some point of the edge, the finger being pressed on some other point or points to fix the position of the nodal lines. If some fine dry sand be sprinkled upon the plate, the sand is thrown off the parts of greatest vibration

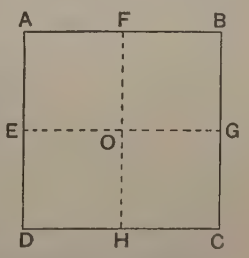

FIG. 683.—Vibration of a plate.

and collects along the nodal lines. If lycopodium powder be used it collects on the parts of greatest vibration. Several of Chladni's

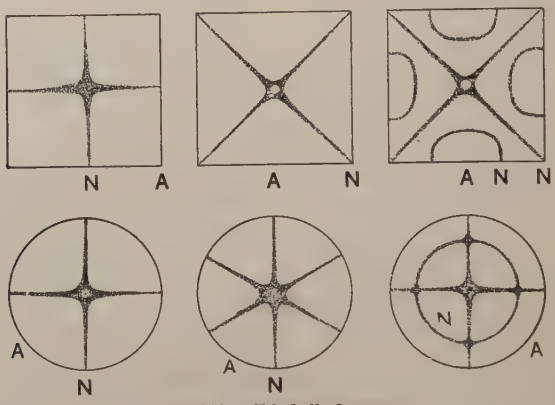

FIG. 684.—Chladni's figures.

figures are given in Fig. 684, the finger being placed at N and the plate bowed at A. The two places on opposite sides of any nodal line are always in opposite phases.

Bells.—The mode of vibration of a bell is very complex, but considering it for a moment to be that of a cylinder, we can see that the vibration is not entirely radial. For if the undisturbed position of the cylinder be indicated by the circle in Fig. 685, one extreme position of the vibration is indicated by the dotted line. A is an antinode, the points NN having no radial motion. They must, however, have a tangential motion, for the arc NA′N is less in length than the arc NAN, and the arc NB′N is longer than NBN. The nodes must, therefore, have a small vibration in a circumferential manner. This explains why passing the finger round the moistened rim of a wine glass can set it in vibration. The motion of the finger causes circumferential motion of the point over which it is passing, and this sets up the radial vibration.

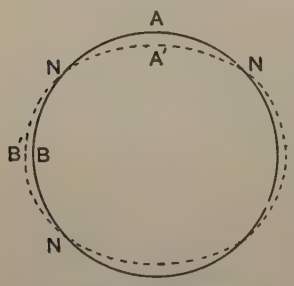

FIG. 685.—Mode of vibration of a bell.

The overtones, which determine the quality of the note of a bell, are not the simple overtones given by a vibrating string or

an organ pipe, and the fundamental note which determines the tone of the bell is not the lowest note, but the lowest but one. The art of constructing bells of high quality is largely empirical, the maker knowing by experience how to remove metal from the bell in order to modify the overtones suitably.

Exercises on Chapter LVIII.

1. How does the pitch of the note emitted by a stretched string depend on (*a*) the stretching force, (*b*) the length of the string, (*c*) the mass of the string per unit length? Describe an experiment for measuring the frequency of a tuning-fork by comparing it with that of a stretched string.

2. Upon what physical properties do (1) the loudness, (2) the pitch, (3) the quality of a musical note depend? How would you show experimentally the existence of overtones in the note emitted by a vibrating string? Sen. Camb. Loc.

3. Describe the possible forms of transverse vibration of a stretched string, and the effect on the quality of the note emitted of plucking or bowing it at different places. L.U.

4. State the laws of vibration of strings, and describe experiments to verify them.

A wire of length 140 cm. and mass 52 grams is stretched by means of a load of 16 kilograms. Calculate the frequency of the fundamental vibration. $g = 981$ cm. per sec. per sec. L.U.

5. Two wires equal in mass and dimensions are stretched on a sounding-board by loads of 8 and 18 pounds respectively. Compare the frequencies of the notes due to their fundamental transverse vibrations.

Explain how, without changing the lengths or tensions of the strings, you could make them emit notes of the same pitch. L.U.

6. Two wires, equal in length and made of the same material, are subjected to tensions in the proportion 1 : 3, the one under the greater tension being the thicker. The thinner wire emits a fundamental note of double the frequency of the other. Compare the diameters of the wires, supposing them to be circular in cross-section. L.U.

7. Describe a sonometer, and give an account of any experiments you have seen done to illustrate the laws of vibrating strings. If a string is sounding a given note, in what ratio must the tension be increased to alter the frequency in the ratio 5 : 2? How much would the string have to be shortened to make the same change without altering the tension? L.U.

8. A bridge is placed under the string of a monochord at a point near the middle, and it is found on plucking the two parts of the string that 3 beats per second are produced when the load stretching the string is 8 kilos. If the load be then increased to 11 kilos., determine the rate of beating of the two parts of the string. L.U.

9. Show by means of diagrams the possible modes of vibration of a stretched string.

If the first overtone of a stretched string 75 cm. long has a frequency of 200 when the string is stretched by a weight of 4 kilos., what is the mass of the string between the supports ? L.U.

10. State the laws of transverse vibrations of strings.

A brass wire, density 8·5, radius 0·02 cm. is stretched between two clamps 90 cm. apart while subjected to an extension of 0·05 cm. per metre. Find the pitch of the lowest note due to transversal vibrations, assuming Young's modulus for the wire to be $9{\cdot}8 \times 10^{11}$ dynes 1 cm^2. Madras University.

11. Explain the stroboscopic method of finding the frequency of a fork. Compare the accuracy of this method with that of other methods you know. Allahabad University.

12. Explain the manner in which reflection of a transverse wave in a stretched string occurs, when the wave reaches a fixed point in the string.

13. The string of a monochord vibrates 100 times a second. Its length is doubled and its tension altered until it makes 150 vibrations a second. What is the ratio of the new tension to the old ? Allahabad University.

14. On what does the pitch of the note given by a stretched string depend ? The lengths of two wires A and B of the same material are in the ratio 2 : 1, and their diameters are as 1 : 2, the cross sections being circular. If the tension in A is 5 kilograms, what must be the tension in B in order that it may emit the same note as A ? L.U.

15. Show how a stationary oscillation on a stretched string can be represented as the sum of two waves travelling with equal velocities in opposite directions along the string. L.U.

16. Explain how you would demonstrate the existence of harmonics in the vibrating wire of a sonometer.

A copper wire one meter long is vibrating in two segments when stretched by a weight of $\frac{1}{4}$ kilogram. Find the frequency of the note if the mass per centimetre is ·01 gram. ($g = 980$ cm. per sec. per sec.) L.U.

CHAPTER LIX

VIBRATION OF AIR IN PIPES

Reflection of compression wave at a rigid wall.—That compression waves are reflected at a rigid wall has already been noted (p. 698), but in order to find the phase of the reflected wave, further consideration is necessary. A longitudinal wave may be represented either by a displacement curve, or by a compression curve, the two being simply related to each other. If the displacement curve is a sine curve, the curve representing compression, and therefore the excess of pressure over the normal, is also a sine curve, whose ordinates are everywhere proportional to the slope of the displacement curve. The compression is represented by a sine curve, a quarter of a wave-length in advance of the displacement curve (p. 686). The compression curve is more useful than the displacement curve in studying the reflection of longitudinal waves in gases.

Let the wave represented by the compression curve C (Fig. 686) be incident upon the wall P. A harmonic pressure is then exerted upon P, and, what is the same thing, P exerts a harmonic pressure upon the gas. The wall therefore gives rise to two waves, one travelling to the right and the other to the left, but the two are not in the same phase, as were the waves produced by a fixed point in a string (p. 728). Remembering that in order to exert a pressure to the left, P would have to move towards the left, we see that the same movement would produce a rarefaction to the right. The two waves would therefore have opposite phases at P, as is the case with waves started upon the two sides of the prong of a tuning-fork (p. 707). The two chain-line curves in Fig. 686 indicate these two waves at

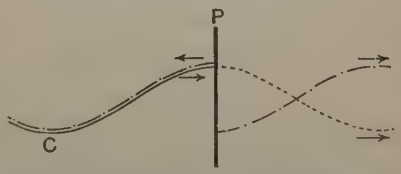

FIG. 686.—Reflection of a compression wave at a rigid wall.

the given instant; the one to the right of P cancels out the continuation of the incident wave (dotted curve), and the one to the left is the reflected wave.

In Fig. 687 four stages in the reflection of a compression wave are shown, from which it will be seen that the incident and reflected waves compound into a steady vibration. The waves to the right of P always have a resultant zero, and consequently need not have any physical existence at all. Also, note that P is a point of maximum variation of pressure and that **a compression is reflected as a compression.**

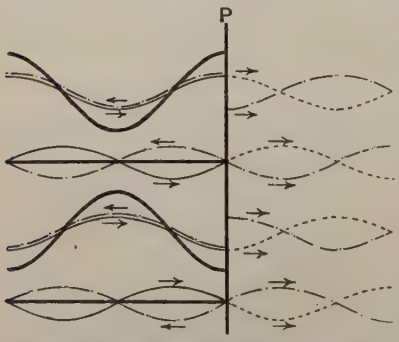

FIG. 687.—Four phases in the reflection of a compression wave at a rigid wall.

The process of construction is, therefore: 1st, continue the incident wave past the obstacle; 2nd, draw a wave which will cancel out this continuation; and 3rd, draw the twin wave to this, which starts simultaneously from P, remembering that in this case the twin waves are in opposite phases at P.

The above process may be represented equally well by equations instead of curves; thus, if the incident compression wave be represented by

$$y = a \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right),$$

the wave to cancel out the continuation of this is

$$y = -a \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right).$$

The twin wave to this is (note the change in sign of the amplitude)

$$y = +a \sin 2\pi \left(\frac{t}{\mathsf{T}} + \frac{x}{\lambda}\right),$$

which is therefore the equation to the reflected wave.

Again, the incident and reflected waves compound into the steady vibration

$$y = a \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right) + a \sin 2\pi \left(\frac{t}{\mathsf{T}} + \frac{x}{\lambda}\right)$$

$$= 2a \cos 2\pi \frac{x}{\lambda} \sin 2\pi \frac{t}{\mathsf{T}}.$$

The variation of pressure is obviously a maximum where $x = 0$, since $\cos 0 = 1$, that is, at the point of reflection, at which point,

$$y = 2a \sin 2\pi \frac{t}{\mathsf{T}}.$$

Closed pipe.—The vibration of the column of air in a closed pipe can now be considered. Let a tuning-fork vibrate over the open end of the pipe AN, which is closed at the end N (Fig 688). As the prong near the pipe moves downwards, it produces a compression which travels down the pipe. This is reflected at N as a compression and returns to A. If, on arriving at A, it finds the prong now moving upwards, there will be a rarefaction produced at A by two simultaneous causes—the reaction of the compression and the movement upwards of the prong. This rarefaction will now travel down the pipe and be reflected as a rarefaction and will arrive at the top as the prong is moving downwards. The wave has travelled four times the length of the pipe, while the prong has made one complete vibration. If l be the length of the pipe, the wave-length of the wave emitted by the fork is $4l = \lambda$, and if n be the frequency of the fork

$$n\lambda = V,$$

or,

$$n = \frac{V}{4l}.$$

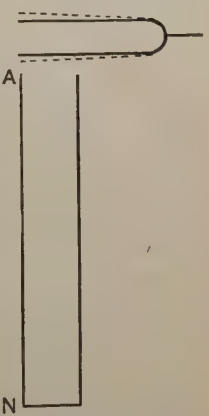

FIG. 688.—Resonance of a closed pipe.

In this case, during the first few vibrations of the fork, the vibration of the column of air in the pipe increases until, eventually, a steady state of vibration is reached. The pipe thus resounds to the fork. Had the length of the pipe been different, the phase relationship between the fork and the compression wave would not have been maintained, and the fork and the wave would have sometimes assisted, sometimes opposed each other, and there would have been no resonance.

Modes of vibration of a closed pipe.—From the fact that the wave travels four times the length of the pipe while the fork makes a complete vibration, it follows that the length of the pipe is one-quarter of the length of a wave. Also, the direct and reflected waves build up into a steady vibration, and the distance between a node and an antinode is a quarter of a wave-length (p. 730). Further, the end A (Fig. 688) is a point of maximum movement of the air, that is, an antinode; the closed end N, although a point of maximum variation of pressure, is obviously a point of zero movement of the air.

Bearing these facts in mind, it is clear that there are other possible modes of vibration of the air in the pipe.

The condition for the column of air to vibrate is, that the frequency must be such that **the open end is an antinode while the closed end is a node, and the nodes must alternate with the antinodes.**

The first three modes of vibration are illustrated in Fig. 689, where the dotted lines represent the displacement curves for the steady vibration resulting from the compounding of the incident and reflected waves. Case (*a*) has already been considered; here $n_0 = V/4l$. The next mode of vibration is (*b*) in which the length of the pipe is three-quarters of a wave-length. Thus, $l = \frac{3}{4}\lambda$; and since

$$n = \frac{V}{\lambda}, \quad n = \frac{3V}{4l} = 3n_0,$$

n_0 being the frequency for the fundamental (*a*). In (*c*) the pipe includes $1\frac{1}{4}$ waves, that is,

$$l = \tfrac{5}{4}\lambda, \quad \text{and} \quad n = \frac{5V}{4l} = 5n_0.$$

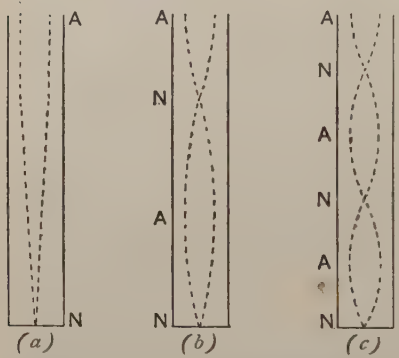

FIG. 689.—Modes of vibration of the air in a closed pipe.

It follows, then, that the possible frequencies of vibration for a closed pipe are in the ratios of the odd numbers

$$1 : 3 : 5 : 7 : \text{etc.}$$

When a pipe is sounded, the first few overtones are always present and give the note its characteristic quality. By blowing the pipe with different pressures of air, the intensity of the overtones changes, and the quality of the note varies.

Reflection at the open end of a pipe.—It is a general fact that a wave of any kind, reaching a discontinuity of any sort, undergoes reflection to some extent. Several cases, such as the reflection at the fixed end of a string and the closed end of a pipe, have been considered. But when a wave, travelling along a pipe, reaches an open end, there is a discontinuity, and reflection occurs. The conditions of this reflection must now be considered. Wherever the pressure in the air is greater than the normal, this excess of pressure is exerted in all directions, and while the wave is travelling in the tube, the

lateral pressure acts upon the sides of the tube, which are supposed to be rigid. On reaching the end of the pipe, however, this lateral pressure is unbalanced by any walls, and expansion can now take place in all directions. The treatment of the previous case enables us to find the form of the reflected wave, remembering that the tendency of the open end is to destroy the excess pressure, that is, to bring the pressure to the normal.

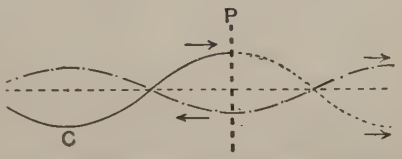

FIG. 690.—Reflection of a compression wave at the open end of a pipe.

Let C (Fig. 690) be the compression curve for a wave travelling along the pipe, reaching the open end at P. The effect of the open end is to produce a tendency to reduce the pressure to the normal, by the air moving outwards as a compression arrives, and inwards as a rarefaction arrives. The wave represented by the chain curve to the right of P then partly neutralises the continuation of the incident wave, but not completely. This is evident, for waves of considerable intensity emerge from the pipe. Thus the reflection by an open end is never complete, that is, the whole wave is not reflected, but part emerges. The twin waves arising at P are therefore in phase with each other at P, for a rushing out of the air from P causes a rarefaction to start in both directions, and, conversely, a rushing in starts compressions.

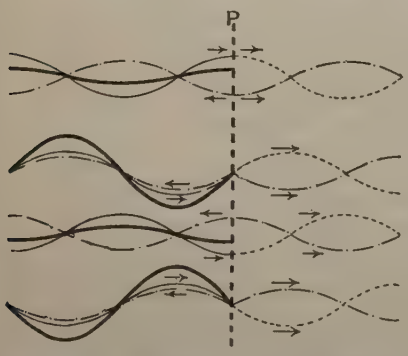

FIG. 691.—Four phases in the reflection of a compression wave at the open end of a pipe.

The compounding of the reflected wave with the incident wave in the tube may be carried out as before (p. 744), four stages in the reflection of one wave-length being given in Fig. 691. It must be remembered that the interference of the waves within the tube is not complete, as the reflected wave is not of the same intensity as the incident wave. Within the tube, we may consider that there is a progressive wave travelling down it, superimposed upon the steady vibration, and that this progressive wave emerges at the end of the tube.

The equations for the waves are then as follows :

Incident compression wave, $y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$. .....(1)

Twin compression waves,

$$\begin{cases} y = -a' \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right), & \text{.....(2)} \\ y = -a' \sin 2\pi \left(\frac{t}{\mathsf{T}} + \frac{x}{\lambda}\right). & \text{.....(3)} \end{cases}$$

Equation (3) represents the reflected wave.
Equation (1) may be written

$$y = (a - a') \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{\mathsf{T}}\right) + a' \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{\mathsf{T}}\right). \quad \text{.....(4)}$$

Combining equations (3) and (4), we obtain for the equation for the state of compression in the pipe,

$$y = (a - a') \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right) + a' \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right) - a' \sin 2\pi \left(\frac{t}{\mathsf{T}} + \frac{x}{\lambda}\right)$$

$$= (a - a') \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right) + 2a' \sin 2\pi \frac{x}{\lambda} \cos 2\pi \frac{t}{\mathsf{T}}.$$

(emerges) (stationary vibration)

The emergent wave is the resultant of the incident wave and the emergent twin wave, that is,

$$y = a \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right) - a' \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right)$$

$$= (a - a') \sin 2\pi \left(\frac{t}{\mathsf{T}} - \frac{x}{\lambda}\right).$$

Modes of vibration of air in an open pipe.—In determining the possible modes of vibration of the air in a pipe open at both ends, it must be remembered that **an open end must always be an antinode,** that is, the amplitude of displacement is a maximum at an open end (p. 746). Hence the simplest mode of vibration possible is that with **an antinode at each end and a node in the middle**, for nodes and antinodes must alternate with each other. It has been seen that the open end is a place of minimum variation of pressure, which is further evidence that it must be an antinode. For, at a node the air is moving in both directions towards it for half a vibration (Fig. 694), and away from it for the other half, and hence a node is the point of maximum variation in pressure.

If follows that the simplest mode of vibration is that of Fig. 692 (*a*). The length of the pipe is the distance between two consecutive antinodes, and is therefore half a wave-length (p. 730). Then, since $\mathsf{V} = n\lambda$, $n_0 = \mathsf{V}/2l$, which is therefore the frequency of the fundamental vibration. The next mode is given in (*b*), where the length

of the pipe is one wave-length, and $n = V/l = 2n_0$. This is the first overtone, and has twice the frequency of the fundamental. In (*c*), $n = 3V/2l = 3n_0$, and so on. Thus the fundamental and overtones of an open pipe have frequencies proportional to the natural numbers 1 : 2 : 3 : 4 : etc., which should be contrasted with those for the closed pipe (p. 746), in which only the frequencies proportional to the odd numbers are present. The remark concerning the presence of the overtones in the case of a closed pipe (p. 746) and the quality of the note also applies here.

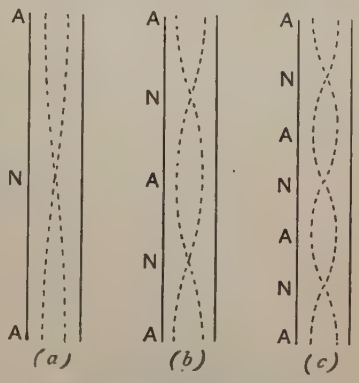

FIG. 692.—Modes of vibration of air in an open pipe.

Manometric flames, etc.—The points of maximum pressure disturbance in an organ pipe may be shown to be at the nodes of displacement by means of manometric flames (p. 708) let into the side of the pipe. On blowing an open pipe gently, the flame B (Fig. 693) when examined in the rotating mirror will present a toothed edge. On blowing the pipe harder, the first overtone (Fig. 692 (*b*)) will be produced and the flames A and C will present teeth, while the flame B will be more nearly quiescent. With a sufficient number of manometric flames, the nodes of Fig. 692 may all be detected when the pipes are suitably blown. It is a general rule that the narrower a pipe and the more forcibly it is blown, the more readily will it give the higher overtones.

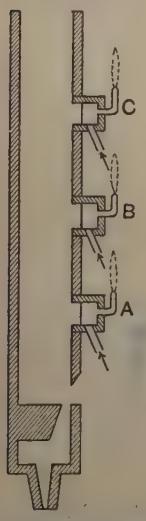

FIG. 693.—Application of manometric flames to pipes.

Another method of finding the positions of the nodes and antinodes is to lower into the pipe a small paper drum upon which some fine sand has been sprinkled. One of the sides of the organ pipe must be of glass, if the movement of the sand upon the paper is to be observed. At a node, the sand is at rest upon the paper, but at an antinode the sand grains are seen to dance about upon the paper.

Cheshire's disc.—The motion of the air in a sounding pipe can be represented very well by

means of a diagram similar to Fig. 633, known as Cheshire's disc. A small circle ABC (Fig. 694) is equally divided into 12 parts by points on the circumference. Perpendiculars are dropped from these points upon the diameter AC and the feet of these perpendiculars are taken successively as the centres of circles of gradually increasing radius. A is the first centre, then the next point to A, and so on until C is reached, and the diameter then retraversed

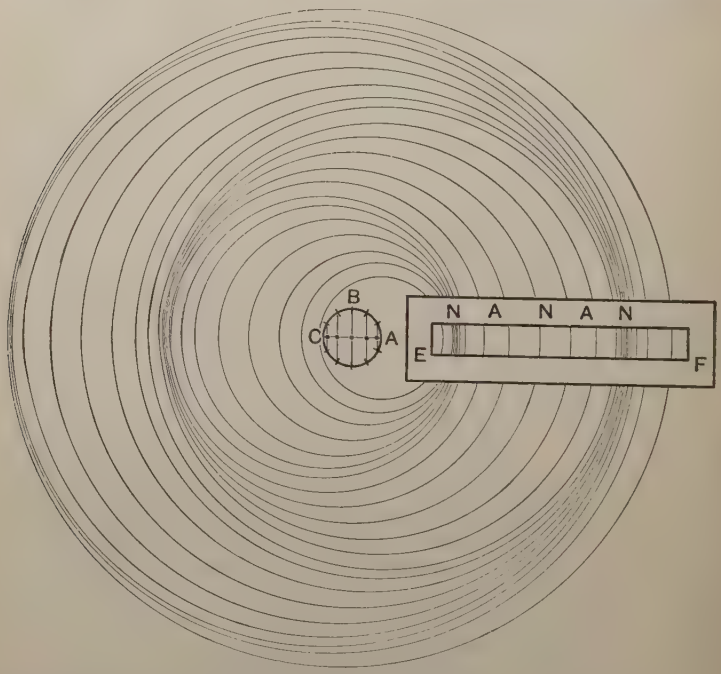

FIG. 694.—Cheshire's disc.

until A is reached again. On covering up all but a strip EF, which represents the pipe, and rotating the circles about the centre of the circle ABC, the lines will vibrate longitudinally, N, N, N being nodes and A, A antinodes. It will be seen that the lines move from both sides towards a node for half a vibration, and away from it for the other half.

End correction of pipe.—It is found that the length of a closed pipe is not exactly one-quarter of the wave-length corresponding to the fundamental; neither is the length of the open pipe exactly

half the wave-length. The reason is that the reflection does not occur exactly in the plane of the open end. From the nature of the reflection at the open end of a pipe, it follows that the process of reflection is not entirely confined to one plane. The effective end of the pipe is, therefore, not at its actual end but a small distance beyond it. It has been found by experiment that for a cylindrical pipe, the effective length may be calculated on the assumption that reflection occurs at a plane situated beyond the open end by a distance $0{\cdot}6 \times$ radius of pipe. It is this effective length and not the actual length of the pipe that must be used in the calculation of the frequencies of the fundamental and overtones on pp. 746 and 748.

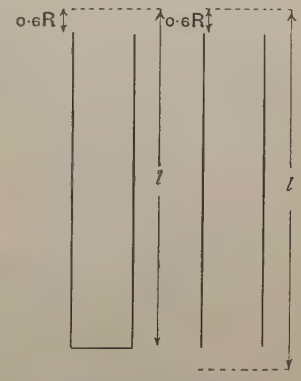

FIG. 695.—End correction at the open end of a pipe.

Thus, for a closed pipe

l = actual length + 0·6 radius,

and for an open pipe

l = actual length + 1·2 radius.

It follows that the overtones of a pipe form a harmonic series, provided that the end correction is strictly constant. This, however, is the case only where the wave-length of the vibration is large in comparison with the diameter of the pipe. Thus, a narrow pipe will have overtones whose frequencies are given very nearly by the numbers 1, 2, 3, 4, etc., in the case of an open pipe; but if the pipe is a wide one, the end correction is greater the higher the frequency of the overtones, and these will not fall exactly in the harmonic series. Also they will not be so readily produced in a wide pipe, which will therefore be poorer in overtones than a narrow one.

Helmholtz showed that it is possible to construct a bell mouth for a pipe which shall have zero end correction, the antinode being in the plane of the end. The side of the bell is in the form of a hyperbola, the radius of the mouth being $\sqrt{2}$ times that of the cylindrical part of the pipe. For a pipe with such a mouth, the overtones would be exactly in the harmonic series. This is of importance in the construction of brass wind instruments, where it is necessary to have the overtones as nearly in the harmonic series as possible.

Conical pipes.—The theory of conical pipes is too complicated to be treated here, but one or two facts concerning them may be

mentioned. With a small angle of cone there is an antinode at the open end, and the antinodes for the overtones are equally spaced along the axis of the pipe. Also, the overtones in the case of a conical pipe **closed at the apex and open at the mouth** form the whole harmonic series, having frequencies 1, 2, 3, 4, etc. This is a contrast to the cylindrical closed pipe, where only the frequencies proportional to the odd numbers are present. Thus, in the case of a conical organ pipe excited by means of a reed (p. 763), which is equivalent to a closed end, the whole harmonic series of overtones may be present.

EXPT. 158.—**Velocity of sound in air by resonance.** A tuning-fork of known frequency is struck and then held over the mouth of a tube

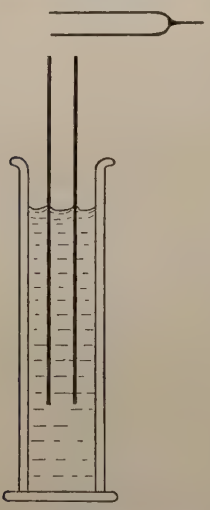

FIG. 696.—Resonance of a column of air.

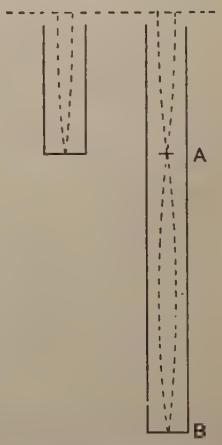

FIG. 697.—Elimination of the end correction of a pipe.

dipping into water (Fig. 696). The length of the column of air in the tube can be varied at will by raising or lowering the tube. A position can be found for which the column of air resounds most loudly to the fork. Measure this length of tube, and add the end correction of 0·6 of the radius, thus obtaining l, the effective length of the pipe. This is one-quarter of the wave-length λ, and since

$$V = n\lambda = 4nl,$$

V, the velocity of sound in air, is determined. This should be corrected for the fact that the air is not as a rule at 0° C. Take the temperature

of the air by means of a Centigrade thermometer and find the velocity at 0° C. from the relation

$$V_t = V_0\sqrt{1+\alpha t}, \quad \text{or} \quad V_T = V_0\sqrt{\frac{T}{273}},$$

T being the temperature upon the absolute scale.

There is another and better way of eliminating the end correction, and that is, to find a second length of tube which resounds to the fork. This length is approximately three times the first length. The difference between the first length and the second is the length AB (Fig. 697), and this is half a wave-length, independently of what the end correction may be. Then, $V = 2n(AB)$.

Longitudinal vibration of rods.—A wooden rod stroked with a resined flannel, or a glass rod stroked with a wet duster, can be set in longitudinal vibration. The ends of the rod being the parts of greatest freedom are antinodes, and hence, for the simplest mode of vibration, there will be a node in the middle of the rod. It will therefore vibrate in a similar manner to the air in an open pipe sounding its fundamental (Fig. 692 (*a*)). It follows that if the rod is clamped, the clamping should be done at its middle point in order not to interfere with its vibration. The overtones are not sufficiently important to take into account.

The velocity of a longitudinal wave in a rod is given by $\sqrt{E/d}$ (p. 695), where E is Young's modulus of elasticity and d is the density of the material of the rod. Again, since $V = n\lambda$ and $\lambda = 2l$, where l is the length of the rod, or the distance between consecutive antinodes,

$$n = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{E}{d}}.$$

Example.—Find the frequency of longitudinal vibrations for a brass rod of length 1½ metre; Young's modulus is $1{\cdot}02 \times 10^{12}$ dynes per sq. cm. and the density is 8·5 grams per c.c.

$$n = \frac{1}{300}\sqrt{\frac{1{\cdot}02 \times 10^{12}}{8{\cdot}5}} = 1154.$$

It is thus seen that the notes given by the longitudinal vibration of rods are usually of high pitch.

In the table on p. 696 the velocity of sound in rods of various materials is given.

Expt. 159.—**Velocity of sound in a rod.** Let a rod of wood or glass be clamped in its middle point A (Fig. 698), and stroked with a resined flannel in the case of wood or metal, or a wet duster in the case of glass,

until it emits a note. With the adjustable bridge, tune a length of the string of a monochord until it gives a note of the same pitch as the rod. Care must be taken that unison is obtained and not the octave. Measure the length of string found and call it l_r. Now move the bridge until a length of the same string gives a note in unison with a standard fork of known frequency. Measure the length of string and call it l_f.

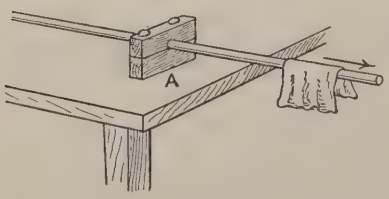

FIG. 698.—Longitudinal vibration of a rod.

Then, $$\frac{\text{frequency of rod}}{\text{frequency of fork}} = \frac{l}{l_r},$$

from which the frequency of the rod can be found. Measure the length of the rod and double it to obtain the wave-length λ.

Then, Velocity of sound in rod $= n\lambda$.

In this way find the velocity of sound in several rods.

Kundt's dust figures.—The longitudinal vibrations of a rod may be used to set a column of air in steady vibration, and the wave-length in air corresponding to the frequency of the rod may be found.

A glass tube AB (Fig. 699), resting on V blocks, is closed at the end B. One end of the rod enters at A, the rod being clamped at its middle

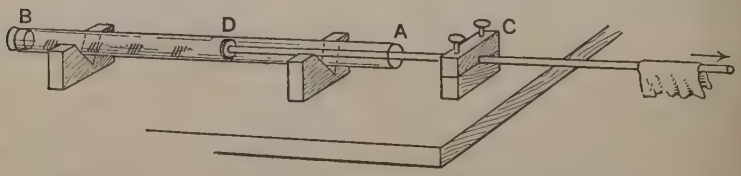

FIG. 699.—Kundt's dust figures.

point C and having a cardboard or metal disc D firmly fixed at the end within the tube. The disc must occupy most of the section of the tube without touching the walls. Before being put in position, the tube must be thoroughly dried by heating, and some lycopodium powder spread along its side. On stroking the rod to cause it to vibrate, compression waves are emitted by D, which travel down the tube and are reflected by B. If the position of the tube is adjusted until B and D are in the respective positions of a node and an antinode, the air between B and D is set into steady vibration.

If the vibration of the air is sufficiently vigorous, the lycopodium collects at the antinodes, and at each antinode forms a number of parallel ridges. The middle of each set of ridges may be taken as the position of the antinode, and in a successful experiment five or six antinodes may be found in this way. Their distance apart can be measured, and remembering that the distance between consecutive antinodes is half the wave-length in air, while the length of the rod is half a wave-length in the rod for the same frequency,

$$\frac{\text{Distance between antinodes in air}}{\text{Length of rod}} = \frac{\text{Velocity of sound in air}}{\text{Velocity of sound in rod}}.$$

If the frequency be found by means of a monochord and standard fork, the velocities of sound in both air and rod can both be found.

EXPT. 160.—**Velocity of sound in air by Kundt's dust figures.** Set up the rod and dried tube with lycopodium powder as in Fig. 699. Stroke the rod vigorously, and slide the tube little by little parallel to its own axis, at the same time turning it a little about its axis, until the dust figures are produced. Measure the distance between the extreme antinodes obtained, and divide by the number of spaces, to get the mean half-wave-length. Obtain the frequency of the rod by tuning a monochord string to it and to a standard fork in turn, and calculate the frequency n as on p. 754.

Then, $\text{Velocity of sound in air} = n \times \lambda$.

Observe the temperature and calculate the velocity of sound at 0° C.

Velocity of sound in gases other than air.—To determine the velocity of sound in a gas other than air, there must be some means of admitting the gas to the Kundt's dust-figure tube so that the air is swept out and replaced by the gas. In the case of hydrogen, or carbon dioxide, an ordinary Kipp's apparatus may be used, but the gas should first be passed through drying tubes. The dust figures may then be obtained as before, and the frequency and velocity calculated. It is usual, however, first to obtain the dust figures with the tube containing air, and then to replace the air by the gas and obtain them again. In this way the ratio of the wave-lengths in the gas and in air is obtained, and since the frequency is the same in both cases, we have

$$\frac{\text{Velocity of sound in gas}}{\text{Velocity of sound in air}} = \frac{\text{Wave-length in gas}}{\text{Wave-length in air}}.$$

The correction for temperature may be made as before (p. 694).

Exercises on Chapter LIX.

1. A resonance tube, 2 cm. in diameter and containing water, gives the loudest note with a fork of frequency 512 when the air column in the tube above the water is 15.5 cm. long. Find (*a*) the wave-length of this note, (*b*) the velocity of sound in air. Sen. Camb. Loc.

2. Two organ pipes, one open and one closed, are giving the same note. What is the ratio of their lengths? What overtones would you expect to find present in each case?

3. State the effect of temperature and density upon the velocity of sound in gases.

A whistle gives a certain note when sounded, the temperature being 18° C. To what must the temperature be raised for it to give a note of $\frac{9}{8}$ the frequency of that at 18° C.? L.U.

4. Describe some method of determining the velocity of sound in air by making use of the phenomenon of resonance. L.U.

5. How do the frequencies of the notes emitted by an organ pipe depend on the temperature of the air?

An organ pipe having been tuned in the morning to give a note of frequency 256 at 15° C. is found in the evening when blown strongly to give a note of 516. Find the temperature in the second case, and state the type of the organ pipe. L.U.

6. Two organ pipes each 3 feet long are sounded on a day when the velocity of sound is 1120 feet per second. One is open at both ends and the other closed at one end. What are the frequencies of their fundamentals and principal overtones?

7. How does a change in temperature alter the pitch of (*a*) an organ pipe, (*b*) a tuning-fork?

Calculate the change of pitch of an open organ pipe 3 feet long when the temperature changes from 10° C. to 15° C. L.U.

8. What change, if any, would take place in pitch and wave-length if the air (*a*) in an organ pipe, (*b*) around a tuning-fork, were replaced by hydrogen? Give reasons in each case. L.U.

9. Describe an experiment by which it can be shown indirectly that sound travels more rapidly in coal-gas than in air. What measurements would be necessary to find the ratio of the velocities? L.U.

10. An organ pipe emits a fundamental note of frequency 120. By blowing it more strongly it can be made to emit a note of frequency 240. Is it a closed or an open pipe? Give reasons for your answer.

11. How are *beats* produced?

Beats are heard three times per second when two open organ pipes, one 2 feet 9 inches in length and the other half an inch longer, are sounded together. Find the velocity of sound in the air at the time of the observation. L.U.

12. Describe an experimental method of determining the effect of change of temperature on the wave-length in air corresponding to a sound of given frequency L.U.

13. A column of air and a tuning-fork produce 4 beats per second when sounding together, the fork giving the lower note and the temperature of the air being 15° C. When the temperature has fallen to 10° C. the two produce 3 beats per second. Find the frequency of the fork.
L.U.

14. Notes sounded by different instruments, although of the same pitch and loudness, possess characteristic differences. To what are these differences due? How does the note emitted by an open pipe differ from that emitted by a closed pipe? L.U.

15. State generally the nature of the reflection of sound at the closed and open end of a pipe respectively, and give reasons for the difference.
Allahabad University.

16. Explain the nature of beats produced when two tuning forks of nearly equal frequencies are sounded together.
Two open pipes without flange, having both a length of 4 m. and diameters 12 cm. and 24 cm., respectively, are observed to give a number of beats when sounded together. Determine which pipe is to be shortened and by how much if they are to be brought into unison.
Velocity of sound = 340 m. per s. and correction = $0{\cdot}5r$, when r is the radius of cross section of the pipe. Madras University.

17. Explain the process of reflection of a harmonic wave in air which occurs when the wave meets a rigid wall. Show that a compression is reflected as a compression, and a rarefaction as a rarefaction.

18. Contrast the possible modes of vibration of air in open and closed organ pipes.
What must be the ratio of the lengths of a closed and an open pipe in order that the third overtone of the open pipe shall be in unison with the second overtone of the closed pipe? L.U.

19. Draw diagrams to illustrate the process of reflection of an air wave at the open end of a pipe. Explain why a compression is reflected as a rarefaction, and a rarefaction as a compression.

20. Describe the motions arising when a tuning-fork sets up a system of progressive sound-waves in air; and explain how such motions differ in general character from the stationary waves occurring in a resonance tube.

CHAPTER LX

THE EAR AND MUSICAL INSTRUMENTS

The human ear.—The ear being the organ by which we become conscious of sound, the following account of the physical nature of the ear is given. Three distinct parts of the ear may be recognised, namely, the **external ear**, by which the sound-wave is collected,

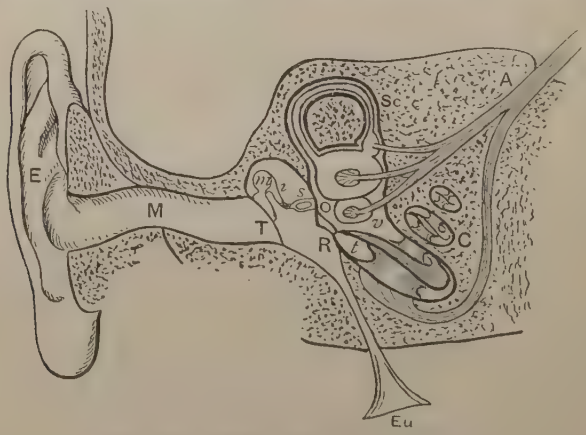

FIG. 700.—Section through the human ear.

the **middle ear**, or **tympanum**, or **drum**, in which the vibrations are transmitted from the external ear to the **internal ear**, or **labyrinth**. The relative positions of these may be seen in the diagrammatic representation of a section through the ear (Fig. 700).

E is the concha, or part external to the head, which serves as a funnel whose narrow end leads to the external auditory meatus M. This is closed at its inner end by the tympanic membrane T, sometimes called the drum of the ear. The drum proper is the cavity bounded upon its outer side by the tympanic membrane, and its

inner side by bony walls, except at two places O and R, across which membranes are stretched. O is called the fenestra ovalis and R the fenestra rotunda. The drum is also in communication with the upper part of the throat by means of the Eustachian tube Eu, which serves to keep the air pressure equal upon the two sides of the tympanic membrane.

The compression waves arriving at the tympanic membrane set it in forced vibration, which vibration is transmitted onwards by three small bones which articulate with each other. The malleus *m* bears against the tympanic membrane. The incus *i* is moved by the malleus and in its turn moves the last of the three, the stapes or stirrup S. The base of the stapes rests against a small oval membrane, the fenestra ovalis O. This is one of the boundaries of the labyrinth, which is a complex set of canals lying within a hard bony part of the skull. It consists essentially of three semicircular canals of which one, Sc, is shown in section, and a spiral canal, the cochlea, shown in section at C. The cochlea is divided throughout its length by a spiral partition in which the ramifications of part of the auditory nerve A end. The canals contain a fluid (lymph) which is continuous from the fenestra ovalis, through the semicircular canals, up one side of the cochlea and down the other, and ends at the small membrane, the fenestra rotunda *r*. The canals are drawn larger than they really are, for the sake of clearness.

It will thus be seen that the vibration of the fenestra ovalis starts waves which travel through the lymph round this complex path, and in doing so cause motion of the bodies in which the ultimate ramifications of the auditory nerve are situated, and so give rise to the sensation of sound. These nerve endings are situated in very complex structures, and the process of audition is not yet fully understood. For a further account of the canals and cochlea with their nerve distributions, the student must be referred to works on anatomy.

The violin.—The four-stringed instruments, violin, viola, violoncello, and double-bass, are similar in form, and differ from all other musical instruments in that the strings are set in vibration by means of a bow. The violin has four strings, *g*, *a*, *d* and *e*, of which *g* is a thick gut loaded with a close spiral of fine copper wire, the *a*, *d* and *e* strings being plain gut of less weight in the case of the higher strings. With this arrangement the tensions in the strings are much nearer equality than would be the case if the strings were all of the same thickness.

The tuning is performed by means of the pegs P (Fig. 701), the strings passing over the bridge B. The various notes are obtained by stopping the strings with the fingers, the string being pressed to the finger-board FF at the appropriate place.

The body of the violin is a sounding box of thin wood, and the vibration of the string sets the walls in vibration, chiefly by means of the bridge B over which the string passes. The strings are set in vibration by bowing at some point between B and F. Owing to the complex shape of the body of the violin, the tone produced is not that of a string stretched upon a plain board. The vibrations of the walls are forced vibrations, since they respond to any note of the string; but they are not entirely forced vibrations, some overtones being produced more loudly than others, since the quality of the note depends entirely upon the construction of the body of the instrument. Owing

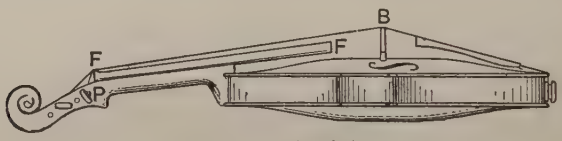

FIG. 701.—The violin.

to the complexity of the vibration of the body, no rules have been evolved for its construction, the maker following the construction that he has found by experience to be the best.

Vibration of a bowed string.—The mode of vibration of a bowed string is peculiar, since it is not a simple harmonic motion. The vibration is set up by drawing the bow across it, the friction between the string and the hairs of the bow causing the vibration. In order to increase this friction rosin is rubbed on the bow. When the bow and string are at rest with respect to each other, the friction is greater than when one is slipping past the other (p. 173). Hence, as the bow grips the string in drawing it across, the string is pulled aside by friction until the restoring force reaches a certain amount, when slipping begins. The string then slips past the bow and executes a half vibration. It then begins to move again in the direction of the bow, is again gripped and carried forward, and the process is repeated.

The peculiar motion of a bowed string may be demonstrated by throwing an image of a small piece of the string upon a screen, making use of a revolving mirror. The string AB (Fig. 702) is bowed vertically, and opposite a part of it, a screen S with a narrow vertical

slot is placed, a strong source of light, such as an arc lamp, being placed to the left of it. By means of the lens L an image of this slot and a small piece of the string is produced upon the screen S_1.

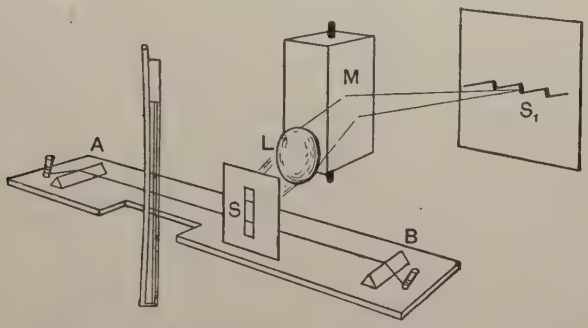

FIG. 702.—Demonstration of the mode of vibration of a bowed string.

The light before reaching the screen falls upon the rotating mirror M, similar to that in Fig. 652. If the string is not bowed, the image of the string upon the screen is drawn out into a horizontal straight line, but when it is bowed this straight line becomes a zig-zag consisting of straight parts. This shows that the vibration of the string is not simple harmonic, for if it were the image on the screen would be a sine curve.

Wind instruments.—A great variety of musical instruments may be classed under the head of 'wind instruments.' In nearly all cases, however, the sounding part is a column of air. In the smaller instruments, such as the flute, the length of the column is varied by having a set of holes along the pipe, which can be opened or closed at will, or in brass instruments, such as the trombone, the length of the pipe can be varied. In the organ, a set of pipes of fixed pitch is provided, and this is consequently an instrument with a fixed keyboard, like the pianoforte. There are two typical methods of setting the column of air in vibration and these will be described separately.

The flue pipe.—Air enters the pipe at A (Fig. 703) and issues from the slot B in a thin sheet or jet which impinges upon the sharp blade C. The energy necessary for maintaining the vibration of the air column is derived from the air under pressure which enters at A.

In order to understand how the column can be maintained in steady vibration by such a jet of air, consider that as the air rushes into the end of the pipe at B to produce a compression, it drives the jet of air coming from A inwards, and since the jet now enters the pipe it increases the compression in it. When the air is leaving the pipe at B during the production of a rarefaction, the jet is driven outwards, and so, by the hydrodynamical principle given on p. 288, tends to increase the rarefaction of the pipe. It is thus obvious that the jet can maintain the vibration in the pipe and also that **the end at which the jet is applied must be considered as an open end of the pipe.**

FIG. 703.—Flue pipe.

The method by which the jet starts the vibration, or causes the pipe to 'speak,' is not so clear. However, some want of symmetry in the direction of the jet, or other accidental circumstance, will cause the jet to be directed either slightly inwards or outwards, and will start a feeble compression or rarefaction which will travel up the pipe and be reflected at the other end. It has been seen above that if the vibration be once started, the energy of the jet tends to increase it, until as much energy is supplied by the jet in a given time as is lost by the radiation of the air waves from the pipe.

Reed pipes.—A reed consists of a flexible strip (of metal, in the case of an organ pipe) which wholly or nearly covers the aperture through which the air passes to the pipe. If the reed completely covers the aperture it is called a **beating** or **striking reed** (Fig. 704) A, but if it nearly, but not quite, closes it, it is called a **free reed** B.

The beating reed is always curved outwards, so that as the air pressure closes it, the closing does not take place suddenly, but is effected gradually from the fixed to the free end. This is necessary, as the note produced is very harsh if the reed closes the aperture throughout its whole length at the same instant.

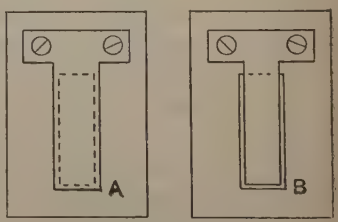

FIG. 704.—Reeds.

On admitting air to the chamber A (Fig. 705) some of it passes round the edges of the reed before it closes, causing a compression to start, which travels up the pipe and is reflected at the open end. This returns as a rarefaction (p. 747), and as it arrives at the lower end, the difference of air pressure inside and outside of the tube still keeps the reed closed. The rarefaction at this end is reflected as a rarefaction, travels to the top and then comes down the pipe

as a compression. When this arrives at the reed, the equalisation of the pressure on the two sides allows the elasticity of the reed to open the aperture and admit more air, and the process is repeated. After the first few oscillations, the state of vibration becomes steady. Since at the reed a rarefaction is reflected as a rarefaction and a compression as a compression, **the beating reed behaves as a closed end of the pipe.** In the case of the free reed, the action is not so clear, but as it vibrates it alternately opens and closes the aperture, allowing the air to enter intermittently. Free reeds are not now used for organ pipes. They are used, however, in the harmonium and American organ; in these instruments there is no pipe attached, the note emitted depending only upon the interruption of the stream of air by the reed. In the case of the organ, the vibration of the air column and the reed react on each other, the resulting note being determined by them both.

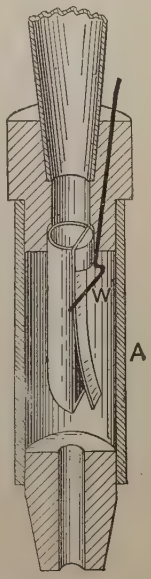

FIG. 705.—Reed pipe.

A free reed is shown in Fig. 706 in which it is seen that air is only admitted in quantity as the reed moves outwards, although a small amount can pass through the small space between the reed and the hole. This is the type of reed used in the harmonium.

Tuning of pipes and reeds.—Closed pipes are usually stopped by a plunger at the upper end, which, being slightly raised or lowered, alters the effective length of the air column. Open pipes are sometimes provided with a flexible metal flap at the upper end, which on being pressed over the end, slightly flattens the note. Sometimes holes are provided near the end, which can be more or less opened or closed by means of flaps or slides. If the pipe is a metal tube,

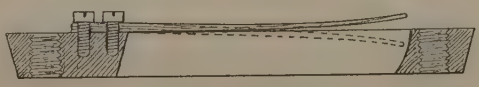
FIG. 706.—Free reed.

the end may be somewhat splayed out to sharpen the note, or bent in to flatten it. Both open and closed pipes, when of the flue type, are frequently provided with metal rings on either side of the lower aperture, which can be bent inwards to flatten the note, or outwards to sharpen it.

In the case of reed pipes, the reed is tuned by means of a wire W (Fig. 705) which can be pushed further down to restrict the motion

of the reed, so that its effective length becomes shorter and its frequency thereby increased. In the case of a free reed, the wire must grip both sides of the reed by being bent round it. The free reed in Fig. 706 is tuned by carefully scraping away metal. If it is desired to sharpen the note, metal is scraped from the tip of the reed, so that its moment of inertia becomes less and the frequency therefore increased (p. 737). In order to flatten the note, metal is scraped from the base of the reed, so that its elastic stiffness is diminished. This lowers the frequency.

The phonograph.—It has been seen that a body subject to a harmonic force executes forced vibrations which are an accurate copy of the applied vibration, when the natural frequency of the body is great compared with that of the applied vibration (p. 715). This is made use of when air vibrations are to be transmitted to some mechanical appliance. The air waves fall upon a thin circular diaphragm whose natural vibration frequency must be high, and the diaphragm is thus set in vibration with a frequency equal to that of the air waves. This method is used in the case of the tympanic membrane of the ear (p. 758), in the telephone, and also in the phonograph. In the phonograph, the back of the diaphragm is provided with a cutting tool, which cuts a trace in a wax cylinder rotating underneath it. Thus the varying depth of the cut in the wax corresponds to the motion of the diaphragm. On replacing the cutting tool by a rounded point and causing this to bear upon the cylinder as it is rotated, the diaphragm is caused to repeat its original motion, and in doing so sets the air in the bell-shaped horn H (Fig. 707) in vibration in a manner similar to the original sound waves, which are thus reproduced. In the original phonograph of Edison, tinfoil was used instead of wax, but the latter soon superseded the tinfoil.

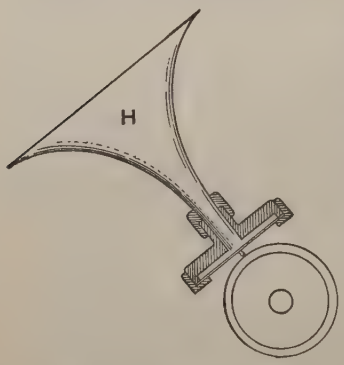

FIG. 707.—The phonograph.

The phonograph has undergone considerable evolution in details but not in essentials. Thus, in the original form the cylinder was driven by hand, or by an electro-motor. It is now almost always

driven by clockwork. Then, too, there is an arrangement for feeding the cutting point, or the reproducer, forward, so that the trace cut in the wax is in the form of a helix running from one end of the cylinder to the other. In the latest types, the cylinder is replaced by a disc, so that the trace is a spiral, running from near the centre to the circumference. In the **gramaphone** the tool makes a wavy lateral trace in the wax, which is followed by a steel point in the reproducer.

Exercises on Chapter LX.

1. Give a short account of the principal parts of the ear by means of which external sound waves are transmitted to the labyrinth.

2. Explain why a bow drawn with uniform velocity across a stretched string maintains the string in vibration. If the bow were greased would the string vibrate? Give reason.

3. Give two methods by which the air in an organ pipe is maintained in vibration.

4. Explain why the lower end of a flue pipe is an antinode and the lower end of a reed pipe is a node.

5. Describe briefly how (*a*) organ pipes, (*b*) metal reeds, may be tuned.

6. Describe how sounds are reproduced in the phonograph, or grama phone.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
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11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 12 4 7 11	15 19 23 15 19 22	27 31 35 26 30 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 11 3 7 10	14 18 21 14 17 20	25 28 32 24 27 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 7 10 3 7 10	13 16 20 13 16 19	23 26 30 22 25 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9 3 6 9	12 15 19 12 15 17	22 25 28 20 23 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 9 3 6 8	11 14 17 11 14 17	20 23 26 19 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8 3 5 8	11 14 16 10 13 16	19 22 24 18 21 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 5 8 2 5 7	10 13 15 10 12 15	18 20 23 17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7 2 5 7	9 12 14 9 11 14	16 19 21 16 18 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7 2 4 6	9 11 13 8 11 13	16 18 20 15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
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45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

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52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
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·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
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·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TRIGONOMETRICAL TABLE.

Angle.	Radians.	Sine.	Tangent.	Cotangent.	Cosine.		
0°	0	0	0	∞	1	1·5708	90°
1	·0175	·0175	·0175	57·2900	·9998	1·5533	89
2	·0349	·0349	·0349	28·6363	·9994	1·5359	88
3	·0524	·0523	·0524	19·0811	·9986	1·5184	87
4	0698	·0698	·0699	14·3006	·9976	1·5010	86
5	·0873	·0872	·0875	11·4301	·9962	1·4835	85
6	·1047	·1045	·1051	9·5144	·9945	1·4661	84
7	·1222	·1219	·1228	8·1443	·9925	1·4486	83
8	·1396	·1392	·1405	7·1154	·9903	1·4312	82
9	·1571	·1564	·1584	6·3138	·9877	1·4137	81
10	·1745	·1736	·1763	5·6713	·9848	1·3963	80
11	·1920	·1908	·1944	5·1446	·9816	1·3788	79
12	·2094	·2079	·2126	4·7046	·9781	1·3614	78
13	·2269	·2250	·2309	4·3315	·9744	1·3439	77
14	·2443	·2419	·2493	4·0108	·9703	1·3265	76
15	·2618	·2588	·2679	3·7321	·9659	1·3090	75
16	·2793	·2756	·2867	3·4874	·9613	1·2915	74
17	·2967	·2924	·3057	3·2709	·9563	1·2741	73
18	·3142	·3090	·3249	3·0777	·9511	1·2566	72
19	·3316	·3256	·3443	2·9042	·9455	1·2392	71
20	·3491	·3420	·3640	2·7475	·9397	1·2217	70
21	·3665	·3584	·3839	2·6051	·9336	1·2043	69
22	·3840	·3746	·4040	2·4751	·9272	1·1868	68
23	·4014	·3907	·4245	2·3559	·9205	1·1694	67
24	·4189	·4067	·4452	2·2460	·9135	1·1519	66
25	·4363	·4226	·4663	2·1445	·9063	1·1345	65
26	·4538	·4384	·4877	2·0503	·8988	1·1170	64
27	·4712	·4540	·5095	1·9626	·8910	1·0996	63
28	·4887	·4695	·5317	1·8807	·8830	1·0821	62
29	·5061	·4848	·5543	1·8040	·8746	1·0647	61
30	·5236	·5000	·5774	1·7321	·8660	1·0472	60
31	·5411	·5150	·6009	1·6643	·8572	1·0297	59
32	·5585	·5299	·6249	1·6003	·8480	1·0123	58
33	·5760	·5446	·6494	1·5399	·8387	·9948	57
34	·5934	·5592	·6745	1·4826	·8290	·9774	56
35	·6109	·5736	·7002	1·4281	·8192	·9599	55
36	·6283	·5878	·7265	1·3764	·8090	·9425	54
37	·6458	·6018	·7536	1·3270	·7986	·9250	53
38	·6632	·6157	·7813	1·2799	·7880	·9076	52
39	·6807	·6293	·8098	1·2349	·7771	·8901	51
40	·6981	·6428	·8391	1·1918	·7660	·8727	50
41	·7156	·6561	·8693	1·1504	·7547	·8552	49
42	·7330	·6691	·9004	1·1106	·7431	·8378	48
43	·7505	·6820	·9325	1·0724	·7314	·8203	47
44	·7679	·6947	·9657	1·0355	·7193	·8029	46
45	·7854	·7071	1·0000	1·0000	·7071	·7854	45
		Cosine.	Cotangent.	Tangent.	Sine.	Radians.	Angle.

ANSWERS

PART II. HEAT

CHAPTER XXIII. p. 322.

4. (*a*) 284° F.; (*b*) 23 °F.; (*c*) −459·4° F.

5. (*a*) 37·78° C.; (*b*) −12·22° C.; (*c*) −51·11° C.

6. −40° C. = −40° F. **9.** −0·52° F. **12.** −1·6° F.

17. Either the Centigrade thermometer should read 43·33°, or the Fahrenheit thermometer should read 113°.

CHAPTER XXIV. p. 331.

2. 1·963 inches. **3.** $10{\cdot}2 \times 10^{-6}$. **4.** 0·01397 inch.

5. 188·9 cm. **6.** 12·0099 sq. feet.

8. True length, 2,000 feet; recorded length, 2000·119 feet; error, +0·119 foot.

9. 40·087 cm.

11. 42,057 lb. wt. (this is probably in excess of the elastic limit, see p. 155).

12. 41,980 lb. wt. (see note to Answer 11). **13.** 0·3305 cubic inch.

CHAPTER XXV. p. 341.

1. 8·403 grams per c.c. **2.** 0·00000936. **3.** 7·78 grams per c.c.

4. 0·02118 c.c. **5.** 0·00000667. **7.** 76·062 cm.

8. 2·624 c.c. **9.** 0·000006. **10.** 0·0000489.

13. 389,600 cub. feet; 39,600 cub. feet. **15.** 18·02 cm.

CHAPTER XXVI. p. 350.

1. 2,545 lb.-deg.-Fah. units; 641,390 calories.

2. 432·2 lb.-deg.-Cent. units; 196,060 calories.

3. 1·504 lb.-deg.-Cent. units; 0·0376 pound.

4. 1,752,000 lb.-deg.-Cent. units.

5. 41·4° C. **6.** 0·0991. **7.** 390·7° C.
8. 0·4933. **9.** 0·6 nearly. **12.** 30·55° C.
13. 936 : 837. **14.** 0·601.

CHAPTER XXVII. p. 364.

1. 1,980,000 foot-lb. ; 1,414 lb.-deg.-Cent. ; 2,545 lb.-deg.-Fah. ; 641,400 calories.
3. 60·6 minutes. **4.** 748·4 lb.-deg.-Cent. **8.** 4·46 per cent.
11. Heat per penny, in lb.-deg.-Cent. : Coal, 67,880 ; petrol, 3,285 ; lighting gas, 8,333.
12. 19,320 lb.-deg.-Cent. **16.** $41{\cdot}5 \times 10^6$ ergs.
17. 6,728 lb.-deg.-Cent. per pound. **18.** 10,920 lb.-deg.-Cent. per pound.

CHAPTER XXVIII. p. 378.

7. $40{\cdot}32 \times 10^6$ calories per hour. **8.** $262{\cdot}1 \times 10^6$ calories per hour.
9. 104·92° C. **13.** 411,500 calories per hour.
16. Temperature of faces in contact, 72° C. ; 6·667 ; 15·24.

CHAPTER XXIX. p. 388.

15. 0·7264.

CHAPTER XXX. p. 399.

2. 1,011 grams wt./sq. cm. **3.** 1,764 lb. wt./sq. inch.
6. 76·025 cm. ; 1·00035. **7.** 29·39 inches of mercury.
10. 75, 50, 37·5, 30, 25, all in lb. wt./sq. inch.
13. 24·87 metres. **15.** 27·72 c.c. **16.** 57·98 cm.

CHAPTER XXXI. p. 416.

1. 266·3° absolute (Cent.). **2.** 441·8 cubic feet. **3.** 530·6° C.
6. 96·08 ; 1,387. **7.** $2{\cdot}87 \times 10^6$. **8.** 947·8° C.
9. 47·45 litres. **11.** 122·3° C. **12.** $41{\cdot}45 \times 10^6$.
13. 3·512 grams. **14.** 578·4 lb. wt. **16.** 0·00128 grams/c.c.
17. 6·93 tons wt. **18.** 308·6 cm. of mercury. **19.** 28·1° C.

CHAPTER XXXII. p. 426.

7. (*a*) 2,117 foot-lb. ; (*b*) 26,460 foot-lb.
8. 10·75 sq. cm. ; 5,376 cm.-kilograms.

9. 33,210 lb.-deg.-Cent. **10.** 9,617 lb.-deg.-Cent.
11. $1{\cdot}69 \times 10^6$ calories. **12.** $41{\cdot}6 \times 10^6$ ergs.
13. 2·418 ; 1·407.

CHAPTER XXXIII. p. 440.

4. (*a*) 22·5 ; (*b*) 12·91 ; (*c*) 17·05 ; all in lb. wt./sq. in.
5. −93·5° C. **7.** 500 cubic inches. **8.** 100 cubic feet.
9. 55 cm. of mercury. **10.** 360 cubic inches.
14. 1·547 lb. wt./sq. inch ; 8. **15.** 0·00203 mm. **16.** 0·65 inch.

CHAPTER XXXIV. p. 456.

6. 3° C. nearly. **7.** 219,500 lb.-deg.-Cent.
10. 307×10^6 foot-lb. ; 155 H.P. **11.** Yes ; 0·39° C.
12. 2,566 calories. **15.** 81·13 calories.
18. 25·1 lb.-deg.-Cent. ; 30·3 cubic feet.
20. 35·45 cm. of mercury. **24.** (*a*) 0·083 ; (*b*) 0·1098.
25. 0·515. **26.** 0·0753.

CHAPTER XXXV. p. 470.

1. 54·85 cm. of mercury.
2. Pressure of the aqueous vapour is 11·8 lb. wt./sq. inch.
3. 226 c.c. **4.** 32·5. **6.** 22·5
9. 646·3 lb.-deg.-Cent. **10.** 17·9 lb. wt.
11. 45·5 c.c. **13.** 29·9 grams. **17.** 65·5 cm.
18. 68·1° C. **19.** 0·209.

CHAPTER XXXVI. p. 480.

2. 7·81 ; 19·2 minutes. **4.** 10·05° C. ; 0·597.
5. 0·603. **7.** 48·2 per cent. ; 6·8° C.
9. 9·4° C. **11.** 8·923 grams ; 0·5.

CHAPTER XXXVII. p. 492.

1. 45·15 calories ; 453·8 calories. **14.** 8·93 lb.-deg.-Cent.
15. 0·00776 grams/c.c. ; 7·76/872.

CHAPTER XXXVIII. p. 500.

1. 20·28 per cent. **6.** 22·07 per cent. ; 15,450,000 foot-lb

CHAPTER XXXIX. p. 518.

3. 73·3 per cent. **4.** 10·23 per cent.
7. 15·9 per cent. ; 21·7 per cent. **12.** 473 H.P.
13. 4·8 B.H.P. ; 5·71 I.H.P. ; 0·91 H.P.
16. 2·78° C. per cm. ; 0·268.

CHAPTER XL. p. 531.

3. 6·64. **4.** 5·39. **5.** 21·3 per cent.
11. 14·82 per cent. **12.** 169·5. **13.** 133 tons.

PART III. LIGHT

CHAPTER XLI. p. 542.

4. Annular. **5.** 2·6 inches. **6.** 3·33 cm. ; 10·67 cm. **8.** 5·6 cm.

CHAPTER XLII. p. 550.

4. Electricity costs twice as much as gas. **5.** 1920 candle-power.
6. 58·6 cm. from the 32 c.p. lamp.
7. At 1·414 feet on other side of screen.
8. 81 candle-power. **9.** 2 feet. **10.** 11·42 per cent.
11. 164 candle-power.

CHAPTER XLIII. p. 560.

4. 2 ft. 10 in. **6.** 60°. **8.** 55°.
12. $\Delta = n\pi - 2(i_1 + i_2 + i_3 + \text{etc.})$.
13. Two mirrors including an angle of 70°. **14.** 270°.

CHAPTER XLIV. p. 571.

2. 79·4 cm. **5.** +51·4 cm. ; 4·28 cm., inverted.
7. (*a*) +100 cm. ; 10 cm. diam., inverted ; (*b*) −60 cm. ; 10 cm. diam., upright.
8. 2·5 cm., inverted ; +22·5 cm.
9. Distance of rod, +45 cm. ; distance of image, +22·5 cm., inverted.
10. (*a*) +27 cm. ; (*b*) +13·5 cm.
11. +5·33 inches ; 0·667 inch. **13.** +11·11 cm.
14. Final image 6 cm. behind convex mirror, 6 cm. long, and inverted.
15. Image formed by one reflection 14 cm., and by two reflections 38 cm. behind plane mirror ; $r = 70$ cm.
16. 515·7 cm. **17.** −46·67 cm.

CHAPTER XLV. p. 585.

3. $r = 28° 8'$. 4. 4·5 feet nearly. 8. 1·467.
9. $r_1 = 13° 11'$; $r_2 = 36° 49'$; $i_2 = 64°$ 11. 38° 41'. 13. 1·667 cm.

CHAPTER XLVI. p. 600.

1. 2·8 inches, inverted, at −11·9 inches from lens.
2. +24 cm. from lens. 4. 23·7 cm., or 126·2 cm. from source.
5. On axis, on side of lens opposite to object, pointing away from lens; 3·33 cm. long; head −20 cm. from lens.
6. Distance of object from nearer lens must be greater than $2f$ or less than $3f/2$.
7. 4 cm.; 4 cm., inverted.
8. $r = 6·6$ inches; −3 or −1·5 inches. 9. −20 cm.
10. $v = -28$ cm. beyond second lens; image real, inverted, and 3·2 cm. high.
11. −60 cm. 12. Infinity; infinity.
13. Image inverted and virtual, twice size of object; +6 inches from the 6 inch lens.
14. At 5·68 cm. from surface of sphere, on radius through object.
15. Image virtual; object +5 cm. from lens.
16. Virtual image, erect; object at +2·667 inches from lens; real image, inverted; object at +5·33 inches from lens.
17. −10 cm.; −6·667 cm.

CHAPTER XLVII. p. 620.

2. +8·18 inches focal length. 5. 2.
6. −22·15 inches focal length. 8. +100 cm. focal length; 8·7 cm.
9. +1·067 cm.; 89. 10. −9·6 inches.
11. +3·75 cm.; 4. 13. 6. 14. Lenses 20 cm. apart; 4.
15. (i) 15; 32 inches; (ii) 17·5; 31·71 inches.
16. −11·6 inches focal length. 17. −33·33 cm. focal length.

CHAPTER XLVIII. p. 634.

7. 2·1°. 8. $D = (\mu - 1)\alpha$. 13. +160·7 cm.; −140·5 cm.
14. ∞; 111·4 cm.; 111·4 cm.; 64·37 cm.

CHAPTER L. p. 654.

6. 65·1°.

CHAPTER LI. p. 660.

2. 201600 miles/sec. 6. 8·333 revs./sec. 7. 4·63 revs./sec.

PART IV. SOUND

CHAPTER LII. p. 668.

4. 365·3.

CHAPTER LIII. p. 677.

6. 880. **8.** $599\frac{14}{15}$ before loading. $599\frac{9}{15}$ after loading.
9. $2{\cdot}75 \times 10^5$ ergs.

CHAPTER LIV. p. 687.

2. 440. **4.** 238·1; 142·8 cm. **7.** 18330; 36·7; 8·97.

CHAPTER LV. p. 704.

2. 4495 ft. and 3·75 feet per sec. **4.** (*a*) No; (*b*) yes.
5. 2240 ft. **6.** $n_1 : n_2 = (V+v) : (V-v)$. **8.** 4·98.
9. 517 ft. **10.** 50 miles per hr.

CHAPTER LVI. p. 717.

7. (1) 2; (2) 4. **10.** 1037.
11. 256, 1st diff.; 1280 1st sum.; 1536 and 1024 self-comb.
13. 260 **14.** 517.

CHAPTER LVIII. p. 741.

4. 23·26. **5.** 2 : 3. **6.** $1 : 2\sqrt{3}$.
7. Tension, 25 : 4; length, 2 : 5. **8.** 3·52 per sec. **9.** 1·308 gram.
10. 42·2. **13.** 1 : 9. **14.** 5 kilos. wt. **16.** 49·5.

CHAPTER LIX. p. 756.

1. 64·4 cm. and 329·8 metres per sec.
2. 2 : 1. **3.** 95·3° C. **5.** 19·4° C. open pipe.
6. 186·7, 373·3, 560, 746·7 open pipe; 93·3, 280, 466·7, 653·3 closed pipe.
7. 1·009. **10.** Open. **11.** 1105·5 ft. per sec.
13. 110·5. **16.** Wide pipe to be shortened by 6 cm. **18.** 5 : 8.

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